calmath2413Alvarez149 Stp

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① $f(x) = 7x^2 - 3x + 5$

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{(7(x+h)^2 - 3(x+h) + 5) - (7x^2 - 3x + 5)}{h} =$$

$$\frac{7(x+h)(x+h) - 3x - 3h + 5 - 7x^2 + 3x - 5}{h} =$$

$$\frac{7(x^2 + xh + xh + h^2) - 3x - 3h + 5 - 7x^2 + 3x - 5}{h} =$$

$$\frac{7(x^2 + 2xh + h^2) - 3x - 3h + 5 - 7x^2 + 3x - 5}{h} =$$

$$\frac{\cancel{7x^2} + 14xh + 7h^2 \cancel{- 3x} - 3h \cancel{+ 5} \cancel{- 7x^2} \cancel{+ 3x} \cancel{- 5}}{h} =$$

$$\frac{14xh + 7h^2 - 3h}{h} =$$

$$\frac{\cancel{h}(14x + 7h - 3)}{\cancel{h}} =$$

$$14x + 7h - 3 =$$

② $f(x) = \frac{7}{x}$

$$\frac{f(x+h) - f(x)}{h} =$$

$$\frac{\frac{7}{x+h} - \frac{7}{x}}{h} =$$

$$\frac{\left(\frac{7}{x+h} - \frac{7}{x}\right)\frac{x(x+h)}{1}}{h\left(\frac{x(x+h)}{1}\right)}$$ mult

$$\frac{\frac{7x\cancel{(x+h)}}{\cancel{(x+h)}} - \frac{7\cancel{x}(x+h)}{\cancel{x}}}{h(x)(x+h)} =$$

$$\frac{7x - 7(x+h)}{h\cdot(x)\cdot(x+h)} =$$

$$\frac{\cancel{7x} - \cancel{7x} - 7h}{h\cdot(x)(x+h)} =$$

$$\frac{-7\cancel{h}}{\cancel{h}(x)(x+h)} =$$

$$\frac{-7}{x(x+h)} =$$

③ $f(x) = 3 - 3x - x^2$

$$\frac{f(x) - f(a)}{x - a} =$$

$$\frac{(3 - 3x - x^2) - (3 - 3(a) - (a)^2)}{x - a} =$$

$$\frac{\cancel{3} - 3x - x^2 - \cancel{3} + 3a + a^2}{x - a} =$$

$$\frac{a^2 - x^2 - 3x + 3a}{x - a} =$$

$$\frac{(a + x)(a - x) - 3(x - a)}{x - a} =$$

$$\frac{(a + x)(-1)(-a + x) - 3(x - a)}{x - a}$$

$$\frac{(a + x)(-1)(x - a) - 3(x - a)}{(x - a)}$$

$$\frac{\cancel{(x - a)}\left((a + x)(-1) - 3\right)}{\cancel{(x - a)}} =$$

$$(a + x)(-1) - 3 =$$

$$-a - x - 3 =$$

④ $s(3) = 172$ and $s(5) = 222$

~~average velocity~~

$$\frac{s(5) - s(3)}{5 - 3} =$$

$$\frac{(222) - (172)}{5 - 3} =$$

$$\frac{222 - 172}{5 - 3} =$$

$$\frac{50}{2} =$$

$$25 =$$

(5) What is the slope of the secant line between the points $(a, f(a))$ and $(b, f(b))$ on the graph f?

$$m_{sec} = \frac{f(b) - f(a)}{b - a}$$

6. What is the slope of the line tangent to the graph of f at (a,f(a))?

Which of the following is the correct formula for the slope of the tangent line?

- A. $m_{tan} = \lim_{t \to a} \frac{f(b) + f(t)}{b - t}$
- B. $m_{tan} = \lim_{t \to a} \frac{f(t) - f(a)}{t - a}$
- C. $m_{tan} = \lim_{t \to a} \frac{f(t) + f(a)}{t - a}$
- D. $m_{tan} = \lim_{t \to a} \frac{f(b) - f(t)}{b - t}$

7. The position of an object moving along a line is given by the function $s(t) = -3t^2 + 9t$. Find the average velocity of the object over the following intervals.

(a) [1, 7]
(b) [1, 6]
(c) [1, 5]
(d) [1, 1 + h] where h > 0 is any real number.

(a) The average velocity of the object over the interval [1, 7] is -15.

(b) The average velocity of the object over the interval [1, 6] is -12.

(c) The average velocity of the object over the interval [1, 5] is -9.

(d) The average velocity of the object over the interval [1, 1 + h] is $-3h+3$.

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1 a $s(t) = -3t^2 + 9t \quad [1, 7]$

$$\frac{s(7)-s(1)}{(7)-(1)} =$$

$$\frac{(-3(7)^2+9(7)) - (-3(1)^2+9(1))}{(7)-(1)} =$$

$$\frac{(-3(7)(7)+9(7)) - (-3(1)(1)+9(1))}{(7)-(1)} =$$

$$\frac{(-3(49)+9(7)) - (-3(1)+9(1))}{(7)-(1)} =$$

$$\frac{(-147+63) - (-3+9)}{(7)-(1)} =$$

$$\frac{(-84) - (6)}{(7)-(1)} =$$

$$\frac{-84-6}{7-1} =$$

$$\frac{-90}{6} =$$

$$-15 =$$

76 $s(t) = -3t^2 + 9t$ $[1, 6]$

$$\frac{s'(6) - s'(1)}{(6)-(1)} =$$

$$\frac{(-3(6)^2 + 9(6)) - (-3(1)^2 + 9(1))}{(6)-(1)} =$$

$$\frac{(-3(6)(6) + 9(6)) - (-3(1)(1) + 9(1))}{(6)-(1)} =$$

$$\frac{(-3(36) + 9(6)) - (-3(1) + 9(1))}{(6)-(1)} =$$

$$\frac{(-108 + 54) - (-3+9)}{(6)-(1)} =$$

$$\frac{(-54) - (6)}{(6)-(1)} =$$

$$\frac{-54-6}{6-1} =$$

$$\frac{-60}{5} =$$

$-12 =$

7c $s(t) = -3t^2 + 9t$ $[1, 5]$

$$\frac{s(5) - s(1)}{(5) - (1)} =$$

$$\frac{(-3(5)^2 + 9(5)) - (-3(1)^2 + 9(1))}{(5) - (1)} =$$

$$\frac{(-3(5)(5) + 9(5)) - (-3(1)(1) + 9(1))}{(5) - (1)} =$$

$$\frac{(-3(25) + 9(5)) - (-3(1) + 9(1))}{(5) - (1)} =$$

$$\frac{(-75 + 45) - (-3 + 9)}{(5) - (1)} =$$

$$\frac{(-30) - (6)}{(5) - (1)} =$$

$$\frac{-30 - 6}{5 - 1} =$$

$$\frac{-36}{4} =$$

$$-9 =$$

1d $s(t) = -3t^2 + 9t$ $[1, 1+h]$

$$\frac{s(1+h) - s(1)}{(1+h) - (1)} =$$

$$\frac{(-3(1+h)^2 + 9(1+h)) - (-3(1)^2 + 9(1))}{(1+h) - (1)} =$$

$$\frac{(-3(1+h)(1+h) + 9(1+h)) - (-3(1)(1) + 9(1))}{(1+h) - (1)} =$$

$$\frac{(-3(1+1h+1h+h^2) + 9(1+h)) - (3(1) + 9(1))}{(1+h) - (1)} =$$

$$\frac{(-3(1+2h+h^2) + 9(1+h)) - (-3+9)}{(1+h) - (1)} =$$

$$\frac{(-3(h^2+2h+1) + 9(1+h)) - (-3+9)}{(1+h) - (1)} =$$

$$\frac{(-3h^2 - 6h - 3 + 9 + 9h) - (6)}{(1+h) - (1)}$$

$$\frac{-3h^2 - 6h - 3 + 9 + 9h - 6}{1 + h - 1} =$$

$$\frac{-3h^2 + 3h}{h} =$$

$$\frac{\cancel{h}(-3h+3)}{\cancel{h}} =$$

$$-3h + 3 =$$

8. For the position function $s(t) = -16t^2 + 100t$, complete the following table with the appropriate average velocities. Then make a conjecture about the value of the instantaneous velocity at $t = 1$.

Time Interval	[1, 2]	[1, 1.5]	[1, 1.1]	[1, 1.01]	[1, 1.001]
Average Velocity	___	___	___	___	___

Complete the following table.

Time Interval	[1, 2]	[1, 1.5]	[1, 1.1]	[1, 1.01]	[1, 1.001]
Average Velocity	52	60	66.4	67.84	67.984

(Type exact answers. Type integers or decimals.)

The value of the instantaneous velocity at $t = 1$ is 68.
(Round to the nearest integer as needed.)

$$\frac{f(2)-f(1)}{2-1} = \frac{(-16(2)^2+100(2)) - (-16(1)^2+100(1))}{2-1} = 52$$

$$\frac{f(1.5)-f(1)}{1.5-1} = \frac{(-16(1.5)^2+100(1.5)) - (-16(1)^2+100(1))}{1.5-1} = 60$$

$$\frac{f(1.1)-f(1)}{1.1-1} = \frac{(-16(1.1)^2+100(1.1)) - (-16(1)^2+100(1))}{1.1-1} = 66.4$$

$$\frac{f(1.01)-f(1)}{1.01-1} = \frac{(-16(1.01)^2+100(1.01)) - (-16(1)^2+100(1))}{1.01-1} = 67.84$$

$$\frac{f(1.001)-f(1)}{1.001-1} = \frac{(-16(1.001)^2+100(1.001)) - (-16(1)^2+100(1))}{1.001-1} = 67.984$$

the value of the instantaneous velocity at $x=1$ is 68

9. For the function $f(x) = 8x^3 - x$, make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at $x = 1$.

Complete the table.

(Round the final answer to three decimal places as needed. Round all intermediate values to four decimal places as needed.)

Interval	Slope of secant line
[1, 2]	55.000
[1, 1.5]	37.000
[1, 1.1]	25.480
[1, 1.01]	23.240
[1, 1.001]	23.000

An accurate conjecture for the slope of the tangent line at $x = 1$ is 23.
(Round to the nearest integer as needed.)

$$\frac{f(2)-f(1)}{2-1} = \frac{(8(2)^3-(1)) - (8(1)^3-(1))}{2-1} = 55,000$$

$$\frac{f(1.5)-f(1)}{1.5-1} = \frac{(8(1.5)^3-(1.5)) - (8(1)^3-(1))}{1.5-1} = 37,000$$

$$\frac{f(1.1)-f(1)}{1.1-1} = \frac{(8(1.1)^3-(1.1)) - (8(1)^3-(1))}{1.1-1} = 25,480$$

$$\frac{f(1.01)-f(1)}{1.01-1} = \frac{(8(1.01)^3-(1.01)) - (8(1)^3-(1))}{1.01-1} = 23.240$$

$$\frac{f(1.001)-f(1)}{1.001-1} = \frac{(8(1.001)^3-(1.001)) - (8(1)^3-(1))}{1.001-1} = 23,000$$

An accurate conjecture for the slope of the tangent line at $x=1$ is 23

10.

Let $f(x) = \frac{x^2 - 36}{x+6}$. **(a)** Calculate f(x) for each value of x in the following table. **(b)** Make a conjecture about the value of

$$\lim_{x \to -6} \frac{x^2 - 36}{x+6}.$$

(a) Calculate f(x) for each value of x in the following table.

x	−5.9	−5.99	−5.999	−5.9999
$f(x) = \frac{x^2 - 36}{x+6}$	−11.9	−11.99	−11.999	−11.9999
x	−6.1	−6.01	−6.001	−6.0001
$f(x) = \frac{x^2 - 36}{x+6}$	−12.1	−12.01	−12.001	−12.0001

(Type an integer or decimal rounded to four decimal places as needed.)

(b) Make a conjecture about the value of $\lim_{x \to -6} \frac{x^2 - 36}{x+6}$.

$$\lim_{x \to -6} \frac{x^2 - 36}{x+6} = \boxed{-12}$$ (Type an integer or a decimal.)

(10) (a) $f(x) = \frac{x^2-36}{x+6}$

$f(-5.9) = \frac{(-5.9)^2-36}{(-5.9)+6} = -11.9$

$f(-5.99) = \frac{(-5.99)^2-36}{(-5.99)+6} = -11.99$

$f(-5.999) = \frac{(-5.999)^2-36}{(-5.999)+6} = -11.999$

$f(-5.9999) = \frac{(-5.9999)^2-36}{(-5.9999)+6} = -11.9999$

$f(-6.1) = \frac{(-6.1)^2-36}{(-6.1)+6} = -12.1$

$f(-6.01) = \frac{(-6.01)^2-36}{(-6.01)+6} = -12.01$

$f(-6.001) = \frac{(-6.001)^2-36}{(-6.001)+6} = -12.001$

$f(-6.0001) = \frac{(-6.0001)^2-36}{(-6.0001)+6} = -12.0001$

(b)

$\lim_{x \to -6} \frac{x^2-36}{x+6} = -12$

10 extra

e

OR

$$\lim_{x \to -6} \frac{x^2-36}{x+6} =$$

formula

a^2-b^2

$(a+b)(a-b)$

$$\lim_{x \to -6} \frac{(x)^2-(6)^2}{x+6} =$$

$$\lim_{x \to -6} \frac{(x+6)(x-6)}{(x+6)} =$$

$$\lim_{x \to -6} \frac{\cancel{(x+6)}(x-6)}{\cancel{(x+6)}} =$$

$$\lim_{x \to -6} (x-6) =$$

$$-6-6=$$

$$-12$$

11. Let $g(t) = \frac{t-49}{\sqrt{t}-7}$.

a. Make two tables, one showing the values of g for t = 48.9, 48.99, and 48.999 and one showing values of g for t = 49.1, 49.01, and 49.001.

b. Make a conjecture about the value of $\lim_{t \to 49} \frac{t-49}{\sqrt{t}-7}$.

a. Make a table showing the values of g for t = 48.9, 48.99, and 48.999.

t	48.9	48.99	48.999
g(t)	13.9929	13.9993	13.9999

(Round to four decimal places.)

Make a table showing the values of g for t = 49.1, 49.01, and 49.001.

t	49.1	49.01	49.001
g(t)	14.0071	14.0007	14.0001

(Round to four decimal places.)

b. Make a conjecture about the value of $\lim_{t \to 49} \frac{t-49}{\sqrt{t}-7}$. Select the correct choice below and fill in any answer boxes in your choice.

○ A. $\lim_{t \to 49} \frac{t-49}{\sqrt{t}-7} =$ 14 (Simplify your answer.)

○ B. The limit does not exist.

11. $g(t) = \frac{t-49}{\sqrt{t}-7}$

$g(48.9) = \frac{48.9-49}{\sqrt{48.9}-7} = \frac{-.1}{-.007146505 2} = 13.9929$

$g(48.99) = \frac{48.99-49}{\sqrt{48.99}-7} = \frac{-.01}{-.0071432216 1} = 13.9993$

$g(48.999) = \frac{48.999-49}{\sqrt{48.999}-7} = \frac{-.001}{-.00714289359} = 13.9999$

$g(49.1) = \frac{49.1-49}{\sqrt{49.1}-7} = \frac{.1}{.007139216 5} = 14.0071$

$g(49.01) = \frac{49.01-49}{\sqrt{49.01}-7} = \frac{.01}{.0071424927 49} = 14.0007$

$g(49.001) = \frac{49.001-49}{\sqrt{49.001}-7} = \frac{.001}{.007142820 7} = 14.0001$

$\lim_{t\to 49} \frac{t-49}{\sqrt{t}-7} = 14$

Or

$\lim_{t\to 49} \frac{t-49}{\sqrt{t}-7} =$

$\lim_{t\to 49} \frac{(t-49)}{(\sqrt{t}-7)}\left(\frac{\sqrt{t}+7}{\sqrt{t}+7}\right) =$ mult

$\lim_{t\to 49} \frac{(t-49)(\sqrt{t}+7)}{(\sqrt{t})^2+\cancel{7\sqrt{t}}-\cancel{7\sqrt{t}}-49} =$

$\lim_{t\to 49} \frac{(t-49)(\sqrt{t}+7)}{\sqrt{t}^2-49} =$

$\lim_{t\to 49} \frac{\cancel{(t-49)}(\sqrt{t}+7)}{\cancel{(t-49)}} =$

$\lim_{t\to 49} \sqrt{t}+7 =$

$\sqrt{49}+7 =$

$7+7 =$

$14 =$

12. If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = M$, where L and M are finite real numbers, then what must be true about L and M in order for $\lim_{x \to a} f(x)$ to exist?

Choose the correct answer below.

- A. $L = M$
- B. $L \neq M$
- C. $L > M$
- D. $L < M$

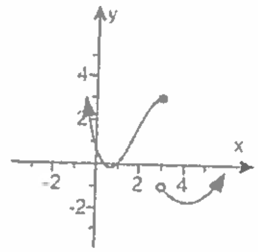
13. Use the graph to find the following limits and function value.

a. $\lim_{x \to 3^-} f(x)$

b. $\lim_{x \to 3^+} f(x)$

c. $\lim_{x \to 3} f(x)$

d. $f(3)$



a. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 3^-} f(x) =$ 3 (Type an integer.)

B. The limit does not exist.

b. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 3^+} f(x) =$ −1 (Type an integer.)

B. The limit does not exist.

c. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

A. $\lim_{x \to 3} f(x) =$ ______ (Type an integer.)

B. The limit does not exist.

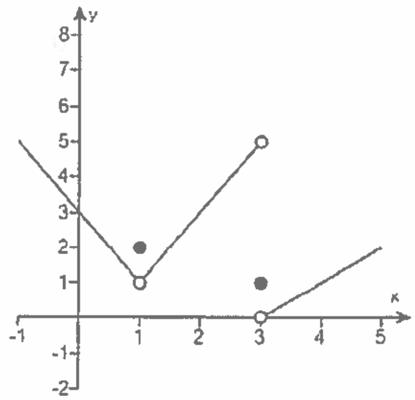
d. Find the function value. Select the correct choice below and fill in any answer boxes in your choice.

A. $f(3) =$ 3 (Type an integer.)

B. The answer is undefined.

14.

Use the graph of f to complete parts (a) through (l). If a limit does not exist, explain why.



(a) Find f(1). Select the correct choice below, and fill in the answer box if necessary.

- A. f(1) = 2
  (Type an integer or a fraction.)
- B. The value of f(1) is undefined.

(b) Find $\lim_{x \to 1^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 1^-} f(x) =$ 1
  (Type an integer or a fraction.)
- B. The limit does not exist because f(x) is not defined for all x < 1.

(c) Find $\lim_{x \to 1^+} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 1^+} f(x) =$ 1
  (Type an integer or a fraction.)
- B. The limit does not exist because f(x) is not defined for all x > 1.

(d) Find $\lim_{x \to 1} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 1} f(x) =$ 1
  (Type an integer or a fraction.)
- B. The limit does not exist because $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$.

(e) Find f(3). Select the correct choice below, and fill in the answer box if necessary.

- A. f(3) = 1
  (Type an integer or a fraction.)
- B. The value of f(3) is undefined.

(f) Find $\lim_{x \to 3^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- A. $\lim_{x \to 3^-} f(x) =$ 5
  (Type an integer or a fraction.)
- B. The limit does not exist because f(x) is not defined for all x < 3.

15. Explain why $\lim_{x \to -2} \frac{x^2+9x+14}{x+2} = \lim_{x \to -2} (x+7)$, and then evaluate $\lim_{x \to -2} \frac{x^2+9x+14}{x+2}$.

✓

Choose the correct answer below.

- A. Since each limit approaches -2, it follows that the limits are equal.
- B. The numerator of the expression $\frac{x^2+9x+14}{x+2}$ simplifies to $x+7$ for all x, so the limits are equal.
- C. Since $\frac{x^2+9x+14}{x+2} = x+7$ whenever $x \neq -2$, it follows that the two expressions evaluate to the same number as x approaches -2.
- D. The limits $\lim_{x \to -2} \frac{x^2+9x+14}{x+2}$ and $\lim_{x \to -2} (x+7)$ equal the same number when evaluated using direct substitution.

Now evaluate the limit.

$\lim_{x \to -2} \frac{x^2+9x+14}{x+2} =$ 5 (Simplify your answer.)

16. Determine the following limit.

$$\lim_{x \to -\infty} 9x^{12}$$

$\lim_{x \to -\infty} 9x^{12} =$ ∞

17. Determine the following limit.

$$\lim_{x \to \infty} x^{-27}$$

$\lim_{x \to \infty} x^{-27} =$ 0

(18) $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ if $f(x) \to 700{,}000$ and $g(x) \to \infty$ as $x \to \infty$

$$\frac{\lim_{x\to\infty}(f(x))}{\lim_{x\to\infty}(g(x))} = \frac{700{,}000}{\infty} =$$

$$0 =$$

(19) $\lim_{x\to\infty} \frac{4+8x+6x^2}{x^2} =$

formula

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to\infty} a = a$$

$$\lim_{x\to\infty}\left(\frac{4}{x^2} + \frac{8x}{x^2} + \frac{6x^2}{x^2}\right) =$$

$$\lim_{x\to\infty}\left(\frac{4}{x^2} + \frac{8}{x} + 6\right) =$$

$$0 + 0 + 6 =$$

$$6 =$$

(20) $\lim_{x \to \infty} \frac{\sin 17x}{5x} =$

Squeeze theorem

$g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, except possibly at $x = c$.

If $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$

then $\lim_{x \to c} f(x) = L$

$-1 \le \sin(17x) \le 1$

$\frac{-1}{5x} \le \frac{\sin(17x)}{5x} \le \frac{1}{+5x}$

$\lim_{x \to \infty} \left(\frac{-1}{5x}\right) \le \lim_{x \to \infty} \frac{\sin(17x)}{5x} \le \lim_{x \to \infty} \left(\frac{1}{+5x}\right)$

$0 \le \lim_{x \to \infty} \frac{\sin(17x)}{5x} \le 0$

$\lim_{x \to \infty} \frac{\sin(17x)}{5x} = 0$

21

$$\lim_{x \to \infty} (5x^7 - 6x^6 + 1) =$$

$$\infty =$$

(22) $\lim_{w \to \infty} \frac{12w^2+5w+1}{\sqrt{9w^4+w^3}}$

$$\lim_{w \to \infty} \left(\frac{12w^2+5w+1}{\sqrt{9w^4+w^3}}\right)\left(\frac{\sqrt{\frac{1}{w^4}}}{\sqrt{\frac{1}{w^4}}}\right)$$ mult

$$\lim_{w \to \infty} \left(\frac{(12w^2+5w+1)\sqrt{\frac{1}{w^4}}}{\sqrt{(9w^4+w^3)\frac{1}{w^4}}}\right) =$$

formula

$$\lim_{w \to \infty} \frac{1}{w} = 0$$

$$\lim_{w \to \infty} \frac{1}{w^n} = 0$$

$$\lim_{w \to \infty} (a) = a$$

$$\lim_{w \to \infty} \left(\frac{(12w^2+5w+1)\frac{1}{w^2}}{\sqrt{\frac{9w^4}{w^4}+\frac{w^3}{w^4}}}\right) =$$

$$\lim_{w \to \infty} \frac{\frac{12w^2}{w^2}+\frac{5w}{w^2}+\frac{1}{w^2}}{\sqrt{9+\frac{1}{w}}} =$$

$$\lim_{w \to \infty} \frac{12+\frac{5}{w}+\frac{1}{w^2}}{\sqrt{9+\frac{1}{w}}} =$$

$$\frac{12+0+0}{\sqrt{9+0}} =$$

$$\frac{12}{\sqrt{9}} =$$

$$\frac{12}{3} =$$

$$4 =$$

(23) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{25x^2+x}}{x}$

$\lim\limits_{x \to -\infty} \left(\dfrac{\sqrt{25x^2+x}}{x}\right)\left(\dfrac{\sqrt{\frac{1}{x^2}}}{\sqrt{\frac{1}{x^2}}}\right)$ mult

$\lim\limits_{x \to -\infty} \left(\dfrac{\sqrt{(25x^2+x)\left(\frac{1}{x^2}\right)}}{x\sqrt{\frac{1}{x^2}}}\right)$

formula

$\lim\limits_{x \to -\infty} \left(\dfrac{\sqrt{\frac{25x^2}{x^2}+\frac{x}{x^2}}}{x\left(\frac{1}{x}\right)}\right)$

$\lim\limits_{x \to \infty} \frac{1}{x} = 0$

$\lim\limits_{x \to \infty} \left(\dfrac{\sqrt{25+\frac{1}{x}}}{1}\right)$

$\lim\limits_{x \to \infty} \sqrt{a} = \sqrt{a}$

$= \dfrac{\sqrt{25+0}}{1} =$

$\lim\limits_{x \to \infty} a = a$

$= \dfrac{\sqrt{25}}{1} =$

$= \sqrt{25} =$

-5

24 $\lim_{x \to \infty}$

$$\lim_{x \to \infty} \frac{7x}{21x+6}$$

$$\lim_{x \to \infty} \frac{(7x)\frac{1}{x}}{(21x+6)\frac{1}{x}} =$$

$$\lim_{x \to \infty} \frac{\frac{7\cancel{x}}{\cancel{x}}}{\frac{21\cancel{x}}{\cancel{x}} + \frac{6}{x}} =$$

$$\lim_{x \to \infty} \frac{7}{21 + \frac{6}{x}} =$$

$$\frac{7}{21+0} =$$

$$\frac{7}{21} =$$

$$\frac{\cancel{7}(1)}{\cancel{7}(3)} =$$

$$\frac{1}{3}$$

formula

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} a = a$$

25

$$\lim_{x\to\infty} \frac{9x^2-8x+9}{3x^2+2} =$$

$$\lim_{x\to\infty} \left(\frac{9x^2-8x+9}{3x^2+2}\right)\frac{\frac{1}{x^2}}{\frac{1}{x^2}} \text{ mult } =$$

$$\lim_{x\to\infty} \frac{\frac{9x^2}{x^2}-\frac{8x}{x^2}+\frac{9}{x^2}}{\frac{3x^2}{x^2}+\frac{2}{x^2}} =$$

formula

$$\lim_{x\to\infty} \frac{1}{x^n}=0$$

$$\lim_{x\to\infty} \frac{1}{x}=0$$

$$\lim_{x\to\infty} (a)=a$$

$$\lim_{x\to\infty} \frac{9-\frac{8}{x}+\frac{9}{x^2}}{3+\frac{2}{x^2}} =$$

$$\frac{9-0+0}{3+0} =$$

$$\frac{9}{3} =$$

$$3 =$$

(26) $\lim_{x\to\infty} f(x)$

$$\lim_{x\to\infty} \frac{5x^3+8}{1-9x^3} =$$

$$\lim_{x\to\infty} \frac{(5x^3+8)\frac{1}{x^3}}{(1-9x^3)\frac{1}{x^3}} =$$

$$\lim_{x\to\infty} \frac{\frac{5\cancel{x^3}}{\cancel{x^3}} + \frac{8}{x^3}}{\frac{1}{x^3} - \frac{9\cancel{x^3}}{\cancel{x^3}}} =$$

$$\lim_{x\to\infty} \frac{5+\frac{8}{x^3}}{\frac{1}{x^3}-9} =$$

$$\frac{5+0}{0-9} =$$

$$\frac{5}{-9} = -$$

formula

$$\lim_{x\to\infty} \frac{1}{x^n} = 0$$

$$\lim_{x\to\infty} \frac{1}{\alpha} = 0$$

$$\lim_{x\to\infty} a = a$$

27. Complete the following sentences.

(a) A function is continuous from the left at a if ______.
(b) A function is continuous from the right at a if ______.

(a) A function is continuous from the left at a if (1) ______.

(b) A function is continuous from the right at a if (2) ______.

(1)
- ○ $\lim_{x\to a^+} f(x) = -f(a)$
- ○ $\lim_{x\to a^+} f(x) = f(a)$
- ○ $\lim_{x\to a^-} f(x) = f(a)$
- ○ $\lim_{x\to a^-} f(x) = -f(a)$

(2)
- ○ $\lim_{x\to a^+} f(x) = f(a)$
- ○ $\lim_{x\to a^+} f(x) = -f(a)$
- ○ $\lim_{x\to a^-} f(x) = f(a)$
- ○ $\lim_{x\to a^-} f(x) = -f(a)$

---

28. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \frac{2x^2 + 7x + 3}{x^2 + 5x}, \; a = -5$$

Select all that apply.

- ☐ A. The function is continuous at a = − 5.
- ☐ B. The function is not continuous at a = − 5 because f(− 5) is undefined.
- ☐ C. The function is not continuous at a = − 5 because $\lim_{x\to -5} f(x)$ does not exist.
- ☐ D. The function is not continuous at a = − 5 because $\lim_{x\to -5} f(x) \neq f(-5)$.

---

29. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \begin{cases} \dfrac{x^2 - 100}{x - 10} & \text{if } x \neq 10 \\ 7 & \text{if } x = 10 \end{cases} ; \; a = 10$$

Select all that apply.

- ☐ A. The function is continuous at a = 10.
- ☐ B. The function is not continuous at a = 10 because f(10) is undefined.
- ☐ C. The function is not continuous at a = 10 because $\lim_{x\to 10} f(x)$ does not exist.
- ☐ D. The function is not continuous at a = 10 because $\lim_{x\to 10} f(x) \neq f(10)$.

---

(30) determine the intervals on which the following function is continuous.

formula

$a^2-b^2=(a+b)(a-b)$

$$f(x)=\frac{x^2-3x+2}{x^2-4}$$

let $x^2-4=0$

$(x)^2-(2)^2=0$

$(x+2)(x-2)=0$

so $x+2=0$ or $x-2=0$

$x+2-2=0-2$ or $x-2+2=0+2$

$x\neq -2$ or $x\neq 2$

$(-\infty,-2)$, $(-2,2)$, $(2,\infty)$

continuous

31.

$$\lim_{x \to 5} \sqrt{x^2+11} =$$

Continuous everywhere

$$\sqrt{(5)^2+11} =$$

$$\sqrt{25+11} =$$

$$\sqrt{36} =$$

$$6 =$$

(32) Suppose x lies in the interval $(3,5)$ with $x \neq 4$. Find the smallest positive value for δ such that the inequality $0 < |x-4| < \delta$ is true for all possible value of x

$$|x-4| < \delta$$

$$-\delta < x-4 < \delta$$

$$-\delta + 4 < x \cancel{-4} \cancel{+4} < \delta + 4$$

$$-\delta + 4 < x < \delta + 4$$

$(3,5)$

$-\delta + 4 \geq 3$ OR $\delta + 4 \leq 5$

$-\delta + 4 - 4 \geq 3 - 4$ OR $\delta + \cancel{4} \cancel{-4} \leq 5 - 4$

$-\delta \geq -1$ Turn all around OR $\delta \leq 1$

$-1(-\delta) \leq -1(-1)$ OR $\delta \leq 1$

$\delta \leq 1$ OR $\delta \leq 1$

$$\delta = 1$$

(35) Find the value of the derivative of the function at the given point.

$f(x) = 4x^2 - 2x$, Point $(-1, 6)$

$f'(x) = 4(2x) - 2$

$f'(x) = 8x - 2$

$f'(-1) = 8(-1) - 2$

$f'(-1) = -8 - 2$

$f'(-1) = -10$

(36) use the definition $m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ to find the slope of the line tangent to the graph of f at P.

$f(x) = x^2 - 3$ Point $P(-2, 1)$

$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} =$

$\lim_{h \to 0} \frac{((a+h)^2 - 3) - ((a)^2 - 3)}{h} =$

$\lim_{h \to 0} \frac{(a+h)(a+h) - 3 - a^2 + 3}{h} =$

$\lim_{h \to 0} \frac{\cancel{a^2} + ah + ah + h^2 \cancel{- 3} \cancel{- a^2} \cancel{+ 3}}{h} =$

$\lim_{h \to 0} \frac{2ah + h^2}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(2a+h)}{\cancel{h}} =$

$\lim_{h \to 0} (2a+h) =$

$2a + 0 =$

$f'(a) = 2a =$ slope

$f'(-2) = 2(-2)$

$f'(-2) = -4 =$ slope $= m_{tan}$

determine the equation of the tangent line at P.

slope $= m = -4$ $(-2, 1)$ with $x_1 = -2$, $y_1 = 1$

$y - y_1 = m(x - x_1)$

$y - (1) = -4(x - (-2))$

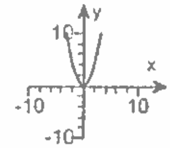
$y - 1 = -4(x + 2)$

$y - 1 = -4x - 8$

$y - 1 + 1 = -4x - 8 + 1$

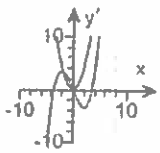
$y = -4x - 7$

37. Match the graph of the function on the right with the graph of its derivative.

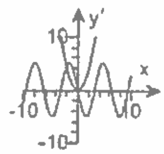


Choose the correct graph of the function (in blue) and its derivative (in red) below.

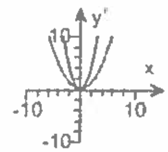
A.



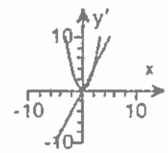
B.



C.



D.



example

Quadratic function

If $y = x^2$

$y' = 2x$

Line function

(38) A line perpendicular to another line or to a tangent line is called a normal line. Find an equation of the line perpendicular to the line that is tangent to the following curve at the given point P.

$y = 3x - 2$ Point $P(1, 1)$

$x_1\ y_1$ formula

slope $= m = 3$

perpendicular slope $= -\frac{1}{3}$

m is perpendicular to $-\frac{1}{m}$

$y - y_1 = m(x - x_1)$

$y - (1) = -\frac{1}{3}(x - (1))$

$y - 1 = -\frac{1}{3}(x - 1)$

$y - 1 = -\frac{1}{3}x + \frac{1}{3}$

$y - \cancel{1} + \cancel{1} = -\frac{1}{3}x + \frac{1}{3} + 1$

$y = -\frac{1}{3}x + \frac{1}{3} + \frac{3}{3}$

$y = -\frac{1}{3}x + \frac{1+3}{3}$

$y = -\frac{1}{3}x + \frac{4}{3}$

39

$$\frac{d}{dx}\left(\frac{x-4}{6x-5}\right)$$

formula

$$y = \frac{f(x)}{g(x)}$$

$$y' = \frac{f'(x)\,g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$\frac{(x-4)'(6x-5) - (x-4)(6x-5)'}{(6x-5)^2} =$$

$$\frac{(1)(6x-5) - (x-4)(6)}{(6x-5)^2} =$$

$$\frac{6x-5-(6x-24)}{(6x-5)^2} =$$

$$\frac{\cancel{6x}-5\ \cancel{-6x}+24}{(6x-5)^2} =$$

$$\frac{19}{(6x-5)^2}$$

(40) $g(x) = \dfrac{2x^2}{x+3}$ find $g'(1)$

formula

$y = \dfrac{f}{g}$

$y' = \dfrac{f'g - fg'}{(g)^2}$

$$g'(x) = \frac{(2x^2)'(x+3) - (2x^2)(x+3)'}{(x+3)^2}$$

$$g'(x) = \frac{(4x)(x+3) - (2x^2)(1)}{(x+3)^2}$$

$$g'(x) = \frac{4x^2 + 12x - 2x^2}{(x+3)^2}$$

$$g'(x) = \frac{2x^2 + 12x}{(x+3)^2}$$

$$g'(1) = \frac{2(1)^2 + 12(1)}{(1+3)^2}$$

$$g'(1) = \frac{2(1)(1) + 12(1)}{(4)^2}$$

$$g'(1) = \frac{2+12}{16}$$

$$g'(1) = \frac{14}{16}$$

$$g'(1) = \frac{\cancel{2}(7)}{\cancel{2}(8)}$$

$$g'(1) = \frac{7}{8}$$

41 $f(x) = (x-3)(3x+4)$

formula
$y = f \cdot g$
$y' = f'g + fg'$

$f'(x) = (x-3)'(3x+4) + (x-3)(3x+4)'$
$f'(x) = (1)(3x+4) + (x-3)(3)$
$f'(x) = 3x+4+3x-9$
$f'(x) = 6x-5$

OR

$f(x) = (x-3)(3x+4)$
$f(x) = 3x^2+4x-9x-12$
$f(x) = 3x^2-5x-12$
$f'(x) = 6x-5-0$
$f'(x) = 6x-5$

42 $h(w) = \frac{5w^6 - w}{w}$

$h'(w) = \frac{(5w^6 - w)'(w) - (5w^6 - w)(w)'}{(w)^2}$

$h'(w) = \frac{(30w^5 - 1)(w) - (5w^6 - w)(1)}{(w)^2}$

$h'(w) = \frac{30w^6 - \cancel{w} - 5w^6 + \cancel{w}}{(w)^2}$

$h'(w) = \frac{25w^6}{w^2}$

$h'(w) = 25w^{6-2}$

$h'(w) = 25w^4$

formula

$y = \frac{f}{g}$

$y' = \frac{f'g - fg'}{g^2}$

$y = x^N$

$y' = Nx^{N-1}$

OR

Simplify

$h(w) = \frac{5w^6 - w}{w}$

$h(w) = \frac{5w^6}{w^1} - \frac{w^1}{w^1}$

$h(w) = 5w^{6-1} - 1$

$h(w) = 5w^5 - 1$

$h'(w) = 25w^4 - 0$

$h'(w) = 25w^4$

43

$$\lim_{x\to 0} \frac{\sin(9x)}{\sin(5x)} =$$

mult by $\frac{\frac{1}{x}}{\frac{1}{x}}$

$$\lim_{x\to 0} \frac{\frac{\sin(9x)}{x}}{\frac{\sin(5x)}{x}} =$$

formula

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{9\sin(9x)}{\frac{9x}{\frac{5\sin(5x)}{5x}}} =$$ mult

$$\lim_{x\to 0} \frac{\sin(mx)}{(mx)} = 1$$

$$\frac{9}{5} \lim_{x\to 0} \frac{\frac{\sin(9x)}{9\cancel{x}}}{\frac{\sin(5x)}{5\cancel{x}}} =$$

$$\frac{9}{5} \frac{\lim_{x\to 0} \frac{\sin(9x)}{9x}}{\lim_{x\to 0} \frac{\sin(5x)}{5x}} =$$

$$\frac{9}{5} \cdot \frac{1}{1} =$$

$$\frac{9}{5}$$

44) $\lim_{x \to 0} \dfrac{\sin(10x)}{\tan(x)}$

$\lim_{x \to 0} \dfrac{\sin(10x)}{\frac{\sin(x)}{\cos(x)}} =$

$\lim_{x \to 0} \dfrac{\frac{\sin(10x)}{1}}{\frac{\sin(x)}{\cos x}} \cdot \left(\dfrac{\frac{1}{x}}{\frac{1}{x}} \right) =$ Mult

$\lim_{x \to 0} \dfrac{\frac{\sin(10x)}{x}}{\frac{\sin x}{\cos x} \frac{1}{x}} =$

$\lim_{x \to 0} \dfrac{\frac{\sin(10x)}{x}}{\frac{1}{\cos x} \cdot \frac{\sin x}{x}} =$

$\lim_{x \to 0} \dfrac{\frac{10\sin(10x)}{10x}}{\frac{1}{\cos x} \cdot \frac{\sin x}{x}} =$

$\dfrac{\lim_{x \to 0} \frac{10\sin(10x)}{10x}}{\lim_{x \to 0} \frac{1}{\cos x} \cdot \lim_{x \to 0} \frac{\sin x}{x}} =$

$\dfrac{10 \lim_{x \to 0} \frac{\sin(10x)}{10x}}{\lim_{x \to 0} \frac{1}{\cos x} \cdot \lim_{x \to 0} \frac{\sin x}{x}} =$

Formula

$\lim_{x \to 0} \dfrac{\sin(x)}{(x)} = 1$

$\lim_{x \to 0} \dfrac{\sin(mx)}{(mx)} = 1$

$\tan x = \dfrac{\sin(x)}{\cos(x)}$

$\dfrac{10(1)}{\frac{1}{\cos(0)} \cdot (1)} =$

$\dfrac{10}{\frac{1}{1}(1)} =$

$\dfrac{10}{1} =$

$10 =$

45 $\lim_{\theta \to 0} \frac{\cos^2\theta - 1}{\theta}$

formula

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$

$\lim_{\theta \to 0} \frac{\cos^2\theta - (1)^2}{\theta}$

$\lim_{\theta \to 0} \frac{(\cos\theta + 1)(\cos\theta - 1)}{\theta} =$

$\lim_{\theta \to 0} (\cos\theta + 1) \cdot \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} =$

formula

$a^2 - b^2 =$

$(a+b)(a-b) =$

$(\cos(0) + 1) \cdot 0 =$

$(1+1) \cdot 0$

$2 \cdot 0 =$

$0 =$

46

$y = 8\sin x + 7\cos x$

$y' = 8\cos(x)(x)' - 7\sin(x)(x)'$

$y' = 8\cos(x)(1) - 7\sin(x)(1)$

$y' = 8\cos(x) - 7\sin(x)$

OR

$$\frac{dy}{dx} = 8\cos(x) - 7\sin(x)$$

formula

$y = \sin(f(x))$

$y' = \cos(f(x))\, f'(x)$

$y = \cos(f(x))$

$y' = -\sin(f(x))\, f'(x)$

(47) $y = e^{-x} \sin x$

Formula

$y = e^{f(x)}$

$y' = e^{f(x)} \cdot f'(x)$

$y' = (e^{-x})'(\sin x) + (e^{-x})(\sin x)'$

$y' = (e^{-x}(-x)')(\sin x) + (e^{-x})(\cos(x)\,(x)')$

$y = \sin(f(x))$

$y' = \cos(f(x))\, f'(x)$

$y' = (e^{-x}(-1))(\sin x) + e^{-x}(\cos(x)\,(1))$

$y = f \cdot g$ Product

$y' = -e^{-x}\sin(x) + e^{-x}\cos(x)$

$y' = f'g + fg'$

$y' = e^{-x}\cos(x) - e^{-x}\sin(x)$ rewrite

$y' = e^{-x}(\cos(x) - \sin(x))$

Or

$$\frac{dy}{dx} = e^{-x}(\cos(x) - \sin(x))$$

(48)

$y = 4\tan(x) + \cot(x)$

$y' = 4\sec^2(x)(x)' - \csc^2(x)(x)'$

$y' = 4\sec^2(x)(1) - \csc^2(x)(1)$

$y' = 4\sec^2(x) - \csc^2(x)$

or

$\frac{dy}{dx} = 4\sec^2 x - \csc^2(x)$

formula

$y = \tan(f(x))$

$y' = \sec^2(f(x)) \cdot f'(x)$

$y = \cot(f(x))$

$y' = -\csc^2(f(x)) \cdot f'(x)$

(49.) Find the equation of the line tangent to the following curve at the given point.

$y = 12x^2 + 7\sin(x)$ point $(0, 0)$
$\qquad\qquad\qquad\qquad\qquad\qquad x_1\ y_1$

$y' = 24x + 7\cos(x)$

$y' = 24(0) + 7\cos(0)$

$y' = 0 + 7(1)$

$y' = 0 + 7$

$y' = 7 \Rightarrow \text{Slope} = m$

$y - y_1 = m(x - x_1)$

$y - (0) = 7(x - (0))$

$y - 0 = 7(x - 0)$

$y = 7(x)$

$y = 7x$

50

$y = e^{kx}$

$y' = e^{kx}(kx)'$

$y' = e^{kx}(k)$

$y' = e^{kx}k$ for any real number k

or

$y' = ke^{kx}$

(57) $h(x) = f(g(x))$ $\quad$ $p(x) = g(f(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$ $\quad$ $p'(x) = g'(f(x)) \cdot f'(x)$

x	1	2	3	4
$f(x)$	2	1	4	3
$f'(x)$	-6	-3	-4	-8
$g(x)$	1	2	3.	4
$g'(x)$	$3/7$	$6/7$	$4/7$	$\frac{1}{7}$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(4) = f'(g(4)) \cdot g'(4)$

$h'(4) = f'(4) \cdot \left(\frac{1}{7}\right)$

$h'(4) = (-8) \cdot \frac{1}{7}$

$h'(4) = \frac{-8}{7}$

$p'(x) = g'(f(x)) \cdot f'(x)$

$p'(3) = g'(f(3)) \cdot f'(3)$

$p'(3) = g'(4) \cdot (-4)$

$p'(3) = \left(\frac{1}{7}\right)(-4) =$ $\quad$ $p'(3) = \frac{-4}{7}$

52

$$y = (3x+2)^7$$

$$y' = 7(3x+2)^{7-1}(3x+2)'$$

$$y' = 7(3x+2)^6(3+0)$$

$$y' = 7(3x+2)^6(3)$$

$$y' = 21(3x+2)^6$$

Chain formula

$$y = (f(x))^N$$

$$y' = N(f(x))^{N-1} \cdot f'(x)$$

formula

$$y = [f(x)]^N$$

$$y' = N[f(x)]^{N-1} \cdot f'(x)$$

$$y = ax$$

$$y' = a$$

$$y = a$$

$$y' = 0$$

53

$y = 7(8x^3+5)^{-4}$

$y' = 7(-4)(8x^3+5)^{-4-1}(8x^3+5)'$

$y' = -28(8x^3+5)^{-5}(24x^2+0)$

$y' = -28(8x^3+5)^{-5}(24x^2)$

$y' = -672x^2(8x^3+5)^{-5}$

$y' = \dfrac{-672x^2}{(8x^3+5)^5}$

formula Chain

$y = [f(x)]^N$

$y' = N[f(x)]^{N-1} \cdot f'(x)$

$y = 9$

$y' = 0$

54 $y = \cos(19t+18)$

$y' = -\sin(19t+18) \cdot (19t+18)'$

$y' = -\sin(19t+18) \cdot (19+0)$

$y' = -\sin(19t+18)(19)$

$y' = -19\sin(19t+18)$

formula

$y = \cos(f(x))$

$y' = -\sin(f(x)) \cdot f'(x)$

$y = a$

$y' = 0$

$y = ax$

$y' = a$

55

$y = \tan(e^x)$

$y' = \sec^2(e^x) \cdot (e^x)'$

$y' = \sec^2(e^x) \cdot (e^x)(x)'$

$y' = \sec^2(e^x) \cdot (e^x)(1)$

$y' = \sec^2(e^x) \cdot (e^x)$

$y' = e^x \sec^2(e^x)$

formula

$y = \tan(f(x))$

$y' = \sec^2(f(x)) \cdot f'(x)$

$y = e^{f(x)}$

$y' = e^{f(x)} \cdot f'(x)$

(56) $y = (\csc(x) + \cot(x))^{19}$

$y' = 19(\csc(x) + \cot(x))^{19-1} (\csc(x) + \cot(x))'$

$y' = 19(\csc(x) + \cot(x))^{18} \cdot (-\csc(x)\cdot\cot(x) - \csc^2 x)$

$y' = 19(\csc(x) + \cot(x))^{18} \cdot (-\csc(x))(\cot(x) + \csc(x))$

$y' = -19\csc(x)(\csc(x) + \cot(x))^{18} \cdot (\cot(x) + \csc(x))$

$y' = -19\csc(x)(\csc(x) + \cot(x))^{18} \cdot (\csc(x) + \cot(x))^{1}$

$y' = -19\csc(x)(\csc(x) + \cot(x))^{18+1}$

$y' = -19\csc(x)(\csc(x) + \cot(x))^{19}$

formulas

$y = (f(x))^N$

$y' = N(f(x))^{N-1} \cdot f'(x)$

$y = \csc(f(x))$

$y' = -\csc(f(x)) \cdot \cot(f(x)) \cdot f'(x)$

$y = \cot(f(x))$

$y' = -\csc^2(f(x)) \cdot f'(x)$

57

$y = \cos(6\sin(x))$

$y' = -\sin(6\sin(x)) \cdot (6\sin(x))'$

$y' = -\sin(6\sin(x)) \cdot (6\cos(x))$

$y' = -6\sin(6\sin(x)) \cdot \cos(x)$

formula

$y = \cos(f(x))$

$y' = -\sin(f(x)) \cdot f'(x)$

$y = \sin(f(x))$

$y' = \cos(f(x)) \cdot f'(x)$

58.

For some equations, such as $x^2 + y^2 = 1$ or $x - y^2 = 1$, it is possible to solve for y and then calculate $\frac{dy}{dx}$. Even in these cases, explain why implicit differentiation is usually a more efficient method for calculating the derivative.

Choose the correct answer below.

- A. Because it produces $\frac{dy}{dx}$ in terms of y only.
- B. Because implicit differentiation gives two or more derivatives.
- C. Because it produces $\frac{dy}{dx}$ in terms of x only.
- D. Because implicit differentiation gives a single unified derivative.

59 Calculate $\frac{dy}{dx}$ using implicit differentiation

$$x = y^2$$

$$1 = 2y\,\frac{dy}{dx}$$

$$\frac{1}{2y} = \frac{\cancel{2y}\,\frac{dy}{dx}}{\cancel{2y}}$$

$$\frac{1}{2y} = \frac{dy}{dx}$$

formula

$$\frac{dy}{dx}\left(y^n\right) =$$

$$= ny^{n-1}\cdot y'$$

or

$$ny^{n-1}\cdot\frac{dy}{dx}$$

(60) Calculate $\frac{dy}{dx}$ using implicit differentiation

$$\sin(y) + y = x$$

$$\cos(y) \cdot \frac{dy}{dx} + 0 = 1$$

$$\cos(y) \cdot \frac{dy}{dx} = 1$$

$$\frac{\cancel{\cos(y)}\,\frac{dy}{dx}}{\cancel{\cos(y)}} = \frac{1}{\cos(y)}$$

formula
$\frac{dy}{dx}\sin(y) =$
$\cos(y)\, y' =$
OR
$\cos(y)\,\frac{dy}{dx} =$

$$\frac{dy}{dx} = \frac{1}{\cos(y)}$$

OR

$$\frac{dy}{dx} = \sec(y)$$

formula

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{1}{\cos(x)} = \sec(x)$$

(61) $x = y^{13}$

$1 = 13y^{13-1} \cdot \frac{dy}{dx}$

$1 = 13y^{12}\frac{dy}{dx}$

$\frac{1}{13y^{12}} = \frac{\cancel{13y^{12}}\frac{dy}{dx}}{\cancel{13y^{12}}}$

$\frac{1}{13y^{12}} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{13y^{12}}$

$\frac{dy}{dx} = \frac{1}{13}y^{-12}$

$\frac{d^2y}{dx^2} = \frac{1}{13}(-12)y^{-12-1}\frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \frac{-12}{13}y^{-13}\frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \frac{-12}{13}y^{-13}\left(\frac{1}{13y^{12}}\right)$ subst

formula

$\frac{d}{dx}(y^n) = ny^{n-1} \cdot y' =$

or

$ny^{n-1} \cdot \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \frac{-12}{13y^{13}} \cdot \frac{1}{13y^{12}}$

$\frac{d^2y}{dx^2} = \frac{-12}{169y^{13+12}}$

$\frac{d^2y}{dx^2} = \frac{-12}{169y^{25}}$

62 (a) use implicit differentiation to find $\frac{dy}{dx}$
(b) find the slope of the curve at the given point.

$x^3+y^3=26$ point $(3,-1)$ x_1, y_1

formula
$\frac{dy}{dx}(y^N) =$
$Ny^{N-1} \cdot y'$
OR
$Ny^{N-1} \cdot \frac{dy}{dx}$

$3x^2+3y^2y'=0$

$\cancel{3x^2}+3y^2y'\cancel{-3x^2}=0-3x^2$

$3y^2y'=-3x^2$

$\frac{\cancel{3y^2}y'}{\cancel{3y^2}}=\frac{-3x^2}{3y^2}$

$y'=\frac{-x^2}{y^2}$

OR

$\frac{dy}{dx}=\frac{-x^2}{y^2}$

$\frac{dy}{dx}=\frac{-x^2}{y^2}$ $(3,-1)$ x, y

eval $\frac{dy}{dx}(3,-1)=\frac{-(3)^2}{(-1)^2}$

$\frac{dy}{dx}(3,-1)=\frac{-(3)(3)}{(-1)(-1)}$

$\frac{dy}{dx}=\frac{-9}{1}$

Slope of curve
$m=-9$

(63) Use implicit differentiation to find $\frac{dy}{dx}$.

$\sin(y) + \sin(x) = 5y$

$\cos(y) \cdot y' + \cos(x)(1) = 5y'$

$y'\cos(y) + \cos(x) = 5y'$

$\cancel{y'\cos(y)} + \cos(x) - \cancel{y'\cos(y)} = 5y' - y'\cos(y)$

$\cos(x) = 5y' - y'\cos(y)$

$\cos(x) = y'(5 - \cos(y))$

$$\frac{\cos(x)}{5-\cos(y)} = \frac{y'\cancel{(5-\cos(y))}}{\cancel{(5-\cos(y))}}$$

$$y' = \frac{\cos(x)}{5-\cos(y)}$$

formula

$\frac{dy}{dx}(\sin(y)) = \cos(y)\,y'$

$\frac{dy}{dx}(\sin(x)) = \cos(x)(1) =$

formula

$y = f \cdot g$

$y' = f'g + fg'$

64

$4\sin(xy) = 5x + 9y$

$4\cos(xy)\,(xy)' = 5 + 9y'$

$4\cos(xy)\,(1(y) + x(y')) = 5 + 9y'$

$4\cos(xy)\,(y + xy') = 5 + 9y'$

$4\cos(xy)\,y + 4\cos(xy)\,xy' = 5 + 9y'$

$\cancel{4\cos(xy)\,y} + 4\cos(xy)\,xy' \cancel{- 4\cos(xy)\,y} = 5 + 9y' - 4\cos(xy)\,y$

$4\cos(xy)\,xy' = 5 + 9y' - 4\cos(xy)\,y$

$4\cos(xy)\,xy' - 9y' = 5 + \cancel{9y'} - 4\cos(xy)\,y \cancel{- 9y'}$

$4\cos(xy)\,xy' - 9y' = 5 - 4\cos(xy)\,y$

$y'(4\cos(xy)\,x - 9) = 5 - 4\cos(xy)\,y$

$y'(4\cos(xy)\,x - 9) = 5 - 4y\cos(xy)$

$$\frac{y'\cancel{(4\cos(xy)x - 9)}}{\cancel{(4\cos(xy)x - 9)}} = \frac{5 - 4y\cos(xy)}{(4\cos(xy)\,x - 9)}$$

$$y' = \frac{5 - 4y\cos(xy)}{4x\cos(xy) - 9}$$

65

$$e^{xy} = 5y$$

$$e^{xy}(xy)' = 5y'$$

$$e^{xy}(1(y) + x(y')) = 5y'$$

$$e^{xy}(y + xy') = 5y'$$

$$ye^{xy} + xy'e^{xy} = 5y'$$

$$ye^{xy} + \cancel{xy'e^{xy} - xy'e^{xy}} = 5y' - xy'e^{xy}$$

$$ye^{xy} = 5y' - xy'e^{xy}$$

$$ye^{xy} = y'(5 - xe^{xy})$$

$$\frac{ye^{xy}}{(5 - xe^{xy})} = \cancel{\frac{y'(5 - xe^{xy})}{(5 - xe^{xy})}}$$

$$\frac{ye^{xy}}{5 - xe^{xy}} = y'$$

$$\frac{dy}{dx} = \frac{ye^{xy}}{5 - xe^{xy}}$$

formula

$$y = f \cdot g$$

$$y' = f'g + fg'$$

$$y = e^{f(x)}$$

$$y' = e^{f(x)} \cdot f'(x)$$

66

$8x^4 + 3y^4 = 11xy$

$32x^3 + 12y^3y' = (11(y) + 11x(y'))$

$32x^3 + 12y^3y' = 11y + 11xy'$

$32x^3 + \cancel{12y^3y'} - \cancel{12y^3y'} = 11y + 11xy' - 12y^3y'$

$32x^3 = 11y + 11xy' - 12y^3y'$

$32x^3 - 11y = \cancel{11y} + 11xy' - 12y^3y' - \cancel{11y}$

$32x^3 - 11y = 11xy' - 12y^3y'$

$32x^3 - 11y = y'(11x - 12y^3)$

$\dfrac{32x^3 - 11y}{(11x - 12y^3)} = \dfrac{y'\cancel{(11x - 12y^3)}}{\cancel{(11x - 12y^3)}}$

$\dfrac{32x^3 - 11y}{11x - 12y^3} = y'$

formula

$\frac{dy}{dx}(y^n) =$

$ny^{n-1} \cdot y'$ or

$ny^{n-1} \cdot \frac{dy}{dx} =$

(67.) $6x^5+7y^5=13xy$

$30x^4+35y^4y'=(13(y)+13x(y'))$

$30x^4+35y^4y'=13y+13xy'$

$30x^4+\cancel{35y^4y'}\cancel{-35y^4y'}=13y+13xy'-35y^4y'$

$30x^4=13y+13xy'-35y^4y'$

$30x^4-13y=\cancel{13y}+13xy'-35y^4y'\cancel{-13y}$

$30x^4-13y=13xy'-35y^4y'$

$30x^4-13y=y'(13x-35y^4)$

$$\frac{30x^4-13y}{(13x-35y^4)}=\frac{y'\cancel{(13x-35y^4)}}{\cancel{(13x-35y^4)}}$$

$$\frac{30x^4-13y}{13x-35y^4}=y'$$

formula

$\frac{dy}{dx}(y^N)=$

$Ny^{N-1}\cdot y'=$

or

$Ny^{N-1}\cdot\frac{dy}{dx}=$

$y=f\cdot g$

$y'=f'g+fg'$

68

$f(x) = \log_b(x)$

$f'(x) = \frac{(x)'}{x} \cdot \frac{1}{\ln(b)}$

$f'(x) = \frac{1}{x} \cdot \frac{1}{\ln(b)}$

$f'(x) = \frac{1}{x\ln(b)}$

formula

$y = \log_b(m(x))$

$y' = \frac{m'(x)}{\ln(b)\, m(x)}$

(69)

$y = \ln\sqrt{x^2+12}$

$y = \ln(x^2+12)^{1/2}$

$y = \frac{1}{2}\ln(x^2+12)$

$y' = \frac{1}{2}\,\frac{(x^2+12)'}{(x^2+12)}$

$y' = \frac{1}{2}\left(\frac{(2x+0)}{x^2+12}\right)$

$y' = \frac{1}{2}\left(\frac{2x}{x^2+12}\right)$

$y' = \frac{\cancel{2}x}{\cancel{2}(x^2+12)}$

$y' = \frac{x}{x^2+12}$

~~formula~~

$y = \ln(f(x))$

$y' = \frac{f'(x)}{f(x)}$

Log ~~laws~~

$\ln(f(x))^n =$

$n\ln f(x) =$

70

$f(x) = g(x)^{h(x)}$

$f(x) = e^{\ln\left(g(x)\right)^{h(x)}}$

$f(x) = e^{h(x)\ln(g(x))}$

formulas

$e^{\ln(x)} = x$

or

$e^{\ln(f(x))} = f(x)$

71

$y = \ln(5x^2+9)$

$y' = \frac{(5x^2+9)'}{(5x^2+9)}$

$y' = \frac{10x+0}{5x^2+9}$

$y' = \frac{10x}{5x^2+9}$

formula

$y = \ln(f(x))$

$y' = \frac{f'(x)}{f(x)}$

72

$y = x^{5\pi}$

$y' = 5\pi x^{5\pi - 1}$

formula

$y = x^n$

$y' = n x^{n-1}$

73

$y = 5^x$

$y' = 5^x \ln(5)$

Formula

$y = a^x$

$y' = a^x \cdot \ln(a)$

(74) $y = 3\log_5(x^3 - 5)$

Formula

$y = \log_b(f(x))$

$y' = \frac{f'(x)}{f(x)} \cdot \frac{1}{\ln(b)}$

$y' = 3\,\frac{(x^3-5)'}{(x^3-5)} \cdot \frac{1}{\ln(5)}$

$y' = 3\,\frac{(3x^2-0)}{(x^3-5)} \cdot \frac{1}{\ln(5)}$

$y' = 3\left(\frac{3x^2}{x^3-5}\right) \cdot \frac{1}{\ln(5)}$

$y' = \frac{9x^2}{x^3-5} \cdot \frac{1}{\ln(5)}$

$y' = \frac{9x^2}{(x^3-5)\ln(5)}$

75.

$y = \log_{13}(x)$

$y' = \frac{(x)'}{x} \cdot \frac{1}{\ln(13)}$

$y' = \frac{1}{x} \cdot \frac{1}{\ln(13)}$

$y' = \frac{1}{x\ln(13)}$

formula

$y = \log_b(f(x))$

$y' = \frac{f'(x)}{f(x)} \cdot \frac{1}{\ln(b)}$

(76) $y = \dfrac{(x+4)^{12}}{(2x-4)^{11}}$

use Logs

formula
$\ln \dfrac{f(x)}{g(x)} =$
$\ln(f(x)) - \ln(g(x))$

$\ln(f(x))^N =$
$N \ln(f(x)) =$

$y = \ln(f(x))$
$y' = \dfrac{f'(x)}{f(x)}$

$\ln(y) = \ln \dfrac{(x+4)^{12}}{(2x-4)^{11}}$

$\ln(y) = \ln(x+4)^{12} - \ln(2x-4)^{11}$

$\ln(y) = 12\ln(x+4) - 11\ln(2x-4)$

$\dfrac{y'}{y} = 12\dfrac{(x+4)'}{x+4} - 11\dfrac{(2x-4)'}{(2x-4)}$

$\dfrac{y'}{y} = 12\left(\dfrac{1}{x+4}\right) - 11\left(\dfrac{2}{2x-4}\right)$

$\dfrac{y'}{y} = \dfrac{12}{x+4} - \dfrac{22}{2x-4}$

$\dfrac{y'}{y} = \dfrac{12}{x+4} - \dfrac{22}{2(x-2)}$

$\dfrac{y'}{y} = \dfrac{12}{x+4} - \dfrac{11}{x-2}$

$\cancel{y}\left(\dfrac{y'}{\cancel{y}}\right) = y\left(\dfrac{12}{x+4} - \dfrac{11}{x-2}\right)$

$y' = \dfrac{(x+4)^{12}}{(2x-4)^{11}}\left[\dfrac{12}{x+4} - \dfrac{11}{x-2}\right]$

77

Formula

$y = \sin^{-1}(x)$

$y' = \dfrac{(x)'}{\sqrt{1-x^2}}$

$y' = \dfrac{1}{\sqrt{1-x^2}}$

$y = \sin^{-1}(f(x))$

$y' = \dfrac{f'(x)}{\sqrt{1-(f(x))^2}}$

$y = \tan^{-1}(x)$

$y' = \dfrac{(x)'}{1+x^2}$

$y' = \dfrac{1}{1+x^2}$

$y = \tan^{-1}(f(x))$

$y' = \dfrac{f'(x)}{1+(f(x))^2}$

$y = \sec^{-1}(x)$

$y' = \dfrac{(x)'}{|x|\sqrt{x^2-1}}$

$y' = \dfrac{1}{|x|\sqrt{x^2-1}}$

$y = \sec^{-1}(f(x))$

$y' = \dfrac{f'(x)}{|f(x)|\sqrt{(f(x))^2-1}}$

78

$$f(x) = \sin^{-1}(3x^5)$$

$$f'(x) = \frac{(3x^5)'}{\sqrt{1-(3x^5)^2}}$$

$$f'(x) = \frac{15x^4}{\sqrt{1-(3^1x^5)^2}}$$

$$f'(x) = \frac{15x^4}{\sqrt{1-(3^2x^{10})}}$$

$$f'(x) = \frac{15x^4}{\sqrt{1-(9x^{10})}}$$

formula

$$y = \sin^{-1}(m(x))$$

$$y' = \frac{m'(x)}{\sqrt{1-(m(x))^2}}$$

79. $y = 4\tan^{-1}(2x)$

$$y' = 4\,\frac{(2x)'}{1+(2x)^2}$$

formula

$$y = \tan^{-1}(m(x))$$

$$y' = \frac{m'(x)}{1+(mx)^2}$$

$$y' = 4\,\frac{2}{1+(2x)(2x)}$$

$$y' = 4\,\frac{2}{1+4x^2}$$

$$y' = \frac{8}{1+4x^2}$$

OR

$$y' = \frac{8}{1+(2x)^2}$$

8c) $f(y) = \cot^{-1}\left(\frac{3}{y^2+2}\right)$

$$f'(y) = -\frac{\left(\frac{3}{y^2+2}\right)'}{1+\left(\frac{3}{y^2+2}\right)^2}$$

$$f'(y) = -\frac{3(y^2+2)^{-1}}{1+\left(\frac{3}{y^2+2}\right)^2}$$

$$f'(y) = -\frac{-3(y^2+2)^{-2}(2y)}{1+\left(\frac{3}{y^2+2}\right)^2}$$

$$f'(y) = \frac{-6y(y^2+2)^{-2}}{1+\left(\frac{3}{y^2+2}\right)^2}$$

$$f'(y) = \frac{6y}{\left(1+\left(\frac{3}{y^2+2}\right)^2\right)\frac{(y^2+2)^2}{1}}$$

$$f'(y) = \frac{6y}{1(y^2+2)^2 + \frac{(3)^2}{\cancel{(y^2+2)^2}} \cdot \cancel{(y^2+2)^2}}$$

$$f'(y) = \frac{6y}{(y^2+2)^2+(3)^2}$$

$$f'(y) = \frac{6y}{(y^2+2)(y^2+2)+9}$$

$$f'(y) = \frac{6y}{y^4+2y^2+2y^2+4+9}$$

$$f'(y) = \frac{6y}{y^4+4y^2+13}$$

format

$y = \cot^{-1}(f(x))$

$$y' = -\frac{f'(x)}{1+(f(x))^2}$$

(81) $f(s) = \cot^{-1}(e^{s})$

$$f'(s) = -\frac{(e^{s})'}{1+(e^{s})^2}$$

$$f'(s) = -\frac{(e^{s})(s)'}{1+(e^{s})^2}$$

$$f'(s) = -\frac{e^{s}(1)}{1+(e^{s})^2}$$

$$f'(s) = \frac{-e^{s}}{1+e^{2s}}$$

formula

$$y = \cot^{-1}(f(x))$$

$$y' = \frac{-f'(x)}{1+(f(x))^2}$$

$$y = e^{f(x)}$$

$$y' = e^{f(x)} \cdot f'(x)$$

82. The sides of a square increase in length at a rate of 3 m/sec.

a. At what rate is the area of the square changing when the sides are 12 m long?
b. At what rate is the area of the square changing when the sides are 20 m long?

a. Write an equation relating the area of a square, A, and the side length of the square, s.

$A = s^2$

Differentiate both sides of the equation with respect to t.

$$\frac{dA}{dt} = (2s)\frac{ds}{dt}$$

$$\frac{dA}{dt} = 2(3)(12) = 72$$ Substitute

The area of the square is changing at a rate of 72 (1) m^2/s when the sides are 12 m long.

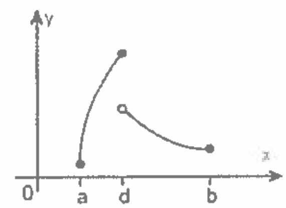
b. The area of the square is changing at a rate of 120 (2) m^2/s when the sides are 20 m long.

$$\frac{dA}{dt} = 2(3)(20) = 120$$ Substitute

(1)
- ○ m
- ✓ m^2/s
- ○ m/s
- ○ m^3/s

(2)
- ○ m^3/s
- ✓ m^2/s
- ○ m
- ○ m/s

83. Determine from the graph whether the function has any absolute extreme values on [a, b].



Where do the absolute extreme values of the function occur on [a, b]?

- ○ **A.** There is no absolute maximum and there is no absolute minimum on [a, b].
- ○ **B.** There is no absolute maximum and the absolute minimum occurs at x = a on [a, b].
- ○ **C.** The absolute maximum occurs at x = d and there is no absolute minimum on [a, b].
- ✓ **D.** The absolute maximum occurs at x = d and the absolute minimum occurs at x = a on [a, b].

84. Find the critical points

$f(x) = 5x^2 - 3x + 1$

$f'(x) = 10x - 3 + 0$

$f'(x) = 10x - 3$

set $10x - 3 = 0$

$10x - \cancel{3} + \cancel{3} = 0 + 3$

$10x = 3$

$\frac{10x}{10} = \frac{3}{10}$

$x = \frac{3}{10}$ critical point

formula

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

85

$f(x) = 2x^2 - 5x - 1$

$f'(x) = 4x - 5 - 0$

$f'(x) = 4x - 5$

$4x - 5 = 0$

$4x - \cancel{5} + \cancel{5} = 0 + 5$

$4x = 5$

$\frac{4x}{4} = \frac{5}{4}$

$x = 5/4$ Critical Point

Formula
$y = x^N$
$y' = Nx^{N-1}$

$y = ax$
$y' = a$

$y = a$
$y' = 0$

(86) $f(x) = \frac{x^3}{3} - 9x$

$f'(x) = \frac{1}{3}(3x^2) - 9$

$f'(x) = x^2 - 9$

formula

$a^2 - b^2$

$(a+b)(a-b)$

set $x^2 - 9 = 0$

$(x)^2 - (3)^2 = 0$

$(x+3)(x-3) = 0$

set $x+3=0$ OR $x-3=0$

$x+3-3 = 0-3$ OR $x-3+3 = 0+3$

$x = -3$ or $x = 3$

Critical points

formula

$y = x^n$

$y' = nx^{n-1}$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

87) $f(x) = -x^2+11$ find absolute extreme $[-2, 4]$

$f'(x) = -2x+0$

$f'(x) = -2x$

set $-2x = 0$

$\frac{-2x}{-2} = \frac{0}{-2}$

$x = 0$

$f(x) = -x^2+11$

$f(-2) = -(-2)^2+11$

$f(-2) = -(-2)(-2)+11$

$f(-2) = -(4)+11$

$f(-2) = -4+11$

$f(-2) = 7$

$f(x) = -x^2+11$

$f(4) = -(4)^2+11$

$f(4) = -(4)(4)+11$

$f(4) = -16+11$

$f(4) = -5$

$f(x) = -x^2+11$

$f(0) = -(0)^2+11$

$f(0) = -(0)(0)+11$

$f(0) = 0+11$

$f(0) = 11$

absolute max $f(0) = 11$

absolute min $f(4) = -5$

88

$S(x) = x^2 + \frac{600}{x}$

$S(x) = x^2 + 600x^{-1}$

$S'(x) = 2x - 600x^{-2}$

$S'(x) = 2x - \frac{600}{x^2}$

set $2x - \frac{600}{x^2} = 0$

$\frac{2x}{1} = \frac{600}{x^2}$ rewrite

$2x(x^2) = 1(600)$

$2x^3 = 600$

$\frac{2x^3}{2} = \frac{600}{2}$

$x^3 = 300$

$\sqrt[3]{x^3} = \sqrt[3]{300}$

$x = 6.694329501$ Critical point

$V = LWH$

$150 = (6.694)(6.694)H$

$150 = 44.809636H$

$\frac{150}{44.809636} = \frac{\cancel{44.809636}H}{\cancel{44.809636}}$

$3.347 = H$

$S(x) = x^2 + \frac{600}{x}$

$S(6.694) = (6.694)^2 + \frac{600}{6.694}$

$S(6.694) = 44.809636 + 89.63250672$

$S(6.694) = 134.442$ ft^2 absolute minimum value of the surface area

---

89

$s = -16t^2 + 32t + 384 \quad 0 \le t \le 6$

$s' = -32t + 32$

set $-32t + 32 = 0$

$-32t + \cancel{32} - \cancel{32} = 0 - 32$

$-32t = -32$

$\frac{-32t}{-32} = \frac{-32}{-32}$

$t = 1$ Critical Point

Max at $t = 1$

formula

$y = x^N$

$y' = Nx^{N-1}$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

(90) $P(n) = n(50 - 0.5n) + 100$

$P(n) = 50n - 0.5n^2 + 100$

$P'(n) = 50 - 0.5(2n) + 0$ formula

$y = x^n$

$y' = nx^{n-1}$

$P'(n) = 50 - n + 0$

$y = ax$

$y' = a$

$P'(n) = 50 - n$

set $50 - n = 0$

$\cancel{50} - n \cancel{- 50} = 0 - 50$

$y = a$

$y' = 0$

$-n = -50$

$\frac{-n}{-1} = \frac{-50}{-1}$

$n = 50$ Critical Point

$n = 50$ to Maximize Profit

If the bus holds a maximum of 44 people, the club should take 44 people to maximize profit

91) At what points c does the conclusion of the Mean Value Theorem hold for $f(x)=x^3$ on the interval $[-13, 13]$

Mean Value theorem

$f(x)=x^3$

$f'(x)=3x^2$

$\frac{f(13)-f(-13)}{(13)-(-13)} =$

$\frac{(13)^3-(-13)^3}{13+13} =$

$\frac{(2197)-(-2197)}{13+13} =$

$\frac{2197+2197}{13+13} =$

$\frac{4394}{26} =$

$169 =$

Set

$3x^2=169$

$\frac{3x^2}{3} = \frac{169}{3}$

$x^2 = \frac{169}{3}$

$\sqrt{x^2} = \pm\sqrt{\frac{169}{3}}$

$x = \pm\frac{\sqrt{169}}{\sqrt{3}}$

$x = \pm\frac{13}{\sqrt{3}}$

$x = \frac{\pm 13\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$x = \pm\frac{13\sqrt{3}}{\sqrt{9}}$

$x = \pm\frac{13\sqrt{3}}{3}$

$x = -\frac{13\sqrt{3}}{3}$ or $x = \frac{13\sqrt{3}}{3}$

(92) ⓐ Determine whether the Mean Value Theorem applies to the function $f(x) = -3 - x^2$ on the interval $[-2, 1]$ ⓑ If so find the point(s) that are guaranteed to exist by the Mean Value theorem.

$f(x) = -3 - x^2$

$f'(x) = -2x$

Mean Value theorem

$$\frac{f(1) - f(-2)}{(1) - (-2)} =$$

$$\frac{(-3 - (1)^2) - (-3 - (-2))^2}{1+2} =$$

$$\frac{(-3 - (1)(1)) - (-3 - (-2)(-2))}{1+2} =$$

$$\frac{(-3 - 1) - (-3 - 4)}{1+2} =$$

$$\frac{(-4) - (-7)}{1+2} =$$

$$\frac{-4 + 7}{1+2} =$$

$$\frac{\cancel{3}}{\cancel{3}} =$$

$1 =$

set

$-2x = 1$

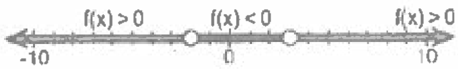
$$\frac{-2x}{-2} = \frac{1}{-2}$$

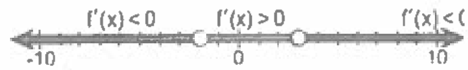
$x = -\frac{1}{2}$

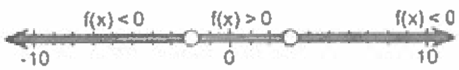
93. Sketch a function that is continuous on $(-\infty,\infty)$ and has the following properties. Use a number line to summarize information about the function.

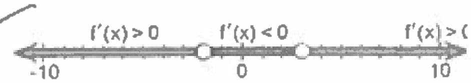
$f'(x) > 0$ on $(-\infty,-2)$; $f'(x) < 0$ on $(-2,3)$; $f'(x) > 0$ on $(3,\infty)$.

Which number line summarizes the information about the function?

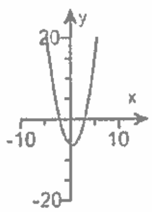
○ A. 

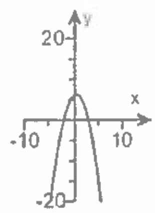
○ B. 

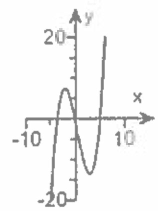
○ C. 

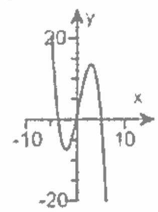
✓ D. 

Which of the following graphs matches the description of the given properties?

○ A. 

○ B. 

✓ C. 

○ D. 

$f'(x) > 0$ on $(-\infty, -2)$ — graph increasing

$f'(x) < 0$ on $(-2, 3)$ — graph decreasing

$f'(x) > 0$ on $(3, \infty)$ — graph increasing

94 $f(x) = -1 + x^2$

$f'(x) = 0 + 2x$

$f'(x) = 2x$

set $2x = 0$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$ critical point

-1 $\qquad$ 1

decreasing $(-\infty, 0)$ $\quad$ 0 $\quad$ increasing $(0, \infty)$

$f'(x) = 2x$

$f'(-1) = 2(-1)$

$f'(-1) = -2 < 0$ decreasing

$f'(x) = 2x$

$f'(1) = 2(1)$

$f'(1) = 2 > 0$ increasing

$\boxed{(0, \infty) \text{ increasing} \quad (-\infty, 0) \text{ decreasing}}$

formula

$y = x^n$
$y' = nx^{n-1}$

$y = ax$
$y' = a$

$y = a$
$y' = 0$

(95) $f(x) = -1 + x - x^2$

$f'(x) = 0 + 1 - 2x$

$f'(x) = 1 - 2x$

set $1 - 2x = 0$

$\cancel{1} - 2x - \cancel{1} = 0 - 1$

$-2x = -1$

$\frac{-2x}{-2} = \frac{-1}{-2}$

$x = \frac{1}{2}$

formula

$y = x^N$
$y' = Nx^{N-1}$

$y = ax$
$y' = a$

$y = a$
$y' = 0$

-1 $\quad$ $\frac{1}{2}$ $\quad$ 1

increasing $(-\infty, \frac{1}{2})$ $\quad$ $<$ $\quad$ decreasing $(\frac{1}{2}, \infty)$

$\boxed{(-\infty, \frac{1}{2}) \text{ increasing} \quad (\frac{1}{2}, \infty) \text{ decreasing}}$

$f'(x) = 1 - 2x$
$f'(-1) = 1 - 2(-1)$
$f'(-1) = 1 + 2$
$f'(-1) = 3 > 0$ increasing

$f'(x) = 1 - 2x$
$f'(1) = 1 - 2(1)$
$f'(1) = 1 - 2$
$f'(1) = -1 < 0$
decreasing

96

$f(x) = 6 - x^2$

$f'(x) = 0 - 2x$

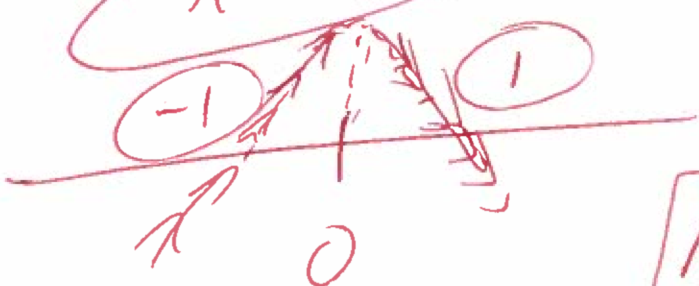
$f'(x) = -2x$

set $-2x = 0$

$\frac{-2x}{-2} = \frac{0}{-2}$

$x = 0$ critical point

CK



Max at $x = 0$

$f'(x) = -2x$

$f'(-1) = -2(-1)$

$f'(-1) = 2 > 0$ incr

$f'(x) = -2x$

$f'(1) = -2(1)$

$f'(1) = -2 < 0$ decreas

formula

$y = x^n$

$y' = n x^{n-1}$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

(97) $f(x) = 2x^3 - 6x^2 + 3$

$f'(x) = 6x^2 - 12x + 0$

$f'(x) = 6x^2 - 12x$

set $6x^2 - 12x = 0$

$6x(x-2) = 0$

set $6x = 0$ OR $x - 2 = 0$

$\frac{6x}{6} = \frac{0}{6}$ OR $x - \cancel{2} + \cancel{2} = 0 + 2$

$x = 0$ OR $x = 2$ Critical Points

max min

0 2

$f'(x) = 6x^2 - 12x$

$f''(x) = 12x - 12$

$f''(x) = 12x - 12$

$f''(0) = 12(0) - 12$

$f''(0) = 0 - 12$

$f''(0) = -12 < 0$ Max

$\cancel{f''(x) = 12 - 12x}$

$\cancel{f''(2) = 12 - 12(2)}$

$\cancel{f''(2) = 12 - 24}$

$\cancel{f''(2) = -12}$

$f''(2) = 12(2) - 12$

$f''(2) = 24 - 12$

$f''(2) = 12 > 0$ min

formula

$y = x^N$

$y' = Nx^{N-1}$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

98. Fill in the blanks: The goal of an optimization problem is to find the maximum or minimum value of the _______ function subject to the _______.

The goal of an optimization problem is to find the maximum or minimum value of the (1) ______ function subject to the (2) ______

(1)
- optimization function
- constraint function
- subjective function
- objective function

(2)
- extreme values.
- constraints.
- variables.
- optimizations.

(99) Use a linear approximation to estimate the following quantity. Choose a value of (a) to produce a small error.

$\ln(1.05)$

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$x = a = 1$

$f'(1) = \frac{1}{1} = 1$

$f(1) = 1$

$L(x) = f(a) + f'(a)(x-a)$ $\quad a = 1$

$L(1.05) = f(1) + f'(1)(1.05-1)$ $\quad x = 1.05$

$\ln(1.05) = \ln(1) + \left(\frac{1}{\cancel{x}}\right)(1.05-1)$

$\ln(1.05) = (0) + (1)(0.05)$

to approximate $\ln(1.05)$

$\ln(1.05) = 0 + 0.05$

$= 0.05$

(100) Consider the following function and express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)\,dx$

formula
$y = e^{f(x)}$
$y' = e^{f(x)} \cdot f'(x)$

$y = e^{10x}$

$\frac{dy}{dx} = e^{10x}(10x)'$

$\frac{dy}{dx} = e^{10x}(10)$

$\frac{dy}{dx} = 10e^{10x}$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = 10e^{10x}(dx)$

$dy = 10e^{10x}\,dx$

101

$f(x) = 2x^3 - 3x$

$y = 2x^3 - 3x$

$$\frac{dy}{dx} = 6x^2 - 3$$

$$\cancel{\frac{dy}{dx}(dx)} = (6x^2 - 3)\,dx$$

$$dy = (6x^2 - 3)\,dx$$

formulas

$y = x^n$
$y' = nx^{n-1}$

$y = ax$
$y' = a$

$y = a$
$y' = 0$

102.

$f(x) = \cot(9x)$

$y = \cot(9x)$

$\frac{dy}{dx} = -\csc^2(9x) \cdot (9x)'$

$\frac{dy}{dx} = -\csc^2(9x) \cdot 9$

$\frac{dy}{dx} = -9\csc^2(9x)$

$\frac{dy}{\cancel{dx}}(\cancel{dx}) = \left(-9\csc^2(9x)\right)dx$

$dy = -9\csc^2(9x)\,dx$

formula

$y = \cot(f(x))$

$y' = -\csc^2(f(x)) \cdot f'(x)$

Use l'Hopital Rule

103 $\lim_{x \to 0} \frac{3\sin(2x)}{5x} =$

$\lim_{x \to 0} \frac{3\cos(2x)\,(2x)'}{(5x)'} =$

$\lim_{x \to 0} \frac{3\cos(2x)\,(2)}{5} =$

$\lim_{x \to 0} \frac{6\cos(2x)}{5} =$

$\frac{6\cos(2(0))}{5} =$

$\frac{6\cos(0)}{5} =$

$\frac{6(1)}{5} =$

$\frac{6}{5} =$

Formula l'Hopital Rule

$\lim_{x \to a} \frac{f(x)}{g(x)} =$

$\lim_{x \to a} \frac{f'(x)}{g'(x)} =$

$y = \sin f(x)$

$y' = \cos f(x)\, f'(x)$

Use L'Hopital's Rule

104

$$\lim_{x \to 0} \frac{4\sin(7x)}{5x} =$$

$$\lim_{x \to 0} \frac{4\cos(7x)(7x)'}{(5x)'} =$$

$$\lim_{x \to 0} \frac{4\cos(7x)\,7}{5} =$$

$$\lim_{x \to 0} \frac{28\cos(7x)}{5} =$$

$$\frac{28\cos(7(0))}{5} =$$

$$\frac{28\cos(0)}{5} =$$

$$\frac{28(1)}{5} =$$

$$\frac{28}{5} =$$

formula

$$\lim_{x \to a} \frac{f(x)}{g(x)} =$$

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} =$$

$y = \sin f(x)$

$y' = \cos f(x)\, f'(x)$

(105) Evaluate the following limit. Use L'Hopital Rule

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} =$$

$$\lim_{u\to\frac{\pi}{4}} \frac{3\tan(u) - 3\cot(u)}{2u - \frac{\pi}{2}} =$$

$$\lim_{u\to\frac{\pi}{4}} \frac{3\sec^2(u) + 3\csc^2(u)}{2} =$$

$$\lim_{u\to\frac{\pi}{4}} \frac{3\frac{1}{\cos^2(u)} + 3\frac{1}{\sin^2(u)}}{2} =$$

$$\frac{3\frac{1}{\cos^2(\frac{\pi}{4})} + 3\frac{1}{\sin^2(\frac{\pi}{4})}}{2} =$$

$$\frac{3\frac{1}{(\frac{\sqrt{2}}{2})^2} + 3\frac{1}{(\frac{\sqrt{2}}{2})^2}}{2} =$$

$$\frac{3\frac{1}{\frac{(\sqrt{2})^2}{(2)^2}} + 3\frac{1}{\frac{(\sqrt{2})^2}{(2)^2}}}{2} =$$

$$\frac{3\frac{1}{\frac{2}{4}} + 3\frac{1}{\frac{2}{4}}}{2} =$$

$$\frac{3\frac{1}{\frac{1}{2}} + 3\frac{1}{\frac{1}{2}}}{2} =$$

$$\frac{3\cdot\frac{1}{1}\left(\frac{2}{1}\right) + 3\left(\frac{1}{1}\right)\frac{2}{1}}{2} =$$

$$\frac{\frac{6}{1} + \frac{6}{1}}{2} =$$

$$\frac{6+6}{2} =$$

$$\frac{12}{2} =$$

$$6$$

106. Use a calculator or program to compute the first 10 iterations of Newton's method when they are applied to the following function with the given initial approximation.

$f(x) = x^2 - 18$; $x_0 = 5$ $f'(x) = 2x$

k	x_k	k	x_k
0	5.000000	6	4.242641
1	4.242641	7	4.242641
2	4.300000	8	4.242641
3	4.242641	9	4.242641
4	4.243023	10	4.242641
5	4.242641		

(Round to six decimal places as needed.)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 5 - \frac{f(5)}{f'(5)}$$

$$x_1 = 5 - \frac{(5)^2 - 18}{2(5)}$$

$$x_1 = 4.30000$$

---

107. Use a calculator or program to compute the first 10 iterations of Newton's method for the given function and initial approximation.

$f(x) = 2 \sin x - 5x - 3$, $x_0 = 1.3$ $f'(x) = 2\cos(x) - 5$

Complete the table.
(Do not round until the final answer. Then round to six decimal places as needed.)

k	x_k	k	x_k
1	-0.396054	6	-0.917677
2	-0.917677	7	-0.917677
3	-0.963849	8	-0.917677
4	-0.917677	9	-0.917677
5	-0.918126	10	-0.917677

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1.3 - \frac{f(1.3)}{f'(1.3)}$$

$$x_1 = 1.3 - \frac{2\sin(1.3) - 5(1.3) - 3}{2\cos(1.3) - 5}$$

$$x_1 = -0.396054$$

108

$$\int (11x^{21} - 7x^{13})\,dx =$$

formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C =$$

$$\frac{11x^{21+1}}{21+1} - \frac{7x^{13+1}}{13+1} + C =$$

$$\frac{11x^{22}}{22} - \frac{7x^{14}}{14} + C =$$

$$\frac{\cancel{11}\,x^{22}}{(\cancel{11})(2)} - \frac{\cancel{7}\,x^{14}}{(\cancel{7})(2)} + C =$$

$$\frac{x^{22}}{2} - \frac{x^{14}}{2} + C =$$

(109) $\int \left(\frac{12}{\sqrt{x}} + 12\sqrt{x}\right) dx$

~~formula~~

$\int x^n dx =$

$\frac{x^{n+1}}{n+1} + C =$

$\int \frac{12}{x^{1/2}} + 12x^{1/2}\, dx =$

$\int 12x^{-1/2} + 12x^{1/2}\, dx =$

$\frac{12x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{12x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$

$\frac{12x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}} + \frac{12x^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$

$\frac{12x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{12x^{3/2}}{\frac{3}{2}} + C =$

$\frac{2}{1}\cdot\left(\frac{12x^{1/2}}{1}\right) + \frac{2}{3}\cdot\frac{12x^{3/2}}{1} + C =$

$24x^{1/2} + \frac{24}{3}x^{3/2} + C =$

$24\sqrt{x} + 8x^{3/2} + C =$

(11c)

$$\int\left(\frac{9}{s^2}+3s^4\right)ds =$$

$$\int(9s^{-2}+3s^4)\,ds =$$

$$\frac{9s^{-2+1}}{-2+1}+\frac{3s^{4+1}}{4+1}+C =$$

$$\frac{9s^{-1}}{-1}+\frac{3s^{5}}{5}+C =$$

$$-9s^{-1}+\frac{3}{5}s^{5}+C$$

formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+C$$

(117) $\int (3x+2)^2 dx =$

$\frac{1}{3}\int (3x+2)^2 (3)\, dx =$

$\frac{1}{3}\frac{(3x+2)^{2+1}}{2+1} + c =$

$\frac{1}{3}\frac{(3x+2)^3}{3} + c =$

$\frac{1}{9}(3x+2)^3 + c =$

$\frac{1}{9}(3x+2)(3x+2)(3x+2) + c =$

$\frac{1}{9}(3x+2)(9x^2+6x+6x+4) + c =$

$\frac{1}{9}(3x+2)(9x^2+12x+4) + c =$

$\frac{1}{9}(27x^3+36x^2+12x+18x^2+24x+8) + c$

$\frac{1}{9}(27x^3+54x^2+36x+8) + c$

$\frac{27x^3}{9} + \frac{54x^2}{9} + \frac{36x}{9} + \frac{8}{9} + c$

$3x^3+6x^2+4x+\frac{8}{9}+c$

$3x^3+6x^2+4x+C_1$

let $C_1 = \frac{8}{9}+c$

Formula

$\int (f(x))^N \cdot f'(x)\, dx =$

$\frac{(f(x))^{N+1}}{N+1} + C =$

OR

(112) $\int 5m(5m^3 - 8m)\,dm =$

$\int (25m^4 - 40m^2)\,dm =$

$\frac{25m^{4+1}}{4+1} - \frac{40m^{2+1}}{2+1} + C =$

$\frac{25m^5}{5} - \frac{40m^3}{3} + C =$

$5m^5 - \frac{40m^3}{3} + C$

Formula

$\int (f(x))^N f'(x)\,dx =$

$\frac{(f(x))^{N+1}}{N+1} + C$

(113) $\int (3x^{2/3} + 2x^{-1/3} + 7)\,dx =$

$$\frac{3x^{2/3+1}}{\frac{2}{3}+1} + \frac{2x^{-1/3+1}}{-\frac{1}{3}+1} + 7x + C =$$

$$\frac{3x^{\frac{2}{3}+\frac{3}{3}}}{\frac{2}{3}+\frac{3}{3}} + \frac{2x^{-\frac{1}{3}+\frac{3}{3}}}{-\frac{1}{3}+\frac{3}{3}} + 7x + C =$$

$$\frac{3x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + 7x + C =$$

$$\frac{3}{5}\cdot 3x^{5/3} + \frac{3}{\cancel{2}}\cdot \cancel{2}x^{2/3} + 7x + C =$$

$$\frac{9}{5}x^{5/3} + 3x^{2/3} + 7x + C =$$

formula

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\int a\,dx = ax + C$$

114

$$\int 4\sqrt[6]{x}\,dx =$$

$$\int 4x^{1/6}\,dx =$$

$$\frac{4x^{1/6+1}}{\frac{1}{6}+1} + C =$$

$$\frac{4x^{\frac{1}{6}+\frac{6}{6}}}{\frac{1}{6}+\frac{6}{6}} + C =$$

$$\frac{4x^{\frac{7}{6}}}{\frac{7}{6}} + C =$$

$$\frac{6}{7}\,4x^{7/6} + C =$$

$$\frac{24}{7}x^{7/6} + C =$$

formul

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

115 $\int (7x+4)(1-x)\,dx =$

$\int (7x - 7x^2 + 4 - 4x)\,dx =$

$\int (-7x^2 + 3x + 4)\,dx =$

$\frac{-7x^{2+1}}{2+1} + \frac{3x^{1+1}}{1+1} + 4x + C =$

$\frac{-7x^3}{3} + \frac{3x^2}{2} + 4x + C =$

Formula

$\int (f(x))^N f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1} + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$\int a\,dx = ax + C =$

(11c) $\int \left(\frac{6}{x^3} + 3 - \frac{2}{x^2} \right) dx =$

$\int 6x^{-3} + 3 - 2x^{-2}) \, dx =$

$\frac{6x^{-3+1}}{-3+1} + 3x - \frac{2x^{-2+1}}{-2+1} + C =$

$\frac{6x^{-2}}{-2} + 3x - \frac{2x^{-1}}{-1} + C =$

$\frac{6x^{-2}}{-2} + 3x + \frac{2x^{-1}}{1} + C =$

$-3x^{-2} + 3x + 2x^{-1} + C =$

$-\frac{3}{x^2} + 3x + \frac{2}{x} + C =$

Formula

$\int x^n dx$

$\frac{x^{n+1}}{n+1} + C$

$\int a \, dx =$

$ax + C =$

(117)

$$\int \frac{3x^5 + 6x^4}{x^3}\,dx =$$

$$\int \frac{3x^5}{x^3} + \frac{6x^4}{x^3}\,dx =$$

$$\int \left(3x^{5-3} + 6x^{4-3}\right)dx =$$

$$\int \left(3x^2 + 6x^1\right)dx =$$

$$\frac{3x^{2+1}}{2+1} + \frac{6x^{1+1}}{1+1} + C =$$

$$\frac{\cancel{3}x^3}{\cancel{3}} + \frac{6x^2}{2} + C$$

$$x^3 + 3x^2 + C =$$

formula

$$\int x^n\,dx =$$

$$\frac{x^{n+1}}{n+1} + C =$$

(118) $\int (\csc^2(4\theta)+5)\,d\theta =$

$\int \csc^2(4\theta)\,d\theta + \int 5\,d\theta =$

$\frac{1}{4}\int \csc^2(4\theta)\,4\,d\theta + \int 5\,d\theta =$

$\frac{1}{4}(-\cot(4\theta)) + 5\theta + C =$

$-\frac{1}{4}\cot(4\theta) + 5\theta + C =$

$\int a\,dx =$

$ax+C =$

Formula

$\int \csc^2(f(x))\cdot f'(x)\,dx$

$-\cot(f(x))+C$

$\cancel{\int a\,dx =}$

$\cancel{ax+C =}$

119

$$\int (\sec^2(x) - 8)\,dx =$$

$$\int (\sec^2(x)(1) - 8)\,dx =$$

$$\tan(x) - 8x + C =$$

formula

$$\int \sec^2(f(x)) \cdot f'(x)\,dx =$$

$$\tan(f(x)) + C =$$

$$\int a\,dx =$$

$$ax + C =$$

120 $\int (5\sec(x)\tan(x) + 2\sec^2(x))\,dx =$

$5\sec(x) + 2\tan(x) + C =$

formula

$\int \sec(f(x))\tan(f(x))\cdot f'(x)\,dx =$

$\sec(f(x)) + C =$

$\int \sec^2(f(x))\cdot f'(x)\,dx =$

$\tan(f(x)) + C =$

(121.) For the following function f, find the antiderivative F that satisfies the given condition.

$f(x) = 6x^3 + 4\sin(x)$, $F(0) = 2$

$\int f(x)\,dx = \int 6x^3 + 4\sin(x)\,dx$

$\int f(x)\,dx = \int 6x^3 + 4\sin(x)(1)\,dx$

$F(x) = \frac{6x^{3+1}}{3+1} - 4\cos x + C$

$F(x) = \frac{6x^4}{4} - 4\cos x + C$ or $\frac{3}{2}x^4 - 4\cos x + C$

Recall

$F(0) = \frac{6(0)^4}{4} - 4\cos(0) + C = 2$

$\frac{6(0)}{4} - 4\cos(0) + C = 2$

$0 - 4(1) + C = 2$

$0 - 4 + C = 2$

$-4 + C = 2$

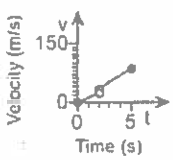
$\cancel{-4} + C \cancel{+4} = 2 + 4$

$C = 6$ $\boxed{F(x) = \frac{3}{2}x^4 - 4\cos(x) + 6}$

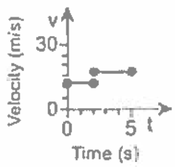
122. Suppose an object moves along a line at 12 m/s for $0 \le t \le 2$ s and at 17 m/s for $2 < t \le 5$ s. Sketch the graph of the velocity function and find the displacement of the object for $0 \le t \le 5$.

Sketch the graph of the velocity function. Choose the correct graph below.

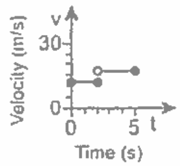
A.



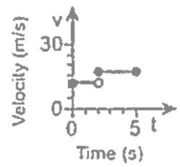
B.



C.

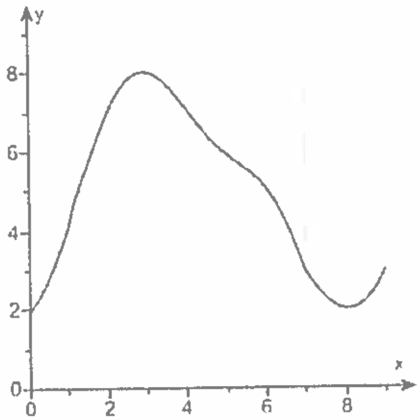


D.



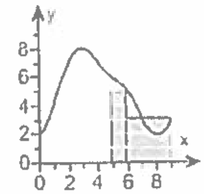
The displacement of the object for $0 \le t \le 5$ is 75 m. (Simplify your answer.)

123\. Approximate the area of the region bounded by the graph of f(x) (shown below) and the x-axis by dividing the interval [0,4] into n = 4 subintervals. Use a right and left Riemann sum to obtain two different approximations. Draw the approximating rectangles.

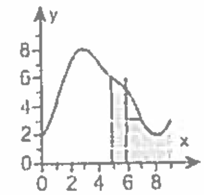


In which graph below are the selected points the right endpoints of the 4 approximating rectangles?

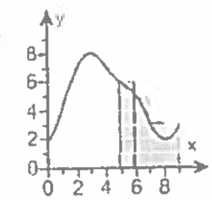
A.



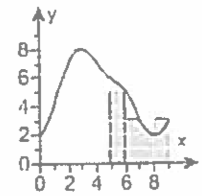
B.



C.



D.

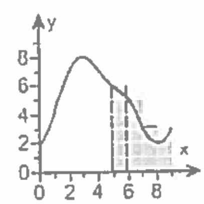


Using the specified rectangles, approximate the area.

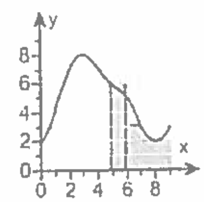
26

In which graph below are the selected points the left endpoints of the 4 approximating rectangles?

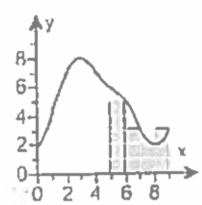
A.



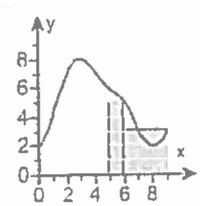
B.



C.



D.



Using the specified rectangles, approximate the area.

21

124. Does the right Riemann sum underestimate or overestimate the area of the region under the graph of a positive decreasing function? Explain.

Choose the correct answer below.

- A. Underestimate; the rectangles all fit under the curve.
- B. Overestimate; the rectangles all fit under the curve.
- C. Underestimate; the rectangles do not fit under the curve.
- D. Overestimate; the rectangles do not fit under the curve.

---

125. The following function is negative on the given interval.

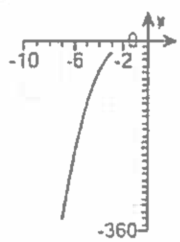
$f(x) = -5 - x^3$; [3,7]

a. Sketch the function on the given interval.

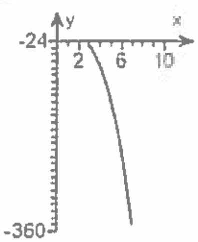
b. Approximate the net area bounded by the graph of f and the x-axis on the interval using a left, right, and midpoint Riemann sum with $n = 4$.

a. Choose the correct graph below.

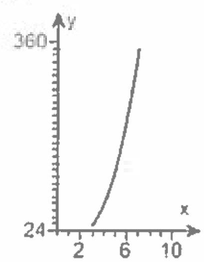
A.



B.



C.



b. The approximate net area using a left Riemann sum is -452.
(Type an integer or a decimal.)

The approximate net area using a midpoint Riemann sum is -595.
(Type an integer or a decimal.)

The approximate net area using a right Riemann sum is -768.
(Type an integer or a decimal.)

126

formula

$$\int x^n dx$$

$$\frac{x^{n+1}}{n+1} + C$$

$$\int a\,dx =$$

$$ax + C$$

$$\int_0^3 (2x+4)\,dx =$$

$$\frac{2x^{1+1}}{1+1} + 4x \Big|_0^3 =$$

$$\frac{2x^2}{2} + 4x \Big|_0^3 =$$

$$x^2 + 4x \Big|_0^3 =$$

$$\left((3)^2 + 4(3)\right) - \left((0)^2 + 4(0)\right) =$$

$$\left((3)(3) + 4(3)\right) - \left((0)(0) + 4(0)\right) =$$

$$(9+12) - (0+0) =$$

$$(21) - (0) =$$

$$21 =$$

127.

Evaluate $\frac{d}{dx}\int_a^x f(t)\,dt$ and $\frac{d}{dx}\int_a^b f(t)\,dt$, where a and b are constants.

$\frac{d}{dx}\int_a^x f(t)\,dt =$ $f(x)$ (Simplify your answer.)

$\frac{d}{dx}\int_a^b f(t)\,dt =$ 0 (Simplify your answer.)

(128) $\int_0^1 (x^2 - 3x + 5)\,dx =$

$\int x^n dx$

$\frac{x^{n+1}}{n+1} + C$

$\int a\,dx =$

$ax + C =$

$\frac{x^{2+1}}{2+1} - \frac{3x^{1+1}}{1+1} + 5x \Big|_0^1 =$

$\frac{x^3}{3} - \frac{3x^2}{2} + 5x \Big|_0^1 =$

$\left(\frac{(1)^3}{3} - \frac{3(1)^2}{2} + 5(1)\right) - \left(\frac{(0)^3}{3} - \frac{3(0)^2}{2} + 5(0)\right) =$

$\left(\frac{1}{3} - \frac{3(1)}{2} + 5(1)\right) - (0 - 0 + 0) =$

$\left(\frac{1}{3} - \frac{3}{2} + 5\right) - (0) =$

$\left(\frac{1}{3}\left(\frac{2}{2}\right) - \frac{3}{2}\left(\frac{3}{3}\right) + \frac{5}{1}\left(\frac{6}{6}\right)\right) - (0) =$

$\left(\frac{2}{6} - \frac{9}{6} + \frac{30}{6}\right) - (0) =$

$\left(-\frac{7}{6} + \frac{30}{6}\right) - (0) =$

$\frac{23}{6} =$

(125) $\int_{-2}^{4} (x^2 - 2x - 8)\, dx =$

formula
$\int x^n dx =$
$\frac{x^{n+1}}{n+1} + C =$

$\int a\,dx =$
$ax + C =$

$\frac{x^{2+1}}{2+1} - \frac{2x^{1+1}}{1+1} - 8x \Big|_{-2}^{4} =$

$\frac{x^3}{3} - \frac{2x^2}{2} - 8x \Big|_{-2}^{4}$

$\frac{x^3}{3} - x^2 - 8x \Big|_{-2}^{4}$

$\left(\frac{(4)^3}{3} - (4)^2 - 8(4) \right) - \left(\frac{(-2)^3}{3} - (-2)^2 - 8(-2) \right)$

$\left(\frac{(4)(4)(4)}{3} - (4)(4) - 8(4) \right) - \left(\frac{(-2)(-2)(-2)}{3} - (-2)(-2) - 8(-2) \right)$

$\left(\frac{64}{3} - 16 - 32 \right) - \left(\frac{-8}{3} - (4) + 16 \right) =$

$\left(\frac{64}{3} - 16 - 32 \right) - \left(-\frac{8}{3} - 4 + 16 \right) =$

$\left(\frac{64}{3} - 48 \right) - \left(-\frac{8}{3} + 12 \right) =$

$\frac{64}{3} - 48 + \frac{8}{3} - 12 =$

$\frac{72}{3} - 60 =$

$\frac{72}{3} - \frac{60}{1}\left(\frac{3}{3}\right) =$

$\frac{72}{3} - \frac{180}{3} =$

$\frac{-108}{3} =$

$-36 =$

130

$$\int_{-9}^{9} (81 - x^2)\,dx$$

~~formula~~

$$\int x^n\,dx$$

$$\frac{x^{n+1}}{n+1} + C$$

$$\int a\,dx$$

$$ax + C$$

$$81x - \frac{x^{2+1}}{2+1} \Big|_{-9}^{9}$$

$$81x - \frac{x^3}{3} \Big|_{-9}^{9} =$$

$$\left(81(9) - \frac{(9)^3}{3}\right) - \left(81(-9) - \frac{(-9)^3}{3}\right) =$$

$$\left(81(9) - \frac{(9)(9)(9)}{3}\right) - \left(81(-9) - \frac{(-9)(-9)(-9)}{3}\right) =$$

$$\left(729 - \frac{729}{3}\right) - \left(-729 - \frac{-729}{3}\right) =$$

$$(729 - \frac{729}{3} - \left(-729 + \frac{729}{3}\right) =$$

$$729 - \frac{729}{3} + 729 - \frac{729}{3} =$$

$$1458 - \frac{729}{3} - \frac{729}{3} =$$

$$1458 - \frac{1458}{3} =$$

$$1458 - 486 =$$

$$972 =$$

131. Is x^{34} an even or odd function? Is $\sin(x^3)$ an even or odd function?

Is x^{34} an even or odd function?

- ○ odd function
- ○ even function

Is $\sin(x^3)$ an even or odd function?

- ○ even function
- ○ odd function

---

132. On which derivative rule is the Substitution Rule based?

Choose the correct answer below.

- ○ **A.** The Product Rule
- ○ **B.** The Constant Multiple Rule
- ○ **C.** The Chain Rule
- ○ **D.** The Quotient Rule

133

$$\int 2x(x^2+8)^{15}\,dx$$

$$\int (x^2+8)^{15}(2x)\,dx \quad \text{rewrite}$$

$$\frac{(x^2+8)^{15+1}}{15+1}+C=$$

$$\frac{(x^2+8)^{16}}{16}+C=$$

$$\frac{1}{16}(x^2+8)^{16}+C=$$

formula

$$\int (f(x))^{n}\cdot f'(x)\,dx=$$

$$\frac{(f(x))^{n+1}}{n+1}+C=$$

$$y=x^n$$

$$y'=nx^{n-1}$$

134.

$$\int -10x \sin(5x^2-2)\,dx =$$

$$\int -\sin(5x^2-2)(+10x)\,dx =$$

$$\cos(5x^2-2) + C$$

Formula

$$\int -\sin(f(x)) \cdot f'(x)\,dx$$

$$\cos(f(x)) + C$$

135

$$\int -8x \sin(4x^2-5)\,dx =$$

$$\int -\sin(4x^2-5)(+8x)\,dx =$$

$$\cos(4x^2-5) + C =$$

formula

$$\int -\sin(f(x))\, f'(x)\,dx =$$

$$\cos(f(x)) + C =$$

136

$$\int (16x+3)\sqrt{8x^2+3x}\,dx =$$

$$\int \sqrt{8x^2+3x}\,(16x+3)\,dx =$$

$$\int (8x^2+3x)^{1/2}(16x+3)\,dx =$$

$$\frac{(8x^2+3x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$$

$$\frac{(8x^2+3x)^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$$

$$\frac{(8x^2+3x)^{3/2}}{3/2} + C =$$

$$\frac{2}{3}(8x^2+3x)^{3/2} + C =$$ rework

formula

$$\int (f(x))^{n} f'(x)\,dx$$

$$\frac{(f(x))^{n+1}}{n+1} + C$$

137

$$\int (9x^8+8)\sqrt{x^9+8x}\,dx =$$

$$\int \sqrt{x^9+8x}\,(9x^8+8)\,dx =$$

$$\int (x^9+8x)^{1/2}(9x^8+8)\,dx =$$

$$\frac{(x^9+8x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$$

$$\frac{(x^9+8x)^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$$

$$\frac{(x^9+8x)^{\frac{3}{2}}}{3/2} + C =$$

$$\frac{2}{3}(x^9+8x)^{3/2} + C =$$

formula

$$\int (f(x))^N \cdot f'(x)\,dx = \frac{(f(x))^{N+1}}{N+1} + C =$$

138 $\int e^{8x+3}\,dx =$

$\frac{1}{8}\int e^{8x+3}(8)\,dx =$ rewrite

formula

$\int e^{f(x)} \cdot f'(x)\,dx =$

$e^{f(x)} + c =$

$\frac{1}{8} e^{8x+3} + c =$

(139) $\int x e^{x^2}\, dx =$

$\int e^{x^2}(x)\, dx =$

$\frac{1}{2}\int e^{x^2}(2x)\, dx =$ rewrite

$\frac{1}{2} e^{x^2} + C =$

formula

$\int e^{f(x)} \cdot f'(x)\, dx =$

$e^{f(x)} + C =$

140

$$\int (x^5+x)^{10}(5x^4+1)\,dx =$$

$$\frac{(x^5+x)^{10+1}}{10+1} + C =$$

$$\frac{(x^5+x)^{11}}{11} + C =$$

formula

$$\int (f(x))^N \cdot f'(x)\,dx =$$

$$\frac{(f(x))^{N+1}}{N+1} + C =$$

(141)

$$\int \sec^2(8x-3)\,dx =$$

$$\frac{1}{8}\int \sec^2(8x-3)(8)\,dx$$ = rewrite

$$\frac{1}{8}\tan(8x-3) + C =$$

~~formula~~

$$\int \sec^2(f(x)) \cdot f'(x)\,dx =$$

$$\tan(f(x)) + C =$$

$y = ax$

$y' = a$

$y = a$

$y' = 0$

(142) $\int \frac{e^{4x}}{e^{4x}+2}\,dx =$

$\frac{1}{4}\int \frac{e^{4x}(4)}{e^{4x}+2}\,dx =$

$\frac{1}{4}\ln\left|e^{4x}+2\right| + C =$

Formula

$\int \frac{f'(x)}{f(x)}\,dx =$

$\ln|f(x)| + C$

$\int e^{f(x)} f'(x)\,dx =$

$e^{f(x)} + C$

143

$$\int_0^{\pi/20} \cos(5x)\,dx$$

formula

$$\int \cos(f(x))\,f'(x)\,dx = \sin(f(x)) + C$$

$$\frac{1}{5}\int_0^{\frac{\pi}{20}} \cos(5x)(5)\,dx$$

$$\frac{1}{5}\sin(5x)\Big|_0^{\frac{\pi}{20}}$$

$$\left(\frac{1}{5}\sin\left(5\left(\frac{\pi}{20}\right)\right)\right) - \left(\frac{1}{5}\sin(5(0))\right) =$$

$$\left(\frac{1}{5}\sin\left(\frac{\cancel{5}\pi}{\cancel{5}(4)}\right)\right) - \left(\frac{1}{5}\sin(0)\right) =$$

$$\left(\frac{1}{5}\sin\left(\frac{\pi}{4}\right)\right) - \left(\frac{1}{5}\sin(0)\right) =$$

$$\left(\frac{1}{5}\left(\frac{\sqrt{2}}{2}\right)\right) - \left(\frac{1}{5}(0)\right) =$$

$$\frac{\sqrt{2}}{10} - 0 =$$

$$\frac{\sqrt{2}}{10} =$$

OR

$$\left(\frac{\sqrt{2}}{10}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) =$$ mult

OR $$\frac{\sqrt{4}}{10\sqrt{2}} =$$

OR $$\frac{2}{10\sqrt{2}} =$$

OR $$\frac{\cancel{2}(1)}{\cancel{2}(5)\sqrt{2}} =$$

OR $$\frac{1}{5\sqrt{2}} =$$

144

$$\int_0^2 6e^{2x}\,dx =$$

$$6\int_0^2 e^{2x}\,dx =$$

$$(6)\left(\tfrac{1}{2}\right)\int_0^2 e^{2x}(2)\,dx =$$

$$3\int_0^2 e^{2x}(2)\,dx =$$

$$3e^{2x}\Big|_0^2 =$$

$$\left(3e^{2(2)}\right) - \left(3e^{2(0)}\right) =$$

$$\left(3e^{4}\right) - \left(3e^{0}\right) =$$

$$\left(3e^{4}\right) - \left(3(1)\right) =$$

$$3e^{4} - 3 =$$

Formula

$$\int e^{f(x)} f'(x)\,dx$$

$$e^{f(x)} + C$$

(145) $\int_0^1 \frac{2x}{(x^2+5)^3}\,dx$

$\int_0^1 2x(x^2+5)^{-3}\,dx$

$\int_0^1 (x^2+5)^{-3}(2x)\,dx$

$\frac{(x^2+5)^{-3+1}}{-3+1}\Big/_0^1$

$\frac{(x^2+5)^{-2}}{-2}\Big/_0^1$

$\frac{1}{-2(x^2+5)^2}\Big/_0^1$

$\left(\frac{1}{-2((1)^2+5)^2}\right) - \left(\frac{1}{-2((0)^2+5)^2}\right) =$

$\frac{1}{-2(1+5)^2} - \left(\frac{1}{-2(0+5)^2}\right) =$

$\left(\frac{1}{-2(6)^2}\right) - \left(\frac{1}{-2(5)^2}\right) =$

$\left(\frac{1}{-2(36)}\right) - \left(\frac{1}{-2(25)}\right) =$

$\left(\frac{1}{-72}\right) - \left(\frac{1}{-50}\right) =$

$\left(\frac{1}{-72}\right) + \frac{1}{50} =$

formula

$\int (f(x))^n \cdot f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + C =$

$\frac{1}{-72}\left(\frac{50}{50}\right) + \frac{1}{50}\left(\frac{72}{72}\right) =$

$\frac{-50}{3600} + \frac{72}{3600} =$

$\frac{-50+72}{3600} =$

$\frac{22}{3600} =$

$\frac{\cancel{2}(11)}{\cancel{2}(1800)} =$

$\frac{11}{1800}$

(14c) $\int \frac{dx}{x^2-2x+82}$

$\int \frac{dx}{x^2-2x+(\frac{1}{2}(-2))^2+82-(\frac{1}{2}(-2))^2} =$ complete square

$\int \frac{dx}{x^2-2x+(-1)^2+82-(-1)^2} =$

$\int \frac{dx}{x^2-2x+1+82-(1)} =$

$\int \frac{dx}{x^2-2x+1+82-1} =$

$\int \frac{dx}{x^2-2x+1+81} =$

$\int \frac{dx}{(x-1)(x-1)+81} =$

$\int \frac{dx}{(x-1)^2+(9)^2} =$

$\int \frac{dx}{(9)^2+(x-1)^2} =$

$\frac{1}{9}\tan^{-1}\left(\frac{x-1}{9}\right)+C =$

Formula

$\int \frac{f'(x)}{(a)^2+(f(x))^2} =$

$\frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right)+C$

OR

$\int \frac{dx}{a^2+x^2}\,dx =$

$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C =$

(147) $\int 11t \cdot e^{t}\, dt$ eval the following integral using integration by parts.

$11t \cdot e^{t} - \int 11 e^{t}\, dx =$

$11t \cdot e^{t} - 11e^{t} + C =$

Derivative D	Integrate I
$11t$	e^{t}
11	e^{t}
0	e^{t}

formula

$\int e^{f(x)} \cdot f'(x)\, dx =$

$e^{f(x)} + C =$

$y = at$

$y' = a$

$y = a$

$y' = 0$

(148) evaluate the following integral using integration by parts.

derivative D	integrate I
$\ln(5x)$	$16x$
$\frac{1}{x}$	$8x^2$

$$\int 16x \ln(5x)\,dx$$

$$8x^2\ln(5x) - \int \left(\frac{1}{x}\right)(8x^2)\,dx$$

$$8x^2\ln(5x) - \int \frac{8x^2}{x}\,dx =$$

$$8x^2\ln(5x) - \int 8x\,dx =$$

$$8x^2\ln(5x) - \frac{8x^{1+1}}{1+1} + C =$$

$$8x^2\ln(5x) - \frac{8x^2}{2} + C =$$

$$8x^2\ln(5x) - 4x^2 + C =$$

formula

$$y = \ln(5x)$$

$$y' = \frac{(5x)'}{5x}$$

$$y' = \frac{5}{5x}$$

$$y' = \frac{1}{x}$$

$$\int 16x\,dx =$$

$$\frac{16x^{1+1}}{1+1} + C =$$

$$\frac{16x^2}{2} + C =$$

$$8x^2 + C$$

(149) If the general solution of a differential equation is $y(t) = Ce^{-2t} + 10$, what is the solution that satisfies the initial condition $y(0) = 5$?

$$y(t) = Ce^{-2t} + 10$$

$$y(0) = Ce^{-2(0)} + 10 = 5$$

$$Ce^{-2(0)} + 10 = 5$$

$$Ce^{0} + 10 = 5$$

$$C(1) + 10 = 5$$

$$C + 10 = 5$$

$$C + \cancel{10} - \cancel{10} = 5 - 10$$

$$C = -5$$

$$y(t) = -5e^{-2t} + 10$$