Student: _____	Instructor: Alfredo Alvarez	Assignment: math 1314 Free Response
Date: _____	Course: Math 1314 Summer 2018	with Help 37

1. Solve the equation by factoring.

   $8x^2 + 10x - 7 = 0$

   The solution set is { _____ }.
   (Use a comma to separate answers as needed.)

2. Use factoring to solve the quadratic equation. Check by substitution or by using a graphing utility and identifying x-intercepts.

   $4x^2 = 11x + 45$

   The solution set is { _____ }.
   (Use commas to separate answers as needed. Type repeated roots only once.)

3. Solve for x using the quadratic formula.

   $x^2 - 2x + 26 = 0$

   The solution set is { _____ }.
   (Type an exact answer, using radicals as needed. Express complex numbers in terms of i. Use a comma to separate answers as needed.)

4. Solve the following equation.

   $4x^2 - 16x + 16 = 0$

   The solution set is { _____ }.
   (Use a comma to separate answers as needed.)

5. Solve the given radical equation. Check all proposed solutions.

   $\sqrt{3x + 28} = x + 6$

   Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

   ○ A. The solution set is { _____ }.
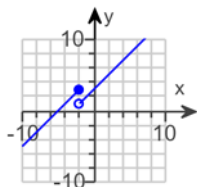   (Use a comma to separate answers as needed.)
   ○ B. There is no solution.

6. The domain of the piecewise function is $(-\infty, \infty)$.
   a. Graph the function.
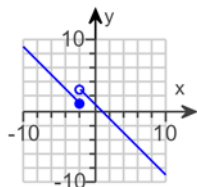   **b.** Use your graph to determine the function's range.

   $$f(x) = \begin{cases} x + 1 & \text{if } x < -2 \\ x - 1 & \text{if } x \geq -2 \end{cases}$$

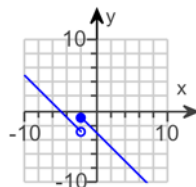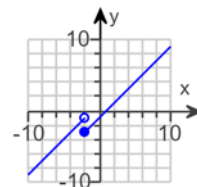   **a.** Choose the correct graph below.

   ○ **A.**   ○ **B.**   ○ **C.**   ○ **D.**

   b. The range of f(x) is [_____]. (Type your answer in interval notation.)

7. Find the difference quotient of f; that is, find $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$, for the following function. Be sure to simplify.

   $$f(x) = x^2 - 6x + 6$$

   $\dfrac{f(x+h) - f(x)}{h} = $ [_____] (Simplify your answer.)

8. Find the domain of the function.

   $$f(x) = \sqrt{20 - 5x}$$

   What is the domain of f?

   [_____] (Type your answer in interval notation.)

9.
First find f + g, f − g, fg and $\dfrac{f}{g}$. Then determine the domain for each function.

$f(x) = 4x^2 + 15x − 54$, $g(x) = x + 6$

(f + g)(x) = [_____] (Simplify your answer.)

What is the domain of f + g?

○ $[0, \infty)$
○ $(3, \infty)$
○ $(-\infty, 3) \cup (3, \infty)$
○ $(-\infty, \infty)$

(f − g)(x) = [_____] (Simplify your answer.)

What is the domain of f − g?

○ $[0, \infty)$
○ $\left(-\infty, \dfrac{18}{5}\right) \cup \left(\dfrac{18}{5}, \infty\right)$
○ $(3, \infty)$
○ $(-\infty, \infty)$

(fg)(x) = [_____]

What is the domain of fg?

○ $(-\infty, \infty)$
○ $\left(\dfrac{30}{7}, \infty\right)$
○ $(-\infty, -6) \cup (-6, \infty)$
○ $\left(-\infty, \dfrac{30}{7}\right) \cup \left(\dfrac{30}{7}, \infty\right)$

$\left(\dfrac{f}{g}\right)(x) = $ [_____] (Simplify your answer.)

What is the domain of $\dfrac{f}{g}$?

○ $(-\infty, -6) \cup (-6, \infty)$
○ $(-6, \infty)$
○ $[0, \infty)$
○ $(-\infty, \infty)$

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10.   For $f(x) = 5 - x$ and $g(x) = 3x^2 + x + 6$, find the following functions.

    a. $(f \circ g)(x)$;   **b.** $(g \circ f)(x)$;   **c.** $(f \circ g)(2)$;   **d.** $(g \circ f)(2)$

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    a. $(f \circ g)(x) = $ [   ]
    (Simplify your answer.)

    b. $(g \circ f)(x) = $ [   ]
    (Simplify your answer.)

    c. $(f \circ g)(2) = $ [   ]

    d. $(g \circ f)(2) = $ [   ]

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11.   Find the distance between the pair of points.

      (6,7) and (3,3)

The distance between the points is [   ] units.
(Round to two decimal places as needed.)

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12.   Find the midpoint of the line segment with the given endpoints.

      (6,2) and (8,4)

The midpoint of the segment is [   ].
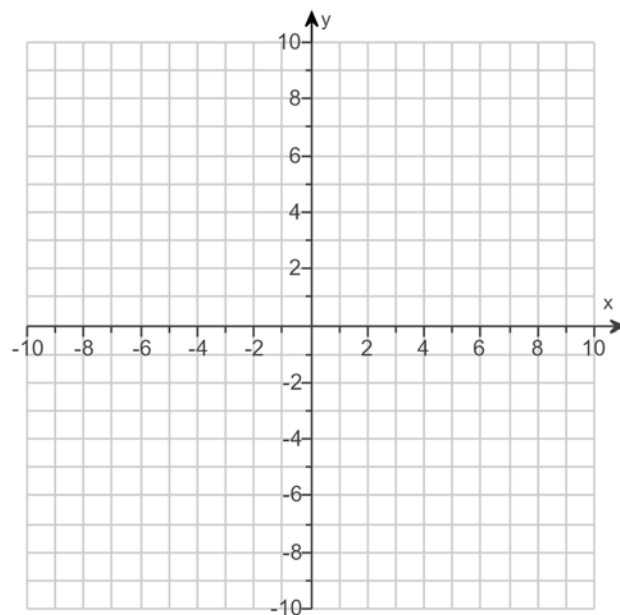(Type an ordered pair.)

13.

Complete the square and write the equation of the circle in standard form. Then determine the center and radius of the circle to graph the equation.

$x^2 + y^2 + 2x + 8y - 8 = 0$

The equation in standard form is [   ].
(Simplify your answer.)

Use the graphing tool to graph the circle.

14.

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation for the parabola's axis of symmetry. Use the parabola to identify the function's domain and range.
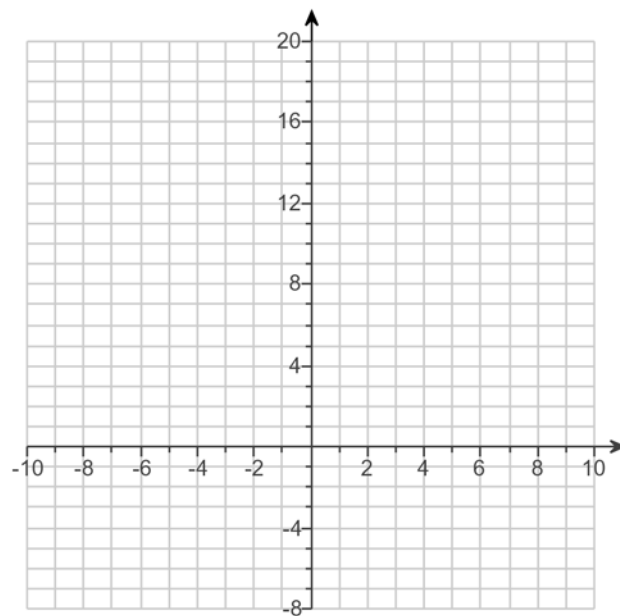
$$f(x) = 2(x + 1)^2 - 1$$

Use the graphing tool to graph the equation. Use the vertex and the y-intercept when drawing the graph.

The axis of symmetry is [   ].
(Type an equation. Simplify your answer)

The domain of f is [   ].
(Type your answer in interval notation.)

The range of f is [   ].
(Type your answer in interval notation.)

15.

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation of the parabola's axis of symmetry. Use the graph to determine the function's domain and range.
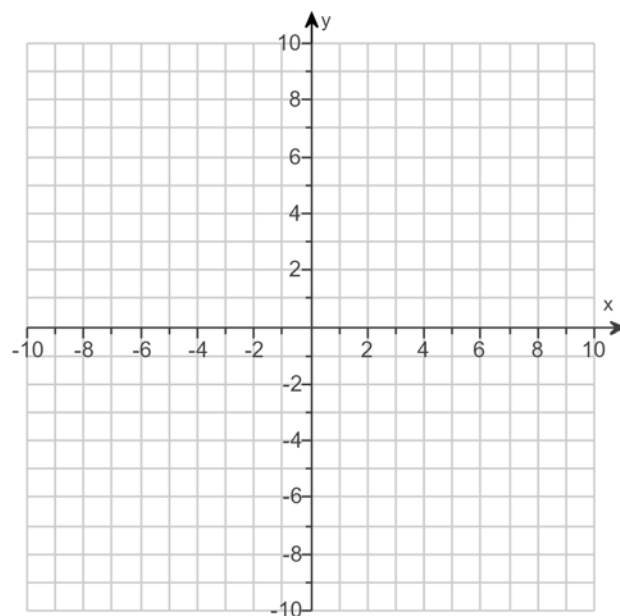
$$f(x) = x^2 + 4x + 3$$

Use the graphing tool to graph the equation. Use the vertex and one of the intercepts when drawing the graph.

The axis of symmetry is [   ].
(Type an equation.)

The domain of f is [   ].
(Type your answer in interval notation.)

The range of f is [   ].
(Type your answer in interval notation.)

16.

Use the vertex and intercepts to sketch the graph of the quadratic function. Give the equation of the parabola's axis of symmetry. Use the graph to determine the domain and range of the function.
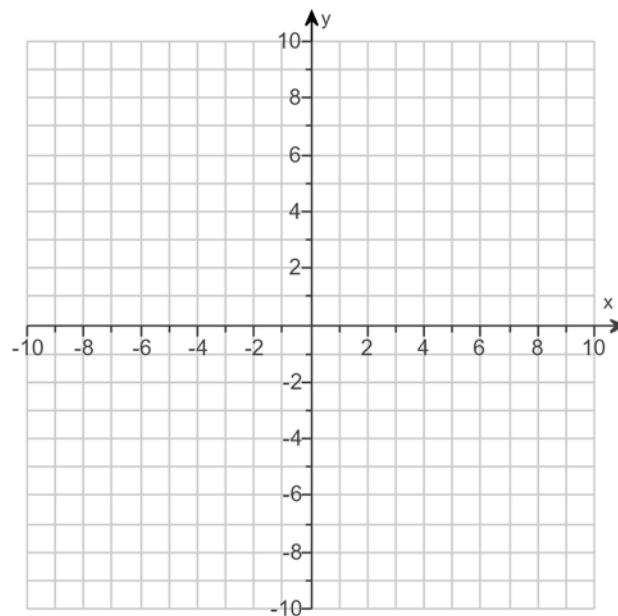
$$f(x) = 8x - x^2 - 7$$

Use the graphing tool to graph the equation. Use the vertex and one of the intercepts to draw the graph.

The axis of symmetry is [   ].
(Type an equation.)

The domain of the function is [   ].
(Type your answer in interval notation.)

The range of the function is [   ].
(Type your answer in interval notation.)

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17. Solve the equation $x^3 - 5x^2 + 2x + 8 = 0$ given that -1 is a zero of $f(x) = x^3 - 5x^2 + 2x + 8$.

The solution set is { [   ] }. (Use a comma to separate answers as needed.)

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18. The following function is given.

$$f(x) = 3x^3 - 7x^2 - 75x + 175$$

a. List all rational zeros that are possible according to the Rational Zero Theorem. Choose the correct answer below.

○ **A.** $\pm 1, \pm 5, \pm 25, \pm 7, \pm 35, \pm 175, \pm \dfrac{1}{3}, \pm \dfrac{5}{3}, \pm \dfrac{25}{3}, \pm \dfrac{7}{3}, \pm \dfrac{35}{3}, \pm \dfrac{175}{3}$

○ **B.** $\pm 1, \pm 3, \pm \dfrac{1}{5}, \pm \dfrac{3}{5}, \pm \dfrac{1}{25}, \pm \dfrac{3}{25}, \pm \dfrac{1}{7}, \pm \dfrac{3}{7}, \pm \dfrac{1}{35}, \pm \dfrac{3}{35}, \pm \dfrac{1}{175}, \pm \dfrac{3}{175}$

○ **C.** $\pm 1, \pm 3, \pm \dfrac{1}{5}, \pm \dfrac{3}{5}, \pm \dfrac{1}{10}, \pm \dfrac{3}{10}, \pm \dfrac{1}{7}, \pm \dfrac{3}{7}, \pm \dfrac{1}{35}, \pm \dfrac{3}{35}, \pm \dfrac{1}{175}, \pm \dfrac{3}{175}$

○ **D.** $\pm 1, \pm 5, \pm 10, \pm 7, \pm 35, \pm 175, \pm \dfrac{1}{3}, \pm \dfrac{5}{3}, \pm \dfrac{10}{3}, \pm \dfrac{7}{3}, \pm \dfrac{35}{3}, \pm \dfrac{175}{3}$

b. Use synthetic division to test several possible rational zeros in order to identify one actual zero.

One rational zero of the given function is [   ].
(Simplify your answer.)

c. Use the zero from part **(b)** to find all the zeros of the polynomial function.

The zeros of the function $f(x) = 3x^3 - 7x^2 - 75x + 175$ are [   ].
(Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed.)

19. The following equation is given.

$$x^3 - 3x^2 - 25x + 75 = 0$$

a. List all rational roots that are possible according to the Rational Zero Theorem.

[   ]

(Use a comma to separate answers as needed.)

b. Use synthetic division to test several possible rational roots in order to identify one actual root.

One rational root of the given equation is [   ].
(Simplify your answer.)

c. Use the root from part (**b.**) and solve the equation.

The solution set of $x^3 - 3x^2 - 25x + 75 = 0$ is {[   ]}.
(Simplify your answer. Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

20. An incomplete graph of the polynomial function $f(x) = -x^3 + 3x^2 + 13x - 15$ is shown on the right.
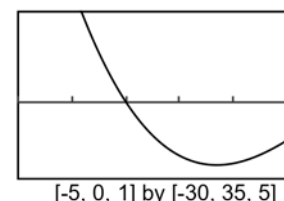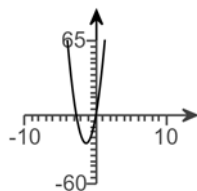
a. Find all zeros of the function.
b. Without using a graphing utility, draw a complete graph of the function.

[-5, 0, 1] by [-30, 35, 5]

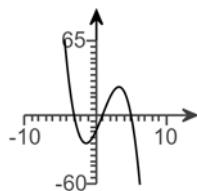**a.** The zeros are [   ].
(Use a comma to separate answers as needed.)

b. Choose the correct graph of the function below. The scale for each graph is $[-10, 10, 1]$ by $[-60, 65, 5]$.
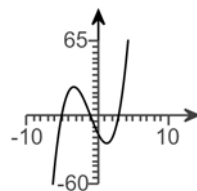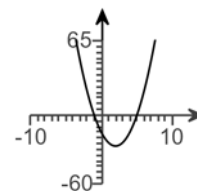
○ **A.**   ○ **B.**   ○ **C.**   ○ **D.**

21. Fill in the blank so that the resulting statement is true.

Based on the synthetic division shown below, the equation of the slant asymptote of $f(x) = \dfrac{6x^2 - 5x + 6}{x - 2}$ is _____.

$$
\begin{array}{r|rrr}
2 & 6 & -5 & 6 \\
 &   & 12 & 14 \\
\hline
 & 6 & 7 & 20 \\
\end{array}
$$

Based on the synthetic division shown, the equation of the slant asymptote of $f(x) = \dfrac{6x^2 - 5x + 6}{x - 2}$ is [   ]. (Type an equation.)

22. Find the vertical asymptotes, if any, and the values of x corresponding to holes, if any, of the graph of the rational function.

$$f(x) = \frac{x - 7}{x^2 - 9x + 14}$$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice. (Type an integer or a fraction. Use a comma to separate answers as needed.)

- ○ **A.** Hole(s) at x = _____
- ○ **B.** Vertical asymptote(s) at x = _____ and hole(s) at x = _____
- ○ **C.** Vertical asymptote(s) at x = _____
- ○ **D.** There are no discontinuities.

23. Find the horizontal asymptote, if any, of the graph of the rational function.

$$f(x) = \frac{15x}{4x^2 + 1}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ○ **A.** The horizontal asymptote is _____ . (Type an equation.)
- ○ **B.** There is no horizontal asymptote.

24. Find the horizontal asymptote, if any, of the graph of the rational function.

$$g(x) = \frac{15x^2}{5x^2 + 4}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ○ **A.** The horizontal asymptote is _____ . (Type an equation.)
- ○ **B.** There is no horizontal asymptote.

25. Find the domain of the logarithmic function.

$$f(x) = \log(14 - x)$$

The domain of $f(x) = \log(14 - x)$ is [_____] .
(Type your answer in interval notation.)

26. Use properties of logarithms to expand the logarithmic expression as much as possible. Evaluate logarithmic expressions without using a calculator if possible.

$$\log_b\left(\frac{x^2 y}{z^9}\right)$$

$$\log_b\left(\frac{x^2 y}{z^9}\right) = [\quad\quad\quad]$$

27. Use properties of logarithms to expand the logarithmic expression as much as possible. Evaluate logarithmic expressions without using a calculator if possible.

$$\ln\left[\frac{x^2\sqrt{x^2+2}}{(x+2)^6}\right]$$

$\ln\left[\dfrac{x^2\sqrt{x^2+2}}{(x+2)^6}\right] = $ [_____]

28. Solve the following exponential equation by expressing each side as a power of the same base and then equating exponents.

$$4^{x+2} = 16^{x-4}$$

The solution set is { [_____] }.

29. Solve the exponential equation. Express the solution in terms of natural logarithms. Then use a calculator to obtain a decimal approximation for the solution.

$$3^{(x-2)} = 469$$

What is the solution in terms of natural logarithms?

The solution set is { [_____] }.
(Use a comma to separate answers as needed. Simplify your answer. Use integers or fractions for any numbers in the expression.)

What is the decimal approximation for the solution?

The solution set is { [_____] }.
(Use a comma to separate answers as needed. Round to two decimal places as needed.)

30. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give an exact answer.

$$\log_5 x + \log_5(4x - 1) = 1$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** The solution set is { [_____] }. (Type an exact answer in simplified form.)
○ **B.** There is no solution.

31. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$$\log_4(x - 4) + \log_4(x + 56) = 4$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ **A.** The solution set is { [_____] }.
(Simplify your answer. Use a comma to separate answers as needed.)
○ **B.** There is no solution.

32. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$\log_6(x + 25) - \log_6(x - 10) = 2$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{$ _____ $\}$.
  (Simplify your answer. Use a comma to separate answers as needed.)
- **B.** There is no solution.

33. Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expressions. Give the exact answer.

$\log x + \log(x + 4) = \log 45$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The solution set is $\{$ _____ $\}$.
  (Simplify your answer. Use a comma to separate answers as needed.)
- **B.** There is no solution.

34. Complete the table for a savings account subject to 4 compoundings yearly.

$$\left[A = P\left(1 + \frac{r}{n}\right)^{nt} \right]$$

Amounted Invested	Number of Compounding Periods	Annual Interest Rate	Accumulated Amount	Time t in Years
$12,500	4	6.25%	$15,000	?

Let A represent the accumulated amount, P the amount invested, n the number of compounding periods, r the annual interest rate, and t the time. Find the time, t.

t = [_____] years

(Do not round until the final answer. Then round to one decimal place as needed.)

35. Solve the given system of equations.

$x + y + 2z = -3$
$x + y + 3z = -4$
$x + 3y + 7z = -14$

Select the correct choice below and fill in any answer boxes within your choice.

- **A.** There is one solution. The solution set is
  $\{($ _____ , _____ , _____ $)\}$. (Simplify your answers.)
- **B.** There are infinitely many solutions.
- **C.** There is no solution.

36.

Find the indicated sum. $\displaystyle\sum_{i=1}^{3} i(i+1)$

$\displaystyle\sum_{i=1}^{3} i(i+1) =$ [_____] (Simplify your answer.)

37.   Use the binomial theorem to expand the binomial.

$(4x-1)^3$

$(4x-1)^3 =$ [_____] (Simplify your answer.)

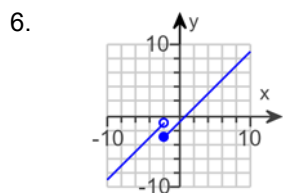1. $\dfrac{1}{2}, -\dfrac{7}{4}$

2. $-\dfrac{9}{4}, 5$

3. $1 + 5i, 1 - 5i$

4. 2

5. A. The solution set is $\{ \boxed{\quad -1 \quad} \}$.(Use a comma to separate answers as needed.)

6.

   D.

   $(-\infty, \infty)$

7. $2x + h - 6$

8. $(-\infty, 4]$

9. $4x^2 + 16x - 48$

   $(-\infty, \infty)$

   $4x^2 + 14x - 60$

   $(-\infty, \infty)$

   $4x^3 + 39x^2 + 36x - 324$

   $(-\infty, \infty)$

   $4x - 9$

   $(-\infty, -6) \cup (-6, \infty)$
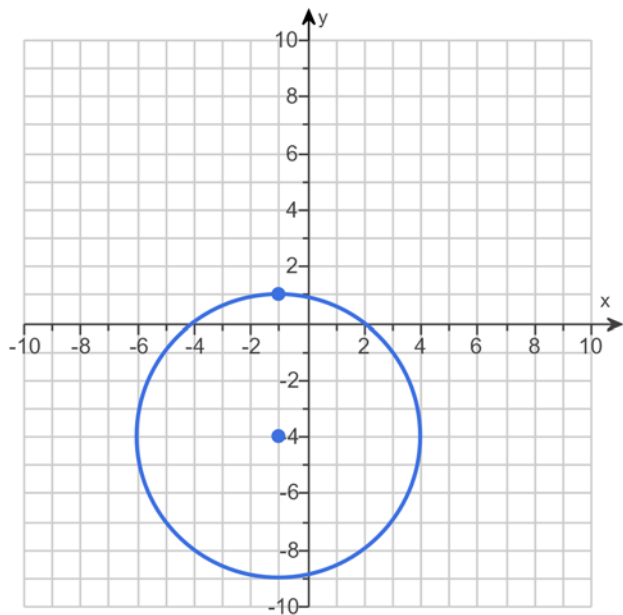
10. $-3x^2 - x - 1$
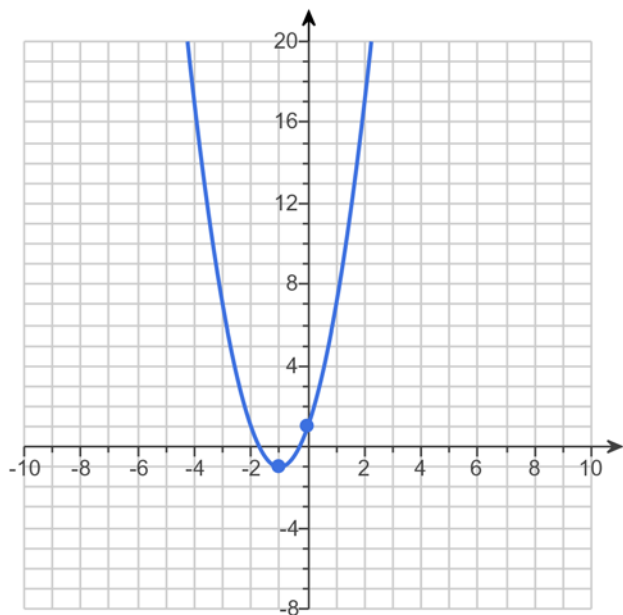
   $3x^2 - 31x + 86$

   -15

   36

11. 5

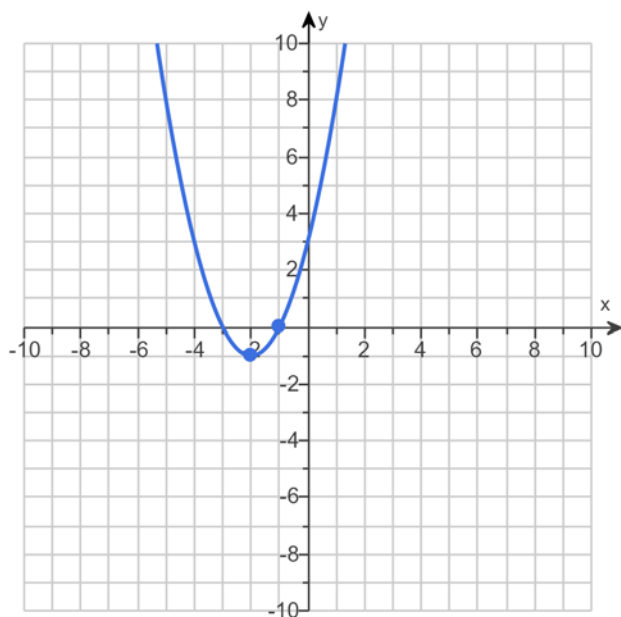12. (7,3)

13. $(x + 1)^2 + (y + 4)^2 = 25$

14.

x = − 1

(− ∞,∞)

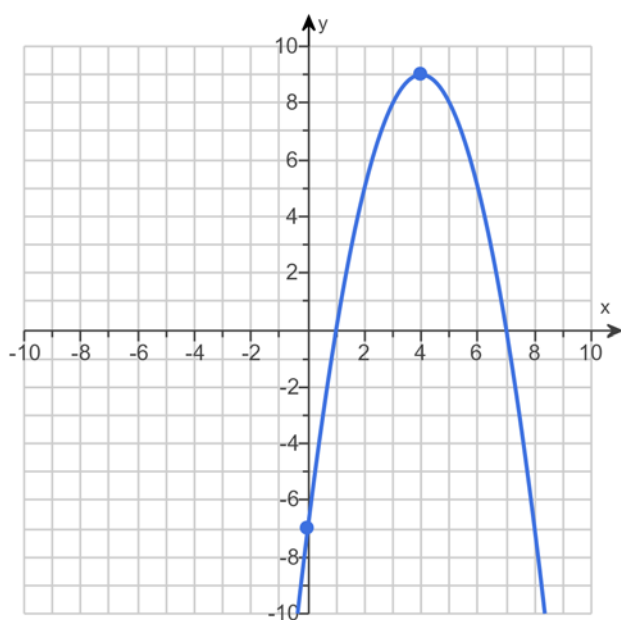[− 1,∞)

15.

$x = -2$

$(-\infty, \infty)$

$[-1, \infty)$

16.

$x = 4$

$(-\infty, \infty)$

$(-\infty, 9]$

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17. $-1, 4, 2$

18.
A. $\pm 1, \pm 5, \pm 25, \pm 7, \pm 35, \pm 175, \pm \dfrac{1}{3}, \pm \dfrac{5}{3}, \pm \dfrac{25}{3}, \pm \dfrac{7}{3}, \pm \dfrac{35}{3}, \pm \dfrac{175}{3}$
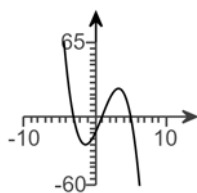
$\dfrac{7}{3}$

$\dfrac{7}{3}, 5, -5$

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19. $1, -1, 5, -5, 75, -75, 3, -3, 15, -15, 25, -25$

3

$3, 5, -5$

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20. $-3, 5, 1$

B.

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21. $y = 6x + 7$

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22. B. Vertical asymptote(s) at x = [**2**] and hole(s) at x = [**7**]

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23. A. The horizontal asymptote is [**y = 0**]. (Type an equation.)

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24. A. The horizontal asymptote is [**y = 3**]. (Type an equation.)

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25. $(-\infty, 14)$

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26. $2\log_b x + \log_b y - 9\log_b z$

27.
$2\ln x + \dfrac{1}{2}\ln\left(x^2 + 2\right) - 6\ln(x + 2)$

---

28. 10

29. $\dfrac{\ln 469}{\ln 3} + 2$

7.60

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30. A. The solution set is $\left\{ \boxed{\dfrac{5}{4}} \right\}$. (Type an exact answer in simplified form.)

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31. A. The solution set is $\left\{ \boxed{8} \right\}$. (Simplify your answer. Use a comma to separate answers as needed.)

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32. A. The solution set is $\left\{ \boxed{11} \right\}$. (Simplify your answer. Use a comma to separate answers as needed.)

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33. A. The solution set is $\left\{ \boxed{5} \right\}$. (Simplify your answer. Use a comma to separate answers as needed.)

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34. 2.9

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35. A.

There is one solution. The solution set is $\left\{ \left(\boxed{2}, \boxed{-3}, \boxed{-1} \right) \right\}$. (Simplify your answers.)

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36. 20

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37. $64x^3 - 48x^2 + 12x - 1$