Prealgebra Textbook
Solutions Manual

Second Edition

Department of Mathematics
College of the Redwoods

2012-2013

ii

Second Edition: 2012-2013

Contents

Second Edition: 2012-2013

The Arithmetic of Numbers

1.1   An Introduction to the Integers

1. Arrange the numbers 2, 8, and 4 on a number line.

Thus, listing the numbers in order from smallest to largest, 2, 4, and 8.

3. Arrange the numbers 1, 8, and 2 on a number line.

Thus, listing the numbers in order from smallest to largest, 1, 2, and 8.

5. Arrange the numbers 0, 4, and 1 on a number line.

Thus, listing the numbers in order from smallest to largest, 0, 1, and 4.

7. Arrange the numbers 4, 9, and 6 on a number line.

Thus, listing the numbers in order from smallest to largest, 4, 6, and 9.

9. Arrange the numbers 0, 7, and 4 on a number line.

Thus, listing the numbers in order from smallest to largest, 0, 4, and 7.

11. Arrange the numbers 1, 6, and 5 on a number line.

Thus, listing the numbers in order from smallest to largest, 1, 5, and 6.

13. On the number line, 3 lies to the left of 8.

Therefore, $3 < 8$.

15. On the number line, 59 lies to the right of 24.

Therefore, $59 > 24$.

17. On the number line, 0 lies to the left of 74.

Therefore, $0 < 74$.

19. On the number line, 1 lies to the left of 81.

Therefore, $1 < 81$.

21. On the number line, 43 lies to the right of 1.

Therefore, $43 > 1$.

23. On the number line, 43 lies to the right of 28.

    Therefore, $43 > 28$.

25. The thousands column is the fourth column from the right. In the number 2,054,867,372, the digit in the thousands column is 7.

27. The hundred thousands column is the sixth column from the right. In the number 8,311,900,272, the digit in the hundred thousands column is 9.

29. The hundred millions column is the ninth column from the right. In the number 9,482,616,000, the digit in the hundred millions column is 4.

31. The ten millions column is the eighth column from the right. In the number 5,840,596,473, the digit in the ten millions column is 4.

33. The hundred millions column is the ninth column from the right. In the number 5,577,422,501, the digit in the hundred millions column is 5.

35. The tens column is the second column from the right. In the number 2,461,717,362, the digit in the tens column is 6.

37. Rounding to the nearest thousand. Identify the rounding digit and the test digit.

Because the test digit is greater than or equal to five, add one to the rounding digit, then make each digit to the right of the rounding digit a zero. Thus,

$$93,857 \approx 94000.$$

39. Rounding to the nearest ten. Identify the rounding digit and the test digit.

Because the test digit is greater than or equal to five, add one to the rounding digit, then make each digit to the right of the rounding digit a zero. Thus,

$$9,725 \approx 9730.$$

41. Rounding to the nearest hundred. Identify the rounding digit and the test digit.

Because the test digit is less than five, leave the rounding digit alone, then make each digit to the right of the rounding digit a zero. Thus,

$$58,739 \approx 58700.$$

43. Rounding to the nearest ten. Identify the rounding digit and the test digit.

Because the test digit is greater than or equal to five, add one to the rounding digit, then make each digit to the right of the rounding digit a zero. Thus,

$$2,358 \approx 2360.$$

45. Rounding to the nearest thousand. Identify the rounding digit and the test digit.

Because the test digit is greater than or equal to five, add one to the rounding digit, then make each digit to the right of the rounding digit a zero. Thus,

$$39,756 \approx 40000.$$

47. Rounding to the nearest ten. Identify the rounding digit and the test digit.

Because the test digit is less than five, leave the rounding digit the same, then make each digit to the right of the rounding digit a zero. Thus,

$$5,894 \approx 5890.$$

49. Rounding to the nearest hundred. Identify the rounding digit and the test digit.

Because the test digit is less than five, leave the rounding digit alone, then make each digit to the right of the rounding digit a zero. Thus,

$$56,123 \approx 56100.$$

51. Rounding to the nearest ten. Identify the rounding digit and the test digit.

Because the test digit is less than five, leave the rounding digit the same, then make each digit to the right of the rounding digit a zero. Thus,

$$5,483 \approx 5480.$$

53. Mark the rounding and test digits.

$$304, \boxed{0} \; \boxed{5} \; 9{,}724$$

Test digit

Rounding digit

The test digit is greater than or equal to 5. The "Rules for Rounding" require that we add 1 to the rounding digit, then make all digits to the right of the rounding digit zeros. Thus, rounded to the nearest hundred thousand,

$$304,059,724 \approx 304,100,000.$$

55. Mark the rounding and test digits.

$$1 \; \boxed{2} \; \boxed{9} \; {,}000$$

Test digit

Rounding digit

The test digit is greater than or equal to 5. The "Rules for Rounding" require that we add 1 to the rounding digit, then make all digits to the right of the rounding digit zeros. Thus, rounded to the nearest ten thousand,

$$129,000 \approx 130,000.$$

57. We note the week of June 1, or 6/15, in the bar chart.

It appears that on June 1, that is, 6/15, a gallon of regular gasoline cost approximately 252 cents.

59.

61. a) Note that the first column in the following bar chart represents the year 2003. Estimate the height of this bar to be approximately 21 according to the provided vertical scale at the left of the bar chart. Hence, the number of pirate attacks in 2003 was approximately 21.

b) In like manner, the last vertical bar represents the year 2008. The height of this bar is approximately 111. Thus, the number of pirate attacks in 2008 was approximately 111.

63. We've circled the point that indicates the number of red M and M's in the bowl.

It appears that there were 9 red M and M's in the bowl.

65. Here is the line plot.

1.2   Order of Operations

1. Start at the number 0, then draw an arrow 3 units to the right, as shown below. Draw a second arrow of length 2, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.

Start   End

$$3 \longrightarrow \quad 2 \longrightarrow$$

0   1   2   3   4   5   6   7   8   9

The number line shows that $3 + 2 = 5$.

3. Start at the number 0, then draw an arrow 3 units to the right, as shown below. Draw a second arrow of length 4, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.

Start   End

$$3 \longrightarrow \quad 4 \longrightarrow$$

0   1   2   3   4   5   6   7   8   9

The number line shows that $3 + 4 = 7$.

5. Start at the number 0, then draw an arrow 4 units to the right, as shown below. Draw a second arrow of length 2, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.

Start   End

$$4 \longrightarrow \quad 2 \longrightarrow$$

0   1   2   3   4   5   6   7   8   9

The number line shows that $4 + 2 = 6$.

7. Start at the number 0, then draw an arrow 2 units to the right, as shown below. Draw a second arrow of length 5, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.

Start   End

$$2 \longrightarrow \quad 5 \longrightarrow$$

0   1   2   3   4   5   6   7   8   9

The number line shows that $2 + 5 = 7$.

9. Start at the number 0, then draw an arrow 4 units to the right, as shown below. Draw a second arrow of length 4, starting at the end of the first arrow. Mark the sum as a shaded dot on the number line.

Start   End

|⊢———— 4 ————→|⊢———— 4 ————→|

0   1   2   3   4   5   6   7   8   9

The number line shows that $4 + 4 = 8$.

11. Because the given identity

$$28 + 0 = 28$$

has the form

$$a + 0 = a,$$

this is an example of the additive identity property of addition.

13. Because the given identity

$$24 + 0 = 24$$

has the form

$$a + 0 = a,$$

this is an example of the additive identity property of addition.

15. Because the given identity

$$(51 + 66) + 88 = 51 + (66 + 88)$$

has the form

$$(a + b) + c = a + (b + c),$$

this is an example of the associative property of addition.

17. Because the given identity

$$64 + 39 = 39 + 64$$

has the form

$$a + b = b + a,$$

this is an example of the commutative property of addition.

19. Because the given identity

$$(70 + 27) + 52 = 70 + (27 + 52)$$

has the form

$$(a + b) + c = a + (b + c),$$

this is an example of the associative property of addition.

21. Because the given identity

$$79 + 0 = 79$$

has the form

$$a + 0 = a,$$

this is an example of the additive identity property of addition.

23. Because the given identity

$$10 + 94 = 94 + 10$$

has the form

$$a + b = b + a,$$

this is an example of the commutative property of addition.

25. Because the given identity

$$47 + 26 = 26 + 47$$

has the form

$$a + b = b + a,$$

this is an example of the commutative property of addition.

27. Because the given identity

$$(61 + 53) + 29 = 61 + (53 + 29)$$

has the form

$$(a + b) + c = a + (b + c),$$

this is an example of the associative property of addition.

29. Start at the number 0, then draw an arrow 8 units to the right, as shown below. Draw a second arrow of length 2, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.

The number line shows that $8 - 2 = 6$.

31. Start at the number 0, then draw an arrow 7 units to the right, as shown below. Draw a second arrow of length 2, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.

The number line shows that $7 - 2 = 5$.

33. Start at the number 0, then draw an arrow 7 units to the right, as shown below. Draw a second arrow of length 4, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.

The number line shows that $7 - 4 = 3$.

35. Start at the number 0, then draw an arrow 9 units to the right, as shown below. Draw a second arrow of length 4, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.

End

Start

$\longleftarrow$ 4 $\longrightarrow$

$\vdash$ 9 $\longrightarrow$

0   1   2   3   4   5   6   7   8   9

The number line shows that $9 - 4 = 5$.

37. Start at the number 0, then draw an arrow 8 units to the right, as shown below. Draw a second arrow of length 5, starting at the end of the first arrow, but pointing to the left. Mark the difference as a shaded dot on the number line.

End

Start

$\longleftarrow$ 5 $\longrightarrow$

$\vdash$ 8 $\longrightarrow$

0   1   2   3   4   5   6   7   8   9

The number line shows that $8 - 5 = 3$.

39. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$16 - 8 + 2 = 8 + 2 \qquad\qquad \textcolor{red}{\text{Subtact: } 16 - 8 = 8.}$$
$$= 10 \qquad\qquad \textcolor{red}{\text{Add: } 8 + 2 = 10.}$$

41. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$20 - 5 + 14 = 15 + 14 \qquad\qquad \textcolor{red}{\text{Subtact: } 20 - 5 = 15.}$$
$$= 29 \qquad\qquad \textcolor{red}{\text{Add: } 15 + 14 = 29.}$$

43. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$15 - 2 + 5 = 13 + 5 \qquad \text{Subtact: } 15 - 2 = 13.$$
$$= 18 \qquad \text{Add: } 13 + 5 = 18.$$

45. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$12 - 5 + 4 = 7 + 4 \qquad \text{Subtact: } 12 - 5 = 7.$$
$$= 11 \qquad \text{Add: } 7 + 4 = 11.$$

47. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$12 - 6 + 4 = 6 + 4 \qquad \text{Subtact: } 12 - 6 = 6.$$
$$= 10 \qquad \text{Add: } 6 + 4 = 10.$$

49. Addition has no priority over subtraction; vice-versa, subtraction has no priority over addition. It is important that you perform additions and subtractions in the order that they appear, as you move from left to right through the expression.

$$15 - 5 + 8 = 10 + 8 \qquad \text{Subtact: } 15 - 5 = 10.$$
$$= 18 \qquad \text{Add: } 10 + 8 = 18.$$

51. A rectangle is a quadrilateral (4 sides) with four right angles. The opposite sides of the rectangle are equal.

To find the perimeter of the rectangle, add up the sides. Therefore, the perimeter is $P = 7 + 9 + 7 + 9 = 32$ in.

53. A rectangle is a quadrilateral (4 sides) with four right angles. The opposite sides of the rectangle are equal.

9 in

8 in   8 in

9 in

To find the perimeter of the rectangle, add up the sides. Therefore, the perimeter is $P = 8 + 9 + 8 + 9 = 34$ in.

55. A rectangle is a quadrilateral (4 sides) with four right angles. The opposite sides of the rectangle are equal.

6 cm

4 cm   4 cm

6 cm

To find the perimeter of the rectangle, add up the sides. Therefore, the perimeter is $P = 4 + 6 + 4 + 6 = 20$ cm.

57. A rectangle is a quadrilateral (4 sides) with four right angles. The opposite sides of the rectangle are equal.

7 cm

4 cm   4 cm

7 cm

To find the perimeter of the rectangle, add up the sides. Therefore, the perimeter is $P = 4 + 7 + 4 + 7 = 22$ cm.

59. A square is a rectangle with 4 equal sides. Thus, each side of the square measures 25 cm.

25 cm

25 cm   25 cm

25 cm

To find the perimeter of the square, add up the sides. Therefore, the perimeter is $P = 25 + 25 + 25 + 25 = 100$ cm.

61. A square is a rectangle with 4 equal sides. Thus, each side of the square measures 16 cm.

16 cm

16 cm   16 cm

16 cm

To find the perimeter of the square, add up the sides. Therefore, the perimeter is $P = 16 + 16 + 16 + 16 = 64$ cm.

63. A square is a rectangle with 4 equal sides. Thus, each side of the square measures 18 in.

To find the perimeter of the square, add up the sides. Therefore, the perimeter is $P = 18 + 18 + 18 + 18 = 72$ in.

65. A square is a rectangle with 4 equal sides. Thus, each side of the square measures 3 in.

To find the perimeter of the square, add up the sides. Therefore, the perimeter is $P = 3 + 3 + 3 + 3 = 12$ in.

67. Align the numbers vertically, then add, starting at the furthest column to the right.

$$
\begin{array}{r}
3005 \\
+5217 \\
\hline
8222
\end{array}
$$

Thus, $3005 + 5217 = 8222$.

69. Align the numbers vertically, then add, starting at the furthest column to the right.

$$
\begin{array}{r}
575 \\
354 \\
+759 \\
\hline
1688
\end{array}
$$

Thus, $575 + 354 + 759 = 1688$.

71. Align the numbers vertically, then add, starting at the furthest column to the right. Add the grouped terms first.

$$
\begin{array}{r}
520 \\
+575 \\
\hline
1095
\end{array}
$$

and

$$
\begin{array}{r}
472 \\
+1095 \\
\hline
1567
\end{array}
$$

Thus, $472 + (520 + 575) = 1567$.

73. Align the numbers vertically, then add, starting at the furthest column to the right. Add the grouped terms first.

$$
\begin{array}{r}
764 \\
+690 \\
\hline
1454
\end{array}
$$

and

$$
\begin{array}{r}
274 \\
+1454 \\
\hline
1728
\end{array}
$$

Thus, $274 + (764 + 690) = 1728$.

75. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 8583 \\ +592 \\ \hline 9175 \end{array}$$

Thus, $8583 + 592 = 9175$.

77. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 899 \\ 528 \\ +116 \\ \hline 1543 \end{array}$$

Thus, $899 + 528 + 116 = 1543$.

79. Align the numbers vertically, then add, starting at the furthest column to the right. Add the grouped terms first.

$$\begin{array}{r} 466 \\ +744 \\ \hline 1210 \end{array}$$

and

$$\begin{array}{r} 1210 \\ +517 \\ \hline 1727 \end{array}$$

Thus, $(466 + 744) + 517 = 1727$.

81. Align the numbers vertically, then add, starting at the furthest column to the right.

$$\begin{array}{r} 563 \\ 298 \\ 611 \\ +828 \\ \hline 2300 \end{array}$$

Thus, $563 + 298 + 611 + 828 = 2300$.

83. Align the numbers vertically, then add, starting at the furthest column to the right.

$$
\begin{array}{r}
607 \\
29 \\
270 \\
+245 \\
\hline
1151
\end{array}
$$

Thus, $607 + 29 + 270 + 245 = 1151$.

85. Align the numbers vertically, then add, starting at the furthest column to the right. Add the grouped terms first.

$$
\begin{array}{r}
86 \\
+557 \\
\hline
643
\end{array}
$$

and

$$
\begin{array}{r}
643 \\
+80 \\
\hline
723
\end{array}
$$

Thus, $(86 + 557) + 80 = 723$.

87. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$
\begin{array}{r}
3493 \\
-2034 \\
\hline
1459
\end{array}
$$

and

$$
\begin{array}{r}
1459 \\
-227 \\
\hline
1232
\end{array}
$$

Thus, $3493 - 2034 - 227 = 1232$.

89. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$
\begin{array}{r}
8338 \\
-7366 \\
\hline
972
\end{array}
$$

Thus, $8338 - 7366 = 972$.

91. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$
\begin{array}{r}
2974 \\
-2374 \\
\hline
600
\end{array}
$$

Thus, $2974 - 2374 = 600$.

93. Align the numbers vertically, then subtract, starting at the furthest column to the right. Subtract the grouped terms first.

$$
\begin{array}{r}
777 \\
-241 \\
\hline
536
\end{array}
$$

and

$$
\begin{array}{r}
3838 \\
-536 \\
\hline
3302
\end{array}
$$

Thus, $3838 - (777 - 241) = 3302$.

95. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$
\begin{array}{r}
5846 \\
-541 \\
\hline
5305
\end{array}
$$

and

$$
\begin{array}{r}
5305 \\
-4577 \\
\hline
728
\end{array}
$$

Thus, $5846 - 541 - 4577 = 728$.

97. Align the numbers vertically, then subtract, starting at the furthest column to the right. Subtract the grouped terms first.

$$\begin{array}{r} 2882 \\ -614 \\ \hline 2268 \end{array}$$

and

$$\begin{array}{r} 3084 \\ -2268 \\ \hline 816 \end{array}$$

Thus, $3084 - (2882 - 614) = 816$.

99. Align the numbers vertically, then subtract, starting at the furthest column to the right. Subtract the grouped terms first.

$$\begin{array}{r} 1265 \\ -251 \\ \hline 1014 \end{array}$$

and

$$\begin{array}{r} 2103 \\ -1014 \\ \hline 1089 \end{array}$$

Thus, $2103 - (1265 - 251) = 1089$.

101. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$\begin{array}{r} 9764 \\ -4837 \\ \hline 4927 \end{array}$$

and

$$\begin{array}{r} 4927 \\ -150 \\ \hline 4777 \end{array}$$

Thus, $9764 - 4837 - 150 = 4777$.

103. Align the numbers vertically, then subtract, starting at the furthest column to the right.

$$
\begin{array}{r}
7095 \\
-226 \\
\hline
6869
\end{array}
$$

Thus, $7095 - 226 = 6869$.

105. To find the total water subsidy, add the numbers 79 and 439. Your answer will be in millions of dollars.

$$79 + 439 = 518$$

Therefore, the total water subsidy is \$518 million.

107. To find how much more the R16 model is, subtract 2,279 from 3,017.

$$3,017 - 2,279 = 738$$

Therefore, the R16 model costs \$738 more.

109. To find the difference between aphelion and perihelion, subtract 147 from 152. Your answer will be in millions of kilometers.

$$152 - 147 = 5$$

Therefore, aphelion is 5 million kilometers further than perihelion.

111. To find how many degrees cooler the sunspots are, subtract 6,300 from 10000. Your answer will be in degrees Fahrenheit.

$$10,000 - 6,300 = 3,700$$

Therefore, sunspots are 3,700 degrees cooler than the surrounding surface of the sun.

113. Add the values for each region.

$$1650 + 450 + 250 + 400 + 500 + 350 = 3600$$

There are an estimated 3600 wild tigers worldwide.

115.

a) Bar chart.

b) Differences between consecutive exams are shown in the following table.

Exam	Difference
1-2	3
2-3	3
3-4	5
4-5	8
5-6	3

The following plot is a line plot of differences between scores on consecutive examinations.

The largest improvement was between Exam #4 and Exam #5, where
Emily improved by 8 points.

1.3   The Rational Numbers

1. Multiplication is equivalent to repeated addition. In this case,

$$2 \cdot 4 = \underbrace{4 + 4}_{2 \text{ times}}$$

Starting at zero, draw 2 arrows of length 4, connected tail to arrowhead and
pointing to the right, as shown in the following figure.

Therefore, $2 \cdot 4 = 8$.

3. Starting at zero, draw 4 arrows of length 2, connected tail to arrowhead
and pointing to the right, as shown in the following figure.

5. Because the identity

$$9 \cdot 8 = 8 \cdot 9$$

has the form

$$a \cdot b = b \cdot a,$$

this identity is an example of the commutative property of multiplication.

7. Because the identity
$$8 \cdot (5 \cdot 6) = (8 \cdot 5) \cdot 6$$
has the form
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$
this identity is an example of the associative property of multiplication.

9. Because the identity
$$6 \cdot 2 = 2 \cdot 6$$
has the form
$$a \cdot b = b \cdot a,$$
this identity is an example of the commutative property of multiplication.

11. Because the identity
$$3 \cdot (5 \cdot 9) = (3 \cdot 5) \cdot 9$$
has the form
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$
this identity is an example of the associative property of multiplication.

13. Because the identity
$$21 \cdot 1 = 21$$
has the form
$$a \cdot 1 = a,$$
this identity is an example of the multiplicative identity property.

15. Because the identity
$$13 \cdot 1 = 13$$
has the form
$$a \cdot 1 = a,$$
this identity is an example of the multiplicative identity property.

17. Use vertical format:

$$
\begin{array}{r}
78 \\
\times\ 3 \\
\hline
234
\end{array}
$$

Thus, $78 \cdot 3 = 234$.

19. Use vertical format:

$$\begin{array}{r} 907 \\ \times\ \ 6 \\ \hline 5442 \end{array}$$

Thus, $907 \cdot 6 = 5442$.

21. Use vertical format:

$$\begin{array}{r} 128 \\ \times\ \ 30 \\ \hline 3840 \end{array}$$

Thus, $128 \cdot 30 = 3840$.

23. Use vertical format:

$$\begin{array}{r} 799 \\ \times\ \ 60 \\ \hline 47940 \end{array}$$

Thus, $799 \cdot 60 = 47940$.

25. Use vertical format:

$$\begin{array}{r} 14 \\ \times\ \ 70 \\ \hline 980 \end{array}$$

Thus, $14 \cdot 70 = 980$.

27. Use vertical format:

$$\begin{array}{r} 34 \\ \times\ \ 90 \\ \hline 3060 \end{array}$$

Thus, $34 \cdot 90 = 3060$.

29. Use vertical format:

$$
\begin{array}{r}
237 \\
\times\ 54 \\
\hline
948 \\
1185 \\
\hline
12798
\end{array}
$$

Thus, $237 \cdot 54 = 12798$.

31. Use vertical format:

$$
\begin{array}{r}
691 \\
\times\ 12 \\
\hline
1382 \\
691 \\
\hline
8292
\end{array}
$$

Thus, $691 \cdot 12 = 8292$.

33. Use vertical format:

$$
\begin{array}{r}
955 \\
\times\ 89 \\
\hline
8595 \\
7640 \\
\hline
84995
\end{array}
$$

Thus, $955 \cdot 89 = 84995$.

35. Use vertical format:

$$
\begin{array}{r}
266 \\
\times\ 61 \\
\hline
266 \\
1596 \\
\hline
16226
\end{array}
$$

Thus, $266 \cdot 61 = 16226$.

37. Use vertical format:

$$
\begin{array}{r}
365 \\
\times\ 73 \\
\hline
1095 \\
2555\ \ \\
\hline
26645
\end{array}
$$

Thus, $365 \cdot 73 = 26645$.

39. Use vertical format:

$$
\begin{array}{r}
955 \\
\times\ 57 \\
\hline
6685 \\
4775\ \ \\
\hline
54435
\end{array}
$$

Thus, $955 \cdot 57 = 54435$.

41. Note that there are 8 rows and 8 columns in the array.

Multiply the number of rows by the number of columns to determine the number of objects in the array.

$$
\begin{aligned}
\text{Number of Objects} &= \text{Number of Rows} \cdot \text{Number of Columns} \\
&= 8 \cdot 8 \\
&= 64
\end{aligned}
$$

Therefore, there are 64 objects in the array.

43. Note that there are 7 rows and 8 columns in the array.

Multiply the number of rows by the number of columns to determine the number of objects in the array.

$$\text{Number of Objects} = \text{Number of Rows} \cdot \text{Number of Columns}$$
$$= 7 \cdot 8$$
$$= 56$$

Therefore, there are 56 objects in the array.

45. The formula for the area of a rectangle is

$$A = LW,$$

where L is the length and W is the width of the rectangle, respectively. Substitute $L = 50\,\text{in}$ and $W = 25\,\text{in}$ and simplify.

$$A = (50\,\text{in})(25\,\text{in}) \qquad \text{Substitute } L = 50\,\text{in and } W = 25\,\text{in.}$$
$$= 1250\,\text{in}^2 \qquad \text{Multiply: } 50 \cdot 25 = 1250.$$

Note also that $\text{in} \cdot \text{in} = \text{in}^2$. Hence, the area of the rectangle is $A = 1250$ square inches.

47. The formula for the area of a rectangle is

$$A = LW,$$

where L is the length and W is the width of the rectangle, respectively. Substitute $L = 47\,\text{in}$ and $W = 13\,\text{in}$ and simplify.

$$A = (47\,\text{in})(13\,\text{in}) \qquad \text{Substitute } L = 47\,\text{in and } W = 13\,\text{in.}$$
$$= 611\,\text{in}^2 \qquad \text{Multiply: } 47 \cdot 13 = 611.$$

Note also that $\text{in} \cdot \text{in} = \text{in}^2$. Hence, the area of the rectangle is $A = 611$ square inches.

49. To find the perimeter, find the sum of the four sides of the rectangle. Hence, the perimeter of the rectangle having length L and width W is

$$P = L + W + L + W.$$

Susbstitute $L = 25$ in and $W = 16$ in and simplify.

$$P = (25\,\text{in}) + (16\,\text{in}) + (25\,\text{in}) + (16\,\text{in}) \qquad L = 25\,\text{in and } W = 16\,\text{in.}$$
$$= 82\,\text{in} \qquad\qquad\qquad\qquad \text{Add: } 25 + 16 + 25 + 16 = 82.$$

Hence the perimeter is $P = 82$ inches.

51. To find the perimeter, find the sum of the four sides of the rectangle. Hence, the perimeter of the rectangle having length L and width W is

$$P = L + W + L + W.$$

Susbstitute $L = 30$ in and $W = 28$ in and simplify.

$$P = (30\,\text{in}) + (28\,\text{in}) + (30\,\text{in}) + (28\,\text{in}) \qquad L = 30\,\text{in and } W = 28\,\text{in.}$$
$$= 116\,\text{in} \qquad\qquad\qquad\qquad \text{Add: } 30 + 28 + 30 + 28 = 116.$$

Hence the perimeter is $P = 116$ inches.

53.
$$19 \cdot 50 = 950 \text{ cents} = \$9.50$$

55.
$$47 \cdot 15 = 705 \text{ dollars}$$

57.
$$24 \cdot 12 \cdot 12 = 3456 \text{ eggs}$$

59.
$$5000 \cdot 4 = 20000 \text{ kilograms}$$

61. The expressions
$$\frac{30}{5}, \quad 30 \div 5, \quad 5\overline{)30}$$
are identical. The expression
$$5 \div 30$$
differs from the remaining three.

63. The expressions
$$\frac{8}{2}, \quad 8 \div 2, \quad 2\overline{)8}$$
are identical. The expression
$$8\overline{)2}$$
differs from the remaining three.

65. The expressions
$$\frac{14}{2}, \quad 14 \div 2, \quad 2\overline{)14}$$
are identical. The expression
$$14\overline{)2}$$
differs from the remaining three.

67. The expressions
$$\frac{24}{3}, \quad 24 \div 3, \quad 3\overline{)24}$$
are identical. The expression
$$3 \div 24$$
differs from the remaining three.

69. When a is a nonzero whole number, $0 \div a = 0$. There are zero groups of 11 in zero. Hence,
$$0 \div 11 = 0.$$

71. Division by zero is undefined. Hence,
$$17 \div 0$$
is undefined.

73. The Multiplication by Zero property says that $a \cdot 0 = 0$ for any whole number a. Hence,
$$10 \cdot 0 = 0.$$

75. Division by zero is undefined. Hence,
$$\frac{7}{0}$$
is undefined.

77. When zero is divided by a nonzero number, the answer is zero. Hence:

$$16\overline{)0}^{\,0}$$

79. When zero is divided by a nonzero whole number, the answer is zero. There are zero groups of 24 in zero. Hence,

$$\frac{0}{24} = 0.$$

81. Division by zero is undefined. Hence,

$$0\overline{)0}$$

is undefined.

83. By long division,

$$44\overline{)2816}^{\,64}$$
$$\frac{264}{176}$$
$$\frac{176}{0}$$

Thus, $\dfrac{2816}{44} = 64$.

85. By long division,

$$83\overline{)2241}^{\,27}$$
$$\frac{166}{581}$$
$$\frac{581}{0}$$

Thus, $\dfrac{2241}{83} = 27$.

87. By long division,

$$
\begin{array}{r}
44 \\
73\overline{)3212} \\
292 \\
\overline{292} \\
292 \\
\overline{0}
\end{array}
$$

Thus, $\dfrac{3212}{73} = 44$.

89. By long division,

$$
\begin{array}{r}
89 \\
98\overline{)8722} \\
784 \\
\overline{882} \\
882 \\
\overline{0}
\end{array}
$$

Thus, $\dfrac{8722}{98} = 89$.

91. By long division,

$$
\begin{array}{r}
15 \\
96\overline{)1440} \\
96 \\
\overline{470} \\
480 \\
\overline{0}
\end{array}
$$

Thus, $\dfrac{1440}{96} = 15$.

93. By long division,

$$
\begin{array}{r}
95 \\
85\overline{)8075} \\
765 \\
\overline{425} \\
425 \\
\overline{0}
\end{array}
$$

Thus, $\dfrac{8075}{85} = 95$.

95. By long division,

$$
\begin{array}{r}
193 \\
92\overline{)17756} \\
92 \\ \hline
855 \\
828 \\ \hline
276 \\
276 \\ \hline
0
\end{array}
$$

Thus, $\dfrac{17756}{92} = 193.$

97. By long division,

$$
\begin{array}{r}
629 \\
19\overline{)11951} \\
114 \\ \hline
55 \\
38 \\ \hline
171 \\
171 \\ \hline
0
\end{array}
$$

Thus, $\dfrac{11951}{19} = 629.$

99. By long division,

$$
\begin{array}{r}
564 \\
32\overline{)18048} \\
160 \\ \hline
204 \\
192 \\ \hline
128 \\
128 \\ \hline
0
\end{array}
$$

Thus, $\dfrac{18048}{32} = 564.$

101. By long division,

$$
\begin{array}{r}
937 \\
31{\overline{\smash{\big)}\,29047}} \\
\underline{279} \\
114 \\
\underline{93} \\
217 \\
\underline{217} \\
0
\end{array}
$$

Thus, $\dfrac{29047}{31} = 937$.

103. By long division,

$$
\begin{array}{r}
426 \\
53{\overline{\smash{\big)}\,22578}} \\
\underline{212} \\
137 \\
\underline{106} \\
318 \\
\underline{318} \\
0
\end{array}
$$

Thus, $\dfrac{22578}{53} = 426$.

105. By long division,

$$
\begin{array}{r}
921 \\
14{\overline{\smash{\big)}\,12894}} \\
\underline{126} \\
29 \\
\underline{28} \\
14 \\
\underline{14} \\
0
\end{array}
$$

Thus, $\dfrac{12894}{14} = 921$.

107. $\dfrac{132}{6} = 22$, so 22 blocks are required.

109. 38 divided by 5 is 7, with a remainder of 3. Therefore, 8 trips are required.

111. The street is $5280 \cdot 4 = 21120$ feet long. 21120 divided by 145 is 145, with a remainder of 95. Therefore, 145 lights are required along the interior of the street. Counting the ends, 147 lights are required.

113. $\dfrac{292}{4} = 73$, so 73 blocks are required.

115. 32 divided by 3 is 10, with a remainder of 2. Therefore, 11 trips are required.

117. The street is $5280 \cdot 2 = 10560$ feet long. 10560 divided by 105 is 100, with a remainder of 60. Therefore, 100 lights are required along the interior of the street. Counting the ends, 102 lights are required.

119. To find the number of articles Eli writes in one week, multiply the numbers 4 and 5. Your answer will be in articles per week.

$$4 \cdot 5 = 20$$

Therefore, Eli writes 20 articles each week.

121. To find the number of yards in 27 laps, multiply 25 by 2 to get 50 yards for one round trip lap. Then multiply that 50 yards by 27.

$$2 \cdot 25 \cdot 27 = 1350$$

Therefore, Wendell swims 1,350 yards when he does 27 laps.

123. To find the minimum amount of hay a horse could eat over a year, multiply the 12 pounds each day by 365 days.

$$12 \cdot 365 = 4,380$$

Therefore, the minimum amout of hay a horse could eat will be 4,380 pounds.

125. To find the cost for non-residents over four years, multiply the 22,000 cost for one year by 4.

$$22,000 \cdot 4 = 88,000$$

Therefore, non-resident undergraduate could expect to pay \$88,000 to attend UC for four years.

127. The formula for the area of a rectangle is

$$A = LW,$$

where L is the length and W is the width of the rectangle, respectively. Substitute $L = 48\,\text{mi}$ and $W = 28\,\text{mi}$ and simplify.

$$\begin{aligned} A &= (48\,\text{mi})(28\,\text{mi}) && \text{Substitute } L = 48\,\text{mi and } W = 28\,\text{mi.} \\ &= 1344\,\text{mi}^2 && \text{Multiply: } 48 \cdot 28 = 1344. \end{aligned}$$

Note also that $\text{mi} \cdot \text{mi} = \text{mi}^2$. Hence, the area of the rectangle is $A = 1344$ square miles.

129. To find the cost to lay the sidewalk, first find the total area in square feet. The total area of the rectangular sidewalk will be the length times the width.

$$\begin{aligned} \text{Area} &= LW \\ &= 80 \cdot 4 \\ &= 320. \end{aligned}$$

The total area is 320 square feet. To find the cost of laying the sidewalk, multiply the area by 8.

$$320 \cdot 8 = 2,560$$

Therefore, the total cost for the sidewalk is $2,560.

131. Pairs of sunspots means two at a time. If the total count of sunspots is 72, dividing by 2 will give us the number of pairs.

$$72 \div 2 = 36$$

There are 36 pairs of sunspots.

1.4 Decimal Notation

1. Because

$$\begin{aligned} 30 &= 1 \cdot 30 \\ 30 &= 2 \cdot 15 \\ 30 &= 3 \cdot 10 \\ 30 &= 5 \cdot 6 \end{aligned}$$

the divisors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30.

3. Because

$$83 = 1 \cdot 83$$

the divisors of 83 are: 1, 83.

5. Because

$$91 = 1 \cdot 91$$
$$91 = 7 \cdot 13$$

the divisors of 91 are: 1, 7, 13, 91.

7. Because

$$75 = 1 \cdot 75$$
$$75 = 3 \cdot 25$$
$$75 = 5 \cdot 15$$

the divisors of 75 are: 1, 3, 5, 15, 25, 75.

9. Because

$$64 = 1 \cdot 64$$
$$64 = 2 \cdot 32$$
$$64 = 4 \cdot 16$$
$$64 = 8 \cdot 8$$

the divisors of 64 are: 1, 2, 4, 8, 16, 32, 64.

11. Because

$$14 = 1 \cdot 14$$
$$14 = 2 \cdot 7$$

the divisors of 14 are: 1, 2, 7, 14.

13. A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8. The number 117 does not end in 0, 2, 4, 6, 8; hence, 117 is **not** divisible by 2.

15. A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8. The number 13 does not end in 0, 2, 4, 6, 8; hence, 13 is **not** divisible by 2.

17. A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8. The number 105 does not end in 0, 2, 4, 6, 8; hence, 105 is **not** divisible by 2.

19. A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8. The number 31 does not end in 0, 2, 4, 6, 8; hence, 31 is **not** divisible by 2.

21. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $5 + 6 + 1 = 12$ is divisible by 3, 561 is divisible by 3.
- Because $5 + 6 + 4 = 15$ is divisible by 3, 564 is divisible by 3.
- Because $8 + 4 + 6 = 18$ is divisible by 3, 846 is divisible by 3.

However,
$$3 + 6 + 4 = 13,$$
which is **not** divisible by 3. Hence, 364 is **not** divisible by 3.

23. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $1 + 8 + 6 = 15$ is divisible by 3, 186 is divisible by 3.
- Because $3 + 1 + 5 = 9$ is divisible by 3, 315 is divisible by 3.
- Because $8 + 0 + 4 = 12$ is divisible by 3, 804 is divisible by 3.

However,
$$5 + 5 + 0 = 10,$$
which is **not** divisible by 3. Hence, 550 is **not** divisible by 3.

25. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $6 + 6 + 3 = 15$ is divisible by 3, 663 is divisible by 3.
- Because $4 + 1 + 4 = 9$ is divisible by 3, 414 is divisible by 3.
- Because $7 + 8 + 9 = 24$ is divisible by 3, 789 is divisible by 3.

However,
$$8 + 2 + 0 = 10,$$
which is **not** divisible by 3. Hence, 820 is **not** divisible by 3.

27. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $3 + 3 + 0 = 6$ is divisible by 3, 330 is divisible by 3.

- Because $6 + 0 + 0 = 6$ is divisible by 3, 600 is divisible by 3.

- Because $1 + 5 + 0 = 6$ is divisible by 3, 150 is divisible by 3.

However,

$$4 + 9 + 3 = 16,$$

which is **not** divisible by 3. Hence, 493 is **not** divisible by 3.

29. A number is divisible by 4 if the last two digits represent a number that is divisible by 4.

- The last two digits of 7648 are 48. Note that 48 is divisible by 4. Hence, 7648 is divisible by 4.

- The last two digits of 4048 are 48. Note that 48 is divisible by 4. Hence, 4048 is divisible by 4.

- The last two digits of 9944 are 44. Note that 44 is divisible by 4. Hence, 9944 is divisible by 4.

However, the last two digits of 3797 are 97. Note that 97 is **not** divisible by 4. Hence, 3797 is **not** divisible by 4.

31. A number is divisible by 4 if the last two digits represent a number that is divisible by 4.

- The last two digits of 4184 are 84. Note that 84 is divisible by 4. Hence, 4184 is divisible by 4.

- The last two digits of 9336 are 36. Note that 36 is divisible by 4. Hence, 9336 is divisible by 4.

- The last two digits of 2460 are 60. Note that 60 is divisible by 4. Hence, 2460 is divisible by 4.

However, the last two digits of 9701 are 1. Note that 1 is **not** divisible by 4. Hence, 9701 is **not** divisible by 4.

33. A number is divisible by 4 if the last two digits represent a number that is divisible by 4.

- The last two digits of 8332 are 32. Note that 32 is divisible by 4. Hence, 8332 is divisible by 4.

- The last two digits of 9816 are 16. Note that 16 is divisible by 4. Hence, 9816 is divisible by 4.

- The last two digits of 7408 are 8. Note that 8 is divisible by 4. Hence, 7408 is divisible by 4.

However, the last two digits of 7517 are 17. Note that 17 is **not** divisible by 4. Hence, 7517 is **not** divisible by 4.

35. A number is divisible by 4 if the last two digits represent a number that is divisible by 4.

- The last two digits of 1244 are 44. Note that 44 is divisible by 4. Hence, 1244 is divisible by 4.

- The last two digits of 7312 are 12. Note that 12 is divisible by 4. Hence, 7312 is divisible by 4.

- The last two digits of 1916 are 16. Note that 16 is divisible by 4. Hence, 1916 is divisible by 4.

However, the last two digits of 7033 are 33. Note that 33 is **not** divisible by 4. Hence, 7033 is **not** divisible by 4.

37. A number is divisible by 5 if and only if it ends with a 0 or 5.

- The number 4120 ends in a 0. Hence, 4120 is divisible by 5.

- The number 8920 ends in a 0. Hence, 8920 is divisible by 5.

- The number 5285 ends in a 5. Hence, 5285 is divisible by 5.

However, the number 9896 does **not** end in a zero or 5. Hence, 9896 is **not** divisible by 5.

39. A number is divisible by 5 if and only if it ends with a 0 or 5.

- The number 8915 ends in a 5. Hence, 8915 is divisible by 5.

- The number 3695 ends in a 5. Hence, 3695 is divisible by 5.

- The number 3005 ends in a 5. Hence, 3005 is divisible by 5.

However, the number 8758 does **not** end in a zero or 5. Hence, 8758 is **not** divisible by 5.

41. A number is divisible by 5 if and only if it ends with a 0 or 5.

- The number 5235 ends in a 5. Hence, 5235 is divisible by 5.

- The number 4240 ends in a 0. Hence, 4240 is divisible by 5.

- The number 4145 ends in a 5. Hence, 4145 is divisible by 5.

However, the number 2363 does **not** end in a zero or 5. Hence, 2363 is **not** divisible by 5.

43. A number is divisible by 5 if and only if it ends with a 0 or 5.

- The number 5550 ends in a 0. Hence, 5550 is divisible by 5.

- The number 4065 ends in a 5. Hence, 4065 is divisible by 5.

- The number 5165 ends in a 5. Hence, 5165 is divisible by 5.

However, the number 1269 does **not** end in a zero or 5. Hence, 1269 is **not** divisible by 5.

45. A number is divisible by 6 if and only if it is divisible by both 2 and 3. A number is divisible by 2 if it ends in 0 2, 4, 6, or 8. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $9 + 9 + 0 = 18$ is divisible by 3, 990 is divisible by 3. Because 990 ends in a 0, it is also divisible by 2. Hence, 990 is divisible by 6.

- Because $5 + 2 + 8 = 15$ is divisible by 3, 528 is divisible by 3. Because 528 ends in a 8, it is also divisible by 2. Hence, 528 is divisible by 6.

- Because $3 + 7 + 2 = 12$ is divisible by 3, 372 is divisible by 3. Because 372 ends in a 2, it is also divisible by 2. Hence, 372 is divisible by 6.

Because 328 ends in a 8, it is divisible by 2. However,

$$3 + 2 + 8 = 13,$$

which is **not** divisible by 3. Hence, 328 is **not** divisible by 3. Because 328 is **not** divisible by both 2 anbd 3, it is not divisible by 6.

47. A number is divisible by 6 if and only if it is divisible by both 2 and 3. A number is divisible by 2 if it ends in 0 2, 4, 6, or 8. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $7 + 4 + 4 = 15$ is divisible by 3, 744 is divisible by 3. Because 744 ends in a 4, it is also divisible by 2. Hence, 744 is divisible by 6.

- Because $1 + 7 + 4 = 12$ is divisible by 3, 174 is divisible by 3. Because 174 ends in a 4, it is also divisible by 2. Hence, 174 is divisible by 6.

- Because $9 + 2 + 4 = 15$ is divisible by 3, 924 is divisible by 3. Because 924 ends in a 4, it is also divisible by 2. Hence, 924 is divisible by 6.

Because 538 ends in a 8, it is divisible by 2. However,

$$5 + 3 + 8 = 16,$$

which is **not** divisible by 3. Hence, 538 is **not** divisible by 3. Because 538 is **not** divisible by both 2 anbd 3, it is not divisible by 6.

49. A number is divisible by 6 if and only if it is divisible by both 2 and 3. A number is divisible by 2 if it ends in 0 2, 4, 6, or 8. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $4 + 7 + 4 = 15$ is divisible by 3, 474 is divisible by 3. Because 474 ends in a 4, it is also divisible by 2. Hence, 474 is divisible by 6.

- Because $6 + 3 + 6 = 15$ is divisible by 3, 636 is divisible by 3. Because 636 ends in a 6, it is also divisible by 2. Hence, 636 is divisible by 6.

- Because $2 + 3 + 4 = 9$ is divisible by 3, 234 is divisible by 3. Because 234 ends in a 4, it is also divisible by 2. Hence, 234 is divisible by 6.

Because 586 ends in a 6, it is divisible by 2. However,

$$5 + 8 + 6 = 19,$$

which is **not** divisible by 3. Hence, 586 is **not** divisible by 3. Because 586 is **not** divisible by both 2 anbd 3, it is not divisible by 6.

51. A number is divisible by 6 if and only if it is divisible by both 2 and 3. A number is divisible by 2 if it ends in 0 2, 4, 6, or 8. A number is divisible by 3 if the sum of its digits is divisible by 3.

- Because $6 + 7 + 8 = 21$ is divisible by 3, 678 is divisible by 3. Because 678 ends in a 8, it is also divisible by 2. Hence, 678 is divisible by 6.

- Because $1 + 6 + 8 = 15$ is divisible by 3, 168 is divisible by 3. Because 168 ends in a 8, it is also divisible by 2. Hence, 168 is divisible by 6.

- Because $7 + 0 + 2 = 9$ is divisible by 3, 702 is divisible by 3. Because 702 ends in a 2, it is also divisible by 2. Hence, 702 is divisible by 6.

Because 658 ends in a 8, it is divisible by 2. However,

$$6 + 5 + 8 = 19,$$

which is **not** divisible by 3. Hence, 658 is **not** divisible by 3. Because 658 is **not** divisible by both 2 anbd 3, it is not divisible by 6.

53. A number is divisible by 8 if the last three digits represent a number that is divisible by 8.

- The last three digits of 2640 are 640. Note that 640 is divisible by 8. Hence, 2640 is divisible by 8.

- The last three digits of 8216 are 216. Note that 216 is divisible by 8. Hence, 8216 is divisible by 8.

- The last three digits of 1792 are 792. Note that 792 is divisible by 8. Hence, 1792 is divisible by 8.

However, the last three digits of 5418 are 418. Note that 418 is **not** divisible by 8. Hence, 5418 is **not** divisible by 8.

55. A number is divisible by 8 if the last three digits represent a number that is divisible by 8.

- The last three digits of 2208 are 208. Note that 208 is divisible by 8. Hence, 2208 is divisible by 8.

- The last three digits of 9016 are 16. Note that 16 is divisible by 8. Hence, 9016 is divisible by 8.

- The last three digits of 3208 are 208. Note that 208 is divisible by 8. Hence, 3208 is divisible by 8.

However, the last three digits of 8506 are 506. Note that 506 is **not** divisible by 8. Hence, 8506 is **not** divisible by 8.

57. A number is divisible by 8 if the last three digits represent a number that is divisible by 8.

- The last three digits of 4712 are 712. Note that 712 is divisible by 8. Hence, 4712 is divisible by 8.

- The last three digits of 7640 are 640. Note that 640 is divisible by 8. Hence, 7640 is divisible by 8.

- The last three digits of 3192 are 192. Note that 192 is divisible by 8. Hence, 3192 is divisible by 8.

However, the last three digits of 2594 are 594. Note that 594 is **not** divisible by 8. Hence, 2594 is **not** divisible by 8.

59. A number is divisible by 8 if the last three digits represent a number that is divisible by 8.

- The last three digits of 1232 are 232. Note that 232 is divisible by 8. Hence, 1232 is divisible by 8.

- The last three digits of 7912 are 912. Note that 912 is divisible by 8. Hence, 7912 is divisible by 8.

- The last three digits of 9808 are 808. Note that 808 is divisible by 8. Hence, 9808 is divisible by 8.

However, the last three digits of 7850 are 850. Note that 850 is **not** divisible by 8. Hence, 7850 is **not** divisible by 8.

61. A number is divisible by 9 if the sum of its digits is divisible by 9.

- Because $2 + 1 + 6 = 9$ is divisible by 9, 216 is divisible by 9.

- Because $2 + 9 + 7 = 18$ is divisible by 9, 297 is divisible by 9.

- Because $4 + 7 + 7 = 18$ is divisible by 9, 477 is divisible by 9.

However,
$$9 + 9 + 1 = 19,$$
which is **not** divisible by 9. Hence, 991 is **not** divisible by 9.

63. A number is divisible by 9 if the sum of its digits is divisible by 9.

- Because $6 + 7 + 5 = 18$ is divisible by 9, 675 is divisible by 9.

- Because $1 + 5 + 3 = 9$ is divisible by 9, 153 is divisible by 9.

- Because $2 + 3 + 4 = 9$ is divisible by 9, 234 is divisible by 9.

However,
$$9 + 3 + 7 = 19,$$
which is **not** divisible by 9. Hence, 937 is **not** divisible by 9.

65. A number is divisible by 9 if the sum of its digits is divisible by 9.

- Because $7 + 8 + 3 = 18$ is divisible by 9, 783 is divisible by 9.

- Because $5 + 9 + 4 = 18$ is divisible by 9, 594 is divisible by 9.

- Because $2 + 1 + 6 = 9$ is divisible by 9, 216 is divisible by 9.

However,
$$9 + 2 + 8 = 19,$$
which is **not** divisible by 9. Hence, 928 is **not** divisible by 9.

67. A number is divisible by 9 if the sum of its digits is divisible by 9.

- Because $4 + 2 + 3 = 9$ is divisible by 9, 423 is divisible by 9.

- Because $8 + 0 + 1 = 9$ is divisible by 9, 801 is divisible by 9.

- Because $9 + 3 + 6 = 18$ is divisible by 9, 936 is divisible by 9.

However,
$$6 + 7 + 6 = 19,$$
which is **not** divisible by 9. Hence, 676 is **not** divisible by 9.

69. The only factors of 19 are 1 and 19. Hence, 19 is a prime number.

71. The only factors of 41 are 1 and 41. Hence, 41 is a prime number.

73. 27 is a composite number. The prime factorization is $27 = 3 \cdot 3 \cdot 3$.

75. 91 is a composite number. The prime factorization is $91 = 7 \cdot 13$.

77. 21 is a composite number. The prime factorization is $21 = 3 \cdot 7$.

79. The only factors of 23 are 1 and 23. Hence, 23 is a prime number.

81. $224 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7$

83. $108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

85. $243 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

87. $160 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

89. $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

91. $360 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

93. $144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

95. $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

97. $216 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

99. Evaluate exponents first. Then multiply.

$$
\begin{aligned}
5^2 \cdot 4^1 &= 25 \cdot 4 && \text{Exponents first: } 5^2 = 25 \text{ and } 4^1 = 4. \\
&= 100 && \text{Multiply: } 25 \cdot 4 = 100.
\end{aligned}
$$

101. The exponent 1 tells us to write the base 0 as a factor 1 times. That is,

$$
\begin{aligned}
0^1 &= (0) && \text{Write the base 1 time.} \\
&= 0 && \text{Multiply.}
\end{aligned}
$$

103. Evaluate exponents first. Then multiply.

$$
\begin{aligned}
3^3 \cdot 0^2 &= 27 \cdot 0 && \text{Exponents first: } 3^3 = 27 \text{ and } 0^2 = 0. \\
&= 0 && \text{Multiply: } 27 \cdot 0 = 0.
\end{aligned}
$$

105. The exponent 1 tells us to write the base 4 as a factor 1 times. That is,

$$
\begin{aligned}
4^1 &= (4) && \text{Write the base 1 time.} \\
&= 4 && \text{Multiply.}
\end{aligned}
$$

107. The exponent 3 tells us to write the base 4 as a factor 3 times. That is,

$$4^3 = (4)(4)(4) \qquad \textcolor{red}{\text{Write the base 3 times.}}$$
$$ = 64 \qquad\qquad \textcolor{red}{\text{Multiply.}}$$

109. Evaluate exponents first. Then multiply.

$$3^3 \cdot 1^2 = 27 \cdot 1 \qquad \textcolor{red}{\text{Exponents first: } 3^3 = 27 \text{ and } 1^2 = 1.}$$
$$ = 27 \qquad\quad \textcolor{red}{\text{Multiply: } 27 \cdot 1 = 27.}$$

111. The formula for the area of a square is

$$A = s^2.$$

Substitute 28 inches for s.

$$A = (28\,\text{in})^2 \qquad \textcolor{red}{\text{Substitute 28 in for } s.}$$
$$ = (28\,\text{in})(28\,\text{in}) \qquad \textcolor{red}{\text{Square.}}$$
$$ = 784\,\text{in}^2 \qquad\quad \textcolor{red}{\text{Multiply.}}$$

Note that $(\text{in})(\text{in}) = \text{in}^2$. Thus, the area of the square is 784 square inches.

113. The formula for the area of a square is

$$A = s^2.$$

Substitute 22 inches for s.

$$A = (22\,\text{in})^2 \qquad \textcolor{red}{\text{Substitute 22 in for } s.}$$
$$ = (22\,\text{in})(22\,\text{in}) \qquad \textcolor{red}{\text{Square.}}$$
$$ = 484\,\text{in}^2 \qquad\quad \textcolor{red}{\text{Multiply.}}$$

Note that $(\text{in})(\text{in}) = \text{in}^2$. Thus, the area of the square is 484 square inches.

115.

Therefore, $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$.

117.

Therefore, $105 = 3 \cdot 5 \cdot 7$.

119.

Therefore, $56 = 2 \cdot 2 \cdot 2 \cdot 7 = 2^3 \cdot 7$

121.

Therefore, $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$

123. Follow the algorithm of Eratosthenes.

   i) Strike out all multiples of 2 (4, 6, 8, etc.)

   ii) The list's next number that has not been struck out is a prime number.

   iii) Strike out from the list all multiples of the number you identified in step (ii).

   iv) Repeat steps (ii) and (iii) until you can no longer strike any more multiples.

   v) All unstruck numbers in the list are primes.

The numbers that remain unstruck are prime. Thus, the primes less than 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

1.5   Algebraic Expressions

1. Order of operations demands that we do multiplications first.

$$5 + 2 \cdot 2 = 5 + 4 \qquad \text{Multiply: } 2 \cdot 2 = 4.$$
$$= 9 \qquad \text{Add: } 5 + 4 = 9.$$

3. Order of operations demands that we do multiplications first.

$$23 - 7 \cdot 2 = 23 - 14 \qquad \text{Multiply: } 7 \cdot 2 = 14.$$
$$= 9 \qquad \text{Subtract: } 23 - 14 = 9.$$

5. Multiply first, then add.

$$4 \cdot 3 + 2 \cdot 5 = 12 + 10 \qquad \text{Multiply: } 4 \cdot 3 = 12 \text{ and } 2 \cdot 5 = 10.$$
$$= 22 \qquad \text{Add: } 12 + 10 = 22.$$

7. Multiply first, then add.

$$6 \cdot 5 + 4 \cdot 3 = 30 + 12 \qquad \text{Multiply: } 6 \cdot 5 = 30 \text{ and } 4 \cdot 3 = 12.$$
$$= 42 \qquad \text{Add: } 30 + 12 = 42.$$

9. Order of operations demands that we do multiplications first.

$$9 + 2 \cdot 3 = 9 + 6 \qquad \text{Multiply: } 2 \cdot 3 = 6.$$
$$= 15 \qquad \text{Add: } 9 + 6 = 15.$$

11. Order of operations demands that we do multiplications first.

$$32 - 8 \cdot 2 = 32 - 16 \qquad \text{Multiply: } 8 \cdot 2 = 16.$$
$$= 16 \qquad \text{Subtract: } 32 - 16 = 16.$$

13. Multiplications and divisions hold no precedence over one another. They must be performed in the order that they appear, moving left to right through the expression.

$$45 \div 3 \cdot 5 = 15 \cdot 5 \qquad \text{Divide: } 45 \div 3 = 15.$$
$$= 75 \qquad \text{Multiply: } 15 \cdot 5 = 75.$$

15. Move left to right, performing multiplications and divisions in the order that they appear.

$$2 \cdot 9 \div 3 \cdot 18 = 18 \div 3 \cdot 18 \qquad \text{Multiply: } 2 \cdot 9 = 18.$$
$$= 6 \cdot 18 \qquad \text{Divide: } 18 \div 3 = 6.$$
$$= 108 \qquad \text{Multiply: } 6 \cdot 18 = 108.$$

17. Multiplications and divisions hold no precedence over one another. They must be performed in the order that they appear, moving left to right through the expression.

$$30 \div 2 \cdot 3 = 15 \cdot 3 \qquad \text{Divide: } 30 \div 2 = 15.$$
$$= 45 \qquad \text{Multiply: } 15 \cdot 3 = 45.$$

19. Additions and subtractions hold no precedence over one another. They must be performed as they appear, moving left to right through the expression.

$$8 - 6 + 1 = 2 + 1 \qquad \text{Subtract: } 8 - 6 = 2.$$
$$= 3 \qquad \text{Add: } 2 + 1 = 3.$$

21. Move left to right, performing multiplications and divisions in the order that they appear.

$$
\begin{aligned}
14 \cdot 16 \div 16 \cdot 19 &= 224 \div 16 \cdot 19 && \text{Multiply: } 14 \cdot 16 = 224. \\
&= 14 \cdot 19 && \text{Divide: } 224 \div 16 = 14. \\
&= 266 && \text{Multiply: } 14 \cdot 19 = 266.
\end{aligned}
$$

23. Move left to right, performing multiplications and divisions in the order that they appear. Then do the same with additions and subtractions.

$$
\begin{aligned}
15 \cdot 17 + 10 \div 10 - 12 \cdot 4 &= 255 + 10 \div 10 - 12 \cdot 4 && \text{Multiply: } 15 \cdot 17 = 255. \\
&= 255 + 1 - 12 \cdot 4 && \text{Divide: } 10 \div 10 = 1. \\
&= 255 + 1 - 48 && \text{Multiply: } 12 \cdot 4 = 48. \\
&= 256 - 48 && \text{Add: } 255 + 1 = 256. \\
&= 208 && \text{Subtract: } 256 - 48 = 208.
\end{aligned}
$$

25. Additions and subtractions hold no precedence over one another. They must be performed as they appear, moving left to right through the expression.

$$
\begin{aligned}
22 - 10 + 7 &= 12 + 7 && \text{Subtract: } 22 - 10 = 12. \\
&= 19 && \text{Add: } 12 + 7 = 19.
\end{aligned}
$$

27. Move left to right, performing multiplications and divisions in the order that they appear. Then do the same with additions and subtractions.

$$
\begin{aligned}
20 \cdot 10 + 15 \div 5 - 7 \cdot 6 &= 200 + 15 \div 5 - 7 \cdot 6 && \text{Multiply: } 20 \cdot 10 = 200. \\
&= 200 + 3 - 7 \cdot 6 && \text{Divide: } 15 \div 5 = 3. \\
&= 200 + 3 - 42 && \text{Multiply: } 7 \cdot 6 = 42. \\
&= 203 - 42 && \text{Add: } 200 + 3 = 203. \\
&= 161 && \text{Subtract: } 203 - 42 = 161.
\end{aligned}
$$

29. Evaluate the expression in the parentheses first, divide, then add.

$$
\begin{aligned}
9 + 8 \div \{4 + 4\} &= 9 + 8 \div 8 && \text{Parens: } 4 + 4 = 8. \\
&= 9 + 1 && \text{Divide: } 8 \div 8 = 1. \\
&= 10 && \text{Add: } 9 + 1 = 10.
\end{aligned}
$$

31. Evaluate the expression in the brackets first, then multiply, then subtract.

$$
\begin{aligned}
7 \cdot [8 - 5] - 10 &= 7 \cdot 3 - 10 && \text{Subtract: } 8 - 5 = 3. \\
&= 21 - 10 && \text{Multiply: } 7 \cdot 3 = 21. \\
&= 11 && \text{Subtract: } 21 - 10 = 11.
\end{aligned}
$$

33. Evaluate the expression in the parentheses first, then divide.

$$
\begin{aligned}
(18 + 10) \div (2 + 2) &= 28 \div 4 && \text{Parens: } 18 + 10 = 28 \text{ and } 2 + 2 = 4. \\
&= 7 && \text{Divide: } 28 \div 4 = 7.
\end{aligned}
$$

35. Evaluate the expression in the parentheses first, then multiply, then subtract.

$$
\begin{aligned}
9 \cdot (10 + 7) - 3 \cdot (4 + 10) &= 9 \cdot 17 - 3 \cdot 14 && \text{Add: } 10 + 7 = 17 \\
& && \text{and } 4 + 10 = 14. \\
&= 153 - 42 && \text{Multiply: } 9 \cdot 17 = 153 \\
& && \text{and } 3 \cdot 14 = 42. \\
&= 111 && \text{Subtract: } 153 - 42 = 111
\end{aligned}
$$

37. Evaluate the expression in the braces first, then multiply, then divide.

$$
\begin{aligned}
2 \cdot \{8 + 12\} \div 4 &= 2 \cdot 20 \div 4 && \text{Add: } 8 + 12 = 20. \\
&= 40 \div 4 && \text{Multiply: } 2 \cdot 20 = 40. \\
&= 10 && \text{Divide: } 40 \div 4 = 10.
\end{aligned}
$$

39. Evaluate the expression in the parentheses first, then multiply, then add.

$$
\begin{aligned}
9 + 6 \cdot (12 + 3) &= 9 + 6 \cdot 15 && \text{Add: } 12 + 3 = 15. \\
&= 9 + 90 && \text{Multiply: } 6 \cdot 15 = 90. \\
&= 99 && \text{Add: } 9 + 90 = 99.
\end{aligned}
$$

41. Evaluate the expression in the innermost parentheses first.

$$
2 + 9 \cdot [7 + 3 \cdot (9 + 5)] = 2 + 9 \cdot [7 + 3 \cdot 14] \qquad \text{Add: } 9 + 5 = 14.
$$

Now evaluate the expression inside the brackets.

$$
\begin{aligned}
&= 2 + 9 \cdot [7 + 42] && \text{Multiply: } 3 \cdot 14 = 42. \\
&= 2 + 9 \cdot 49 && \text{Add: } 7 + 42 = 49. \\
&= 2 + 441 && \text{Multiply: } 9 \cdot 49 = 441. \\
&= 443 && \text{Add: } 2 + 441 = 443.
\end{aligned}
$$

43. Evaluate the expression in the innermost parentheses first.

$$7 + 3 \cdot [8 + 8 \cdot (5 + 9)] = 7 + 3 \cdot [8 + 8 \cdot 14] \qquad \text{Add: } 5 + 9 = 14.$$

Now evaluate the expression inside the brackets.

$$= 7 + 3 \cdot [8 + 112] \qquad \text{Multiply: } 8 \cdot 14 = 112.$$
$$= 7 + 3 \cdot 120 \qquad \text{Add: } 8 + 112 = 120.$$
$$= 7 + 360 \qquad \text{Multiply: } 3 \cdot 120 = 360.$$
$$= 367 \qquad \text{Add: } 7 + 360 = 367.$$

45. When parentheses are nested, evaluate the innermost parentheses first.

$$6 - 5[11 - (2 + 8)] = 6 - 5[11 - 10] \qquad \text{Inner parens first: } 2 + 8 = 10.$$
$$= 6 - 5[1] \qquad \text{Brackets next: } 11 - 10 = 1.$$
$$= 6 - 5 \qquad \text{Multiply: } 5[1] = 5.$$
$$= 1 \qquad \text{Subtract: } 6 - 5 = 1.$$

47. When parentheses are nested, evaluate the innermost parentheses first.

$$11 - 1[19 - (2 + 15)] = 11 - 1[19 - 17] \qquad \text{Inner parens first: } 2 + 15 = 17.$$
$$= 11 - 1[2] \qquad \text{Brackets next: } 19 - 17 = 2.$$
$$= 11 - 2 \qquad \text{Multiply: } 1[2] = 2.$$
$$= 9 \qquad \text{Subtract: } 11 - 2 = 9.$$

49. Evaluate the expressions in the innermost parentheses first.

$$4\{7[9 + 3] - 2[3 + 2]\} = 4\{7[12] - 2[5]\} \qquad \text{Add: } 9 + 3 = 12 \text{ and } 3 + 2 = 5.$$

Now evaluate the expression inside the braces.

$$= 4\{84 - 10\} \qquad \text{Multiply: } 7[12] = 84$$
$$\qquad\qquad\qquad \text{and } 2[5] = 10.$$
$$= 4\{74\} \qquad \text{Subtract: } 84 - 10 = 74.$$
$$= 296 \qquad \text{Multiply: } 4\{74\} = 296.$$

51. Evaluate the expression in the innermost parentheses first.

$$9 \cdot [3 + 4 \cdot (5 + 2)] = 9 \cdot [3 + 4 \cdot 7] \qquad \text{Add: } 5 + 2 = 7.$$

Now evaluate the expression inside the brackets.

$$= 9 \cdot [3 + 28] \qquad \text{Multiply: } 4 \cdot 7 = 28.$$
$$= 9 \cdot 31 \qquad \text{Add: } 3 + 28 = 31.$$
$$= 279 \qquad \text{Multiply: } 9 \cdot 31 = 279.$$

53. Evaluate the expressions in the innermost parentheses first.

$3\{8[6 + 5] - 8[7 + 3]\} = 3\{8[11] - 8[10]\}$ Add: $6 + 5 = 11$ and $7 + 3 = 10$.

Now evaluate the expression inside the braces.

$$= 3\{88 - 80\} \qquad \text{Multiply: } 8[11] = 88$$
$$\text{and } 8[10] = 80.$$
$$= 3\{8\} \qquad \text{Subtract: } 88 - 80 = 8.$$
$$= 24 \qquad \text{Multiply: } 3\{8\} = 24.$$

55. Evaluate the expression in the innermost parentheses first.

$$3 \cdot [2 + 4 \cdot (9 + 6)] = 3 \cdot [2 + 4 \cdot 15] \qquad \text{Add: } 9 + 6 = 15.$$

Now evaluate the expression inside the brackets.

$$= 3 \cdot [2 + 60] \qquad \text{Multiply: } 4 \cdot 15 = 60.$$
$$= 3 \cdot 62 \qquad \text{Add: } 2 + 60 = 62.$$
$$= 186 \qquad \text{Multiply: } 3 \cdot 62 = 186.$$

57. Subtract inside the parentheses first, then evaluate the exponent.

$$(5 - 2)^2 = (3)^2 \qquad \text{Parentheses: } (5 - 2) = (3).$$
$$= 9 \qquad \text{Exponent: } (3)^2 = 9.$$

59. Add inside the parentheses first, then evaluate the exponent.

$$(4 + 2)^2 = (6)^2 \qquad \text{Parentheses: } (4 + 2) = (6).$$
$$= 36 \qquad \text{Exponent: } (6)^2 = 36.$$

61. Evaluate the exponents first, then add.

$$2^3 + 3^3 = 8 + 3^3 \qquad \text{Exponent: } 2^3 = 8.$$
$$= 8 + 27 \qquad \text{Exponent: } 3^3 = 27.$$
$$= 35 \qquad \text{Add: } 8 + 27 = 35.$$

63. Evaluate the exponents first, then subtract.

$$2^3 - 1^3 = 8 - 1^3 \qquad \text{Exponent: } 2^3 = 8.$$
$$= 8 - 1 \qquad \text{Exponent: } 1^3 = 1.$$
$$= 7 \qquad \text{Subtract: } 8 - 1 = 7.$$

65. Evaluate the exponent first, then multiply, then add.

$$
\begin{aligned}
12 \cdot 5^2 + 8 \cdot 9 + 4 &= 12 \cdot 25 + 8 \cdot 9 + 4 & &\text{Exponent: } 5^2 = 25. \\
&= 300 + 8 \cdot 9 + 4 & &\text{Multiply: } 12 \cdot 25 = 300. \\
&= 300 + 72 + 4 & &\text{Multiply: } 8 \cdot 9 = 72. \\
&= 376 & &\text{Add: } 300 + 72 + 4 = 376.
\end{aligned}
$$

67. Evaluate the exponent first, then multiply, then subtract and add working left to right.

$$
\begin{aligned}
9 - 3 \cdot 2 + 12 \cdot 10^2 &= 9 - 3 \cdot 2 + 12 \cdot 100 & &\text{Exponent: } 10^2 = 100. \\
&= 9 - 6 + 12 \cdot 100 & &\text{Multiply: } 3 \cdot 2 = 6. \\
&= 9 - 6 + 1200 & &\text{Multiply: } 12 \cdot 100 = 1200. \\
&= 3 + 1200 & &\text{Subtract: } 9 - 6 = 3. \\
&= 1203 & &\text{Add: } 3 + 1200 = 1203.
\end{aligned}
$$

69. The parenthetical expression must be evaluated first, then the exponent, then the subtraction.

$$
\begin{aligned}
4^2 - (13 + 2) &= 4^2 - 15 & &\text{Parentheses first: } 13 + 2 = 15. \\
&= 16 - 15 & &\text{Exponent next: } 4^2 = 16. \\
&= 1 & &\text{Subtract: } 16 - 15 = 1.
\end{aligned}
$$

71. The parenthetical expression must be evaluated first, then the exponent, then the subtraction.

$$
\begin{aligned}
3^3 - (7 + 12) &= 3^3 - 19 & &\text{Parentheses first: } 7 + 12 = 19. \\
&= 27 - 19 & &\text{Exponent next: } 3^3 = 27. \\
&= 8 & &\text{Subtract: } 27 - 19 = 8.
\end{aligned}
$$

73. We must evaluate the innermost grouping symbols first.

$$
\begin{aligned}
19 + 3[12 - (2^3 + 1)] &= 19 + 3[12 - (8 + 1)] & &\text{Exponent first: } 2^3 = 8. \\
&= 19 + 3[12 - 9] & &\text{Add: } 8 + 1 = 9. \\
&= 19 + 3[3] & &\text{Brackets next: } 12 - 9 = 3. \\
&= 19 + 9 & &\text{Multiply: } 3[3] = 9. \\
&= 28 & &\text{Add: } 19 + 9 = 28.
\end{aligned}
$$

75. We must evaluate the innermost grouping symbols first.

$$
\begin{aligned}
17 + 7[13 - (2^2 + 6)] &= 17 + 7[13 - (4 + 6)] && \text{Exponent first: } 2^2 = 4. \\
&= 17 + 7[13 - 10] && \text{Add: } 4 + 6 = 10. \\
&= 17 + 7[3] && \text{Brackets next: } 13 - 10 = 3. \\
&= 17 + 21 && \text{Multiply: } 7[3] = 21. \\
&= 38 && \text{Add: } 17 + 21 = 38.
\end{aligned}
$$

77. The parenthetical expression must be evaluated first, then the exponent, then the subtraction.

$$
\begin{aligned}
4^3 - (12 + 1) &= 4^3 - 13 && \text{Parentheses first: } 12 + 1 = 13. \\
&= 64 - 13 && \text{Exponent next: } 4^3 = 64. \\
&= 51 && \text{Subtract: } 64 - 13 = 51.
\end{aligned}
$$

79. We must evaluate the innermost grouping symbols first.

$$
\begin{aligned}
5 + 7[11 - (2^2 + 1)] &= 5 + 7[11 - (4 + 1)] && \text{Exponent first: } 2^2 = 4. \\
&= 5 + 7[11 - 5] && \text{Add: } 4 + 1 = 5. \\
&= 5 + 7[6] && \text{Brackets next: } 11 - 5 = 6. \\
&= 5 + 42 && \text{Multiply: } 7[6] = 42. \\
&= 47 && \text{Add: } 5 + 42 = 47.
\end{aligned}
$$

81. We must simplify numerator and denominator separately, then divide.

$$
\begin{aligned}
\frac{13 + 35}{3(4)} &= \frac{48}{12} && \text{Numerator: } 13 + 35 = 48. \\
&&& \text{Denominator: } 3(4) = 12. \\
&= 4 && \text{Divide: } 48/12 = 4.
\end{aligned}
$$

83. In the numerator, we need to evaluate the parentheses first. To do so, multiply first, then subtract.

$$
\begin{aligned}
\frac{64 - (8 \cdot 6 - 3)}{4 \cdot 7 - 9} &= \frac{64 - (48 - 3)}{4 \cdot 7 - 9} && \text{Multiply: } 8 \cdot 6 = 48. \\
&= \frac{64 - 45}{4 \cdot 7 - 9} && \text{Subtract: } 48 - 3 = 45.
\end{aligned}
$$

In the denominator, we must multiply first.

$$
= \frac{64 - 45}{28 - 9} \qquad \text{Multiply: } 4 \cdot 7 = 28.
$$

Now we can subtract in both numerator and denominator.

$$= \frac{19}{19}$$ Numerator: $64 - 45 = 19$.

Numerator: $28 - 9 = 19$.

$$= 1$$ Divide: $19/19 = 1$.

85. Simplify numerator and denominator separately, then divide.

$$\frac{2 + 13}{4 - 1} = \frac{15}{3}$$ Numerator: $2 + 13 = 15$.

Denominator: $4 - 1 = 3$.

$$= 5$$ Divide: $15/3 = 5$.

87. Simplify numerator and denominator separately, then divide.

$$\frac{17 + 14}{9 - 8} = \frac{31}{1}$$ Numerator: $17 + 14 = 31$.

Denominator: $9 - 8 = 1$.

$$= 31$$ Divide: $31/1 = 31$.

89. We must simplify numerator and denominator separately, then divide.

$$\frac{37 + 27}{8(2)} = \frac{64}{16}$$ Numerator: $37 + 27 = 64$.

Denominator: $8(2) = 16$.

$$= 4$$ Divide: $64/16 = 4$.

91. In the numerator, we need to evaluate the parentheses first. To do so, multiply first, then subtract.

$$\frac{40 - (3 \cdot 7 - 9)}{8 \cdot 2 - 2} = \frac{40 - (21 - 9)}{8 \cdot 2 - 2}$$ Multiply: $3 \cdot 7 = 21$.

$$= \frac{40 - 12}{8 \cdot 2 - 2}$$ Subtract: $21 - 9 = 12$.

In the denominator, we must multiply first.

$$= \frac{40 - 12}{16 - 2}$$ Multiply: $8 \cdot 2 = 16$.

Now we can subtract in both numerator and denominator.

$$= \frac{28}{14}$$ Numerator: $40 - 12 = 28$.

Numerator: $16 - 2 = 14$.

$$= 2$$ Divide: $28/14 = 2$.

93. Distribute, multiply, then add.

$$5 \cdot (8 + 4) = 5 \cdot 8 + 5 \cdot 4 \qquad \text{Distribute 5 times each term in parens.}$$
$$= 40 + 20 \qquad \text{Multiply: } 5 \cdot 8 = 40 \text{ and } 5 \cdot 4 = 20.$$
$$= 60 \qquad \text{Add: } 40 + 20 = 60.$$

95. Distribute, multiply, then subtract.

$$7 \cdot (8 - 3) = 7 \cdot 8 - 7 \cdot 3 \qquad \text{Distribute 7 times each term in parens.}$$
$$= 56 - 21 \qquad \text{Multiply: } 7 \cdot 8 = 56 \text{ and } 7 \cdot 3 = 21.$$
$$= 35 \qquad \text{Add: } 56 - 21 = 35.$$

97. Distribute, multiply, then subtract.

$$6 \cdot (7 - 2) = 6 \cdot 7 - 6 \cdot 2 \qquad \text{Distribute 6 times each term in parens.}$$
$$= 42 - 12 \qquad \text{Multiply: } 6 \cdot 7 = 42 \text{ and } 6 \cdot 2 = 12.$$
$$= 30 \qquad \text{Add: } 42 - 12 = 30.$$

99. Distribute, multiply, then add.

$$4 \cdot (3 + 2) = 4 \cdot 3 + 4 \cdot 2 \qquad \text{Distribute 4 times each term in parens.}$$
$$= 12 + 8 \qquad \text{Multiply: } 4 \cdot 3 = 12 \text{ and } 4 \cdot 2 = 8.$$
$$= 20 \qquad \text{Add: } 12 + 8 = 20.$$

101. First, expand 62 as $60 + 2$, then apply the distributive property.

$$9 \cdot 62 = 9 \cdot (60 + 2) \qquad \text{Expand: } 62 = 60 + 2.$$
$$= 9 \cdot 60 + 9 \cdot 2 \qquad \text{Distribute the 9.}$$
$$= 540 + 18 \qquad \text{Multiply: } 9 \cdot 60 = 540 \text{ and } 9 \cdot 2 = 18.$$
$$= 558 \qquad \text{Add: } 540 + 18 = 558.$$

103. First, expand 58 as $50 + 8$, then apply the distributive property.

$$3 \cdot 58 = 3 \cdot (50 + 8) \qquad \text{Expand: } 58 = 50 + 8.$$
$$= 3 \cdot 50 + 3 \cdot 8 \qquad \text{Distribute the 3.}$$
$$= 150 + 24 \qquad \text{Multiply: } 3 \cdot 50 = 150 \text{ and } 3 \cdot 8 = 24.$$
$$= 174 \qquad \text{Add: } 150 + 24 = 174.$$

Chapter 2

The Integers

2.1   An Introduction to the Integers

1. Locate the number 4 on the number line, then move three units to the left to locate the second number.

Hence, the number 1 lies three units to the left of the number 4.

3. Locate the number 6 on the number line, then move three units to the left to locate the second number.

Hence, the number 3 lies three units to the left of the number 6.

5. Locate the number 0 on the number line, then move two units to the right to locate the second number.

Hence, the number 2 lies two units to the right of the number 0.

7. Locate the number 1 on the number line, then move two units to the right to locate the second number.

Hence, the number 3 lies two units to the right of the number 1.

9. Locate the number 6 on the number line, then move four units to the left to locate the second number.

Hence, the number 2 lies four units to the left of the number 6.

11. Locate the number -5 on the number line, then move two units to the right to locate the second number.

Hence, the number -3 lies two units to the right of the number -5.

13.

  i) Arrange the integers 6, 1, -3, and -5 on a number line.

  ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: -5, -3, 1, 6

15.

i) Arrange the integers 5, −6, 0, and 2 on a number line.

ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: −6, 0, 2, 5

17.

i) Arrange the integers −3, −5, 3, and 5 on a number line.

ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: −5, −3, 3, 5

19.

i) Arrange the integers −5, 4, 2, and −3 on a number line.

ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: −5, −3, 2, 4

21.

  i) Arrange the integers 3, 5, -5, and -1 on a number line.

  ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: -5, -1, 3, 5

23.

  i) Arrange the integers -2, -4, 3, and -6 on a number line.

  ii) The order on the number line, left to right, is also from smallest to largest. Therefore, in order, smallest to largest: -6, -4, -2, 3

25. If you locate the numbers -4 and 0 on a number line, you can see that -4 lies to the left of 0.

Hence, $-4 < 0$.

27. If you locate the numbers -2 and -1 on a number line, you can see that -2 lies to the left of -1.

Hence, $-2 < -1$.

29. If you locate the numbers -3 and -1 on a number line, you can see that -3 lies to the left of -1.

Hence, $-3 < -1$.

31. If you locate the numbers 3 and 6 on a number line, you can see that 3 lies to the left of 6.

Hence, $3 < 6$.

33. If you locate the numbers -3 and -6 on a number line, you can see that -3 lies to the right of -6.

Hence, $-3 > -6$.

35. If you locate the numbers -1 and 4 on a number line, you can see that -1 lies to the left of 4.

Hence, $-1 < 4$.

37. The opposite of -4 is 4. That is, $-(-4) = 4$.

39. The number 7 is 7 units from the origin.

Hence, $|7| = 7$.

41. The number 5 is 5 units from the origin.

Hence, $|5| = 5$.

43. You must first take the absolute value of -11, which is 11. Then you must take the opposite of this result. That is,

$$-|-11| = -(11) \qquad \text{First: } |-11| = 11.$$
$$= -11 \qquad \text{Second: The opposite of 11 is } -11.$$

Hence, $|-11| = -11$.

45. The number -5 is 5 units from the origin.

Hence, $|-5| = 5$.

47. You must first take the absolute value of -20, which is 20. Then you must take the opposite of this result. That is,

$$-|-20| = -(20) \qquad \text{First: } |-20| = 20.$$
$$= -20 \qquad \text{Second: The opposite of 20 is } -20.$$

Hence, $|-20| = -20$.

49. The number -4 is 4 units from the origin.

Hence, $|-4| = 4$.

51. The opposite of -2 is 2. That is, $-(-2) = 2$.

53. Plot the integer 2 on the number line, then move 2 units to the left and 2 units to the right to find the required integers.

Thus, 0 and 4 are 2 units away from 2.

55. Plot the integer -3 on the number line, then move 4 units to the left and 4 units to the right to find the required integers.

Thus, -7 and 1 are 4 units away from -3.

57. Plot the integer -2 on the number line, then move 3 units to the left and 3 units to the right to find the required integers.

Thus, -5 and 1 are 3 units away from -2.

59. Plot the integer 3 on the number line, then move 2 units to the left and 2 units to the right to find the required integers.

Thus, 1 and 5 are 2 units away from 3.

61. Plot the integer 0 on the number line, then move 3 units to the left and 3 units to the right to find the required integers.

Thus, -3 and 3 are 3 units away from 0.

63. Plot the integer 0 on the number line, then move 2 units to the left and 2 units to the right to find the required integers.

Thus, -2 and 2 are 2 units away from 0.

65. Set a vertical number line with 0 representing sea level, positive numbers representing heights above sea lever, and negative number representing depths below sea level. Thus, 2,350 feet above sea level can be represented by the positive integer 2,350.

67. Read the heights of the bars to create the following table of profit and loss.

Month	Profit/Loss
Jan	8
Feb	-4
Mar	-3
Apr	-2
May	2
Jun	5

Use the data in the table to create the following line graph.

69. We've circled the low temperature in the plot. Note that the low temperature is approximately $-8°$ Fahrenheit and it occurred on January 16.

71.

2.2   Adding Integers

1. The vector starts at -4 and finishes at 0. The vector points to the right and has length 4. Hence, this vector represents the integer 4.

3. The vector starts at 0 and finishes at 6. The vector points to the right and has length 6. Hence, this vector represents the integer 6.

5. The vector starts at 1 and finishes at -4. The vector points to the left and has length 5. Hence, this vector represents the integer -5.

7. The vector starts at 6 and finishes at 0. The vector points to the left and has length 6. Hence, this vector represents the integer -6.

9. The vector starts at -4 and finishes at 6. The vector points to the right and has length 10. Hence, this vector represents the integer 10.

11. The vector starts at 2 and finishes at -5. The vector points to the left and has length 7. Hence, this vector represents the integer -7.

13. To add a negative and a positive integer, subtract the smaller magnitude from the larger magnitude ($15 - 1 = 14$), then prefix the sign of the integer with the larger magnitude. Thus,

$$-15 + 1 = -14.$$

15. To add a positive and a negative integer, subtract the smaller magnitude from the larger magnitude $(18 - 10 = 8)$, then prefix the sign of the integer with the larger magnitude. Thus,

$$18 + (-10) = 8.$$

17. To add two negative integers, (1) add their magnitudes $(10 + 12 = 22)$, and (2) prefix their common negative sign. Thus,

$$-10 + (-12) = -22$$

19. To add two positive integers, (1) add their magnitudes $(5 + 10 = 15)$, and (2) prefix their common negative sign. Thus,

$$5 + 10 = 15$$

21. To add two positive integers, (1) add their magnitudes $(2 + 5 = 7)$, and (2) prefix their common negative sign. Thus,

$$2 + 5 = 7$$

23. To add a positive and a negative integer, subtract the smaller magnitude from the larger magnitude $(19 - 15 = 4)$, then prefix the sign of the integer with the larger magnitude. Thus,

$$19 + (-15) = 4.$$

25. To add two negative integers, (1) add their magnitudes $(2 + 7 = 9)$, and (2) prefix their common negative sign. Thus,

$$-2 + (-7) = -9$$

27. To add a negative and a positive integer, subtract the smaller magnitude from the larger magnitude $(16 - 6 = 10)$, then prefix the sign of the integer with the larger magnitude. Thus,

$$-6 + 16 = 10.$$

29. To add two negative integers, (1) add their magnitudes $(11 + 6 = 17)$, and (2) prefix their common negative sign. Thus,

$$-11 + (-6) = -17$$

31. To add a positive and a negative integer, subtract the smaller magnitude from the larger magnitude $(14-9=5)$, then prefix the sign of the integer with the larger magnitude. Thus,

$$14 + (-9) = 5.$$

33. To add two positive integers, (1) add their magnitudes $(10+11=21)$, and (2) prefix their common negative sign. Thus,

$$10 + 11 = 21$$

35. To add a negative and a positive integer, subtract the smaller magnitude from the larger magnitude $(13 - 1 = 12)$, then prefix the sign of the integer with the larger magnitude. Thus,

$$-13 + 1 = -12.$$

37. The identity

$$-1 + (3 + (-8)) = (-1 + 3) + (-8)$$

is an example of the associative property of addition.

39. The identity

$$7 + (-7) = 0$$

is an example of the additive inverse property.

41. The identity

$$15 + (-18) = -18 + 15$$

is an example of the commutative property of addition.

43. The identity

$$-15 + 0 = -15$$

is an example of the additive identity property.

45. The identity

$$-7 + (1 + (-6)) = (-7 + 1) + (-6)$$

is an example of the associative property of addition.

47. The identity
$$17 + (-2) = -2 + 17$$
is an example of the commutative property of addition.

49. The identity
$$-4 + 0 = -4$$
is an example of the additive identity property.

51. The identity
$$19 + (-19) = 0$$
is an example of the additive inverse property.

53. Because
$$18 + (-18) = 0,$$
the additive inverse of 18 is -18.

55. Because
$$12 + (-12) = 0,$$
the additive inverse of 12 is -12.

57. Because
$$-16 + 16 = 0,$$
the additive inverse of -16 is 16. Alternatively, the additive inverse of -16 is $-(-16)$, which equals 16.

59. Because
$$11 + (-11) = 0,$$
the additive inverse of 11 is -11.

61. Because
$$-15 + 15 = 0,$$
the additive inverse of -15 is 15. Alternatively, the additive inverse of -15 is $-(-15)$, which equals 15.

63. Because
$$-18 + 18 = 0,$$
the additive inverse of -18 is 18. Alternatively, the additive inverse of -18 is $-(-18)$, which equals 18.

65. Because the commutative and associative properties allow us to add in any order, we add the positive integers first, then the negatives. Then we take a final sum.

$$
\begin{aligned}
6 + (-1) + 3 + (-4) &= 9 + (-5) && \text{Add positives: } 6 + 3 = 9; \\
&&& \text{Add negatives: } -1 + (-4) = -5. \\
&= 4 && \text{Add: } 9 + (-5) = 4.
\end{aligned}
$$

67. Perform the additions in the order that they occur, moving from left to right.

$$
\begin{aligned}
15 + (-1) + 2 &= 14 + 2 && \text{Add first two terms: } 15 + (-1) = 14. \\
&= 16 && \text{Add: } 14 + 2 = 16.
\end{aligned}
$$

69. Perform the additions in the order that they occur, moving from left to right.

$$
\begin{aligned}
-17 + 12 + 3 &= -5 + 3 && \text{Add first two terms: } -17 + 12 = -5. \\
&= -2 && \text{Add: } -5 + 3 = -2.
\end{aligned}
$$

71. Perform the additions in the order that they occur, moving from left to right.

$$
\begin{aligned}
7 + 20 + 19 &= 27 + 19 && \text{Add first two terms: } 7 + 20 = 27. \\
&= 46 && \text{Add: } 27 + 19 = 46.
\end{aligned}
$$

73. Because the commutative and associative properties allow us to add in any order, we add the positive integers first, then the negatives. Then we take a final sum.

$$
\begin{aligned}
4 + (-8) + 2 + (-5) &= 6 + (-13) && \text{Add positives: } 4 + 2 = 6; \\
&&& \text{Add negatives: } -8 + (-5) = -13. \\
&= -7 && \text{Add: } 6 + (-13) = -7.
\end{aligned}
$$

75. Because the commutative and associative properties allow us to add in any order, we add the positive integers first, then the negatives. Then we take a final sum.

$$7 + (-8) + 2 + (-1) = 9 + (-9) \qquad \text{Add positives: } 7 + 2 = 9;$$
$$\text{Add negatives: } -8 + (-1) = -9.$$
$$= 0 \qquad \text{Add: } 9 + (-9) = 0.$$

77. Because the commutative and associative properties allow us to add in any order, we add the positive integers first, then the negatives. Then we take a final sum.

$$9 + (-3) + 4 + (-1) = 13 + (-4) \qquad \text{Add positives: } 9 + 4 = 13;$$
$$\text{Add negatives: } -3 + (-1) = -4.$$
$$= 9 \qquad \text{Add: } 13 + (-4) = 9.$$

79. Perform the additions in the order that they occur, moving from left to right.

$$9 + 10 + 2 = 19 + 2 \qquad \text{Add first two terms: } 9 + 10 = 19.$$
$$= 21 \qquad \text{Add: } 19 + 2 = 21.$$

81. To find how much is in Gerry's account now, begin with the original deposit. Withdraws are represented by negative integers and deposits are represented by positive integers. Add together all the amounts withrawn and deposited.

$$215 + (-40) + (-75) + (-20) + 185 \qquad \text{Represent the deposits and withdraws as integers.}$$
$$= \big[215 + 185\big] + \big[(-40) + (-75) + (-20)\big] \qquad \text{Use commutative and associative properties}$$
$$\text{to rearrange integers.}$$
$$= 400 + (-135) \qquad \text{Add positives: } 215 + 185 = 400;$$
$$\text{Add negatives: } -40 + (-75) + (-20) = -135.$$
$$= 265 \qquad \text{Add integers with different signs.}$$

Therefore, Gerry has $265 in his account.

83. We read the following values from the bar chart.

Month	Jan	Feb	Mar	Apr	May	Jun
Profit/Loss	10	6	8	-2	-4	6

Positive integers represent profits; negative integers represent losses. To find the net profit or loss, sum the integers in the table.

$$
\begin{aligned}
10 + 6 + 8 + (-2) + (-4) + 6 &= (10 + 6 + 8 + 6) + (-2 + (-4)) && \text{Reorder and regroup.} \\
&= 30 + (-6) && \text{Sum positives and negatives.} \\
&= 24 && \text{Add.}
\end{aligned}
$$

Because the result is positive and the profit and loss data is scaled in thousands of dollars, there was a net profit of $24,000.

2.3   Subtracting Integers

1. Subtraction means "add the opposite," so change the difference into a sum.

$$
\begin{aligned}
16 - 20 &= 16 + (-20) && \text{Add the opposite.} \\
&= -4 && \text{Add.}
\end{aligned}
$$

3. Subtraction means "add the opposite," so change the difference into a sum.

$$
\begin{aligned}
10 - 12 &= 10 + (-12) && \text{Add the opposite.} \\
&= -2 && \text{Add.}
\end{aligned}
$$

5. Subtraction means "add the opposite," so change the difference into a sum.

$$14 - 11 = 14 + (-11) \qquad \text{Add the opposite.}$$
$$= 3 \qquad \text{Add.}$$

7. Subtraction means "add the opposite," so change the difference into a sum.

$$7 - (-16) = 7 + 16 \qquad \text{Add the opposite.}$$
$$= 23 \qquad \text{Add.}$$

9. Subtraction means "add the opposite," so change the difference into a sum.

$$-4 - (-9) = -4 + 9 \qquad \text{Add the opposite.}$$
$$= 5 \qquad \text{Add.}$$

11. Subtraction means "add the opposite," so change the difference into a sum.

$$8 - (-3) = 8 + 3 \qquad \text{Add the opposite.}$$
$$= 11 \qquad \text{Add.}$$

13. Subtraction means "add the opposite," so change the difference into a sum.

$$2 - 11 = 2 + (-11) \qquad \text{Add the opposite.}$$
$$= -9 \qquad \text{Add.}$$

15. Subtraction means "add the opposite," so change the difference into a sum.

$$-8 - (-10) = -8 + 10 \qquad \text{Add the opposite.}$$
$$= 2 \qquad \text{Add.}$$

17. Subtraction means "add the opposite," so change the difference into a sum.

$$13 - (-1) = 13 + 1 \qquad \text{Add the opposite.}$$
$$= 14 \qquad \text{Add.}$$

19. Subtraction means "add the opposite," so change the difference into a sum.

$$-4 - (-2) = -4 + 2 \qquad \text{Add the opposite.}$$
$$= -2 \qquad \text{Add.}$$

21. Subtraction means "add the opposite," so change the difference into a sum.

$$7 - (-8) = 7 + 8 \qquad \text{Add the opposite.}$$
$$= 15 \qquad \text{Add.}$$

23. Subtraction means "add the opposite," so change the difference into a sum.

$$-3 - (-10) = -3 + 10 \qquad \text{Add the opposite.}$$
$$= 7 \qquad \text{Add.}$$

25. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$14 - 12 - 2 = 14 + (-12) + (-2) \qquad \text{Add the opposite of 12, which is } -12.$$
$$\text{Add the opposite of 2, which is } -2.$$
$$= 2 + (-2) \qquad \text{Add: } 14 + (-12) = 2.$$
$$= 0 \qquad \text{Add: } 2 + (-2) = 0.$$

27. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$-20 - 11 - 18 = -20 + (-11) + (-18) \qquad \text{Add the opposite of 11, which is } -11.$$
$$\text{Add the opposite of 18, which is } -18.$$
$$= -31 + (-18) \qquad \text{Add: } -20 + (-11) = -31.$$
$$= -49 \qquad \text{Add: } -31 + (-18) = -49.$$

29. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$5 - (-10) - 20 = 5 + 10 + (-20) \qquad \text{Add the opposite of } -10, \text{ which is 10.}$$
$$\text{Add the opposite of 20, which is } -20.$$
$$= 15 + (-20) \qquad \text{Add: } 5 + 10 = 15.$$
$$= -5 \qquad \text{Add: } 15 + (-20) = -5.$$

31. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$
\begin{aligned}
-14 - 12 - 19 &= -14 + (-12) + (-19) && \text{Add the opposite of 12, which is } -12. \\
&&& \text{Add the opposite of 19, which is } -19. \\
&= -26 + (-19) && \text{Add: } -14 + (-12) = -26. \\
&= -45 && \text{Add: } -26 + (-19) = -45.
\end{aligned}
$$

33. First, change the subtraction signs to "adding the opposite," then add, moving left to right.

$$
\begin{aligned}
-11 - (-7) - (-6) &= -11 + 7 + 6 && \text{Add the opposite of } -7, \text{ which is 7.} \\
&&& \text{Add the opposite of } -6, \text{ which is 6.} \\
&= -4 + 6 && \text{Add: } -11 + 7 = -4. \\
&= 2 && \text{Add: } -4 + 6 = 2.
\end{aligned}
$$

35. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$
\begin{aligned}
-2 - (-6 - (-5)) &= -2 - (-6 + 5) && \text{Add the opposite of } -5, \text{ or 5.} \\
&= -2 - (-1) && \text{Add: } -6 + 5 = -1. \\
&= -2 + 1 && \text{Add the opposite of } -1, \text{ or 1.} \\
&= -1 && \text{Add: } -2 + 1 = -1.
\end{aligned}
$$

37. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$
\begin{aligned}
(-5 - (-8)) - (-3 - (-2)) &= (-5 + 8) - (-3 + 2) && \text{Add the opposite of } -8, \text{ or 8.} \\
&&& \text{Add the opposite of } -2, \text{ or 2.} \\
&= 3 - (-1) && \text{Add: } -5 + 8 = 3. \\
&&& \text{Add: } -3 + 2 = -1. \\
&= 3 + 1 && \text{Add the opposite of } -1, \text{ or 1.} \\
&= 4 && \text{Add: } 3 + 1 = 4.
\end{aligned}
$$

39. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$(6 - (-9)) - (3 - (-6)) = (6 + 9) - (3 + 6)$$ Add the opposite of -9, or 9.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ Add the opposite of -6, or 6.

$$= 15 - 9$$ Add: $6 + 9 = 15$.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ Add: $3 + 6 = 9$.

$$= 15 + (-9)$$ Add the opposite of 9, or -9.

$$= 6$$ Add: $15 + (-9) = 6$.

41. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$-1 - (10 - (-9)) = -1 - (10 + 9)$$ Add the opposite of -9, or 9.

$$= -1 - 19$$ Add: $10 + 9 = 19$.

$$= -1 + (-19)$$ Add the opposite of 19, or -19.

$$= -20$$ Add: $-1 + (-19) = -20$.

43. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$3 - (-8 - 17) = 3 - (-8 + (-17))$$ Add the opposite of 17, or -17.

$$= 3 - (-25)$$ Add: $-8 + (-17) = -25$.

$$= 3 + 25$$ Add the opposite of -25, or 25.

$$= 28$$ Add: $3 + 25 = 28$.

45. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$13 - (16 - (-1)) = 13 - (16 + 1)$$ Add the opposite of -1, or 1.

$$= 13 - 17$$ Add: $16 + 1 = 17$.

$$= 13 + (-17)$$ Add the opposite of 17, or -17.

$$= -4$$ Add: $13 + (-17) = -4$.

47. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$(7 - (-8)) - (5 - (-2)) = (7 + 8) - (5 + 2) \quad \text{Add the opposite of } -8, \text{ or } 8.$$

<div style="text-align:right; color:red;">Add the opposite of -2, or 2.</div>

$$= 15 - 7 \qquad \text{Add: } 7 + 8 = 15.$$

<div style="text-align:right; color:red;">Add: $5 + 2 = 7$.</div>

$$= 15 + (-7) \qquad \text{Add the opposite of } 7, \text{ or } -7.$$

$$= 8 \qquad \text{Add: } 15 + (-7) = 8.$$

49. First, evaluate what's inside the parentheses. Change the subtraction to "adding the opposite," then add.

$$(6 - 4) - (-8 - 2) = (6 + (-4)) - (-8 + (-2)) \quad \text{Add the opposite of } 4, \text{ or } -4.$$

<div style="text-align:right; color:red;">Add the opposite of 2, or -2.</div>

$$= 2 - (-10) \qquad \text{Add: } 6 + (-4) = 2.$$

<div style="text-align:right; color:red;">Add: $-8 + (-2) = -10$.</div>

$$= 2 + 10 \qquad \text{Add the opposite of } -10, \text{ or } 10.$$

$$= 12 \qquad \text{Add: } 2 + 10 = 12.$$

51. The temperature change is found by subtracting the first temperature from the second temperature.

Change in Temperature	=	Latter Measurement	−	Former Measurement
	=	65° F	−	42° F
	=	23° F		

Hence, the change in temperature is 23° F.

53. The temperature change is found by subtracting the first temperature from the second temperature.

Change in Temperature	=	Latter Measurement	−	Former Measurement
	=	51° F	−	30° F
	=	21° F		

Hence, the change in temperature is 21° F.

55. To find the change in temperature, subtract the earlier temperature from the later temperature.

Change in Temperature	=	Latter Temperature	−	Former Temperature
	=	$-19°$ F	−	$(-2)°$ F
	=	$-19°$ F	+	$2°$ F
	=	$-17°$ F		

Hence, the change in temperature is $-17°$ F.

57. To find the distance the message must travel, subtract the lower elevation from the higher elevation.

$$22,500 - (-1,600) = 22,500 + 1,600 \qquad \text{To subtract, add the opposite.}$$
$$= 24,100 \qquad \text{Add the integers.}$$

The distance the message must travel is 24,100 feet.

59. To find how much lower the shore of the Dead Sea is from Death Valley, subtract the Dead Sea's lower elevation from Death Valley's higher elevation.

$$-282 - (-1,371) = -282 + 1,371 \qquad \text{To subtract, add the opposite.}$$
$$= 1,089 \qquad \text{Add the integers.}$$

The Dead Sea at the Israel-Jordan border is -1,089 feet lower than Death Valley.

2.4   Multiplication and Division of Integers

1. The identity
$$(-2)\big[(-16)(13)\big] = \big[(-2)(-16)\big](13)$$
has the form
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c.$$
Hence, this is an example of the associative property of multiplication.

3. The identity
$$(-17)(-10) = (-10)(-17)$$
has the form
$$a \cdot b = b \cdot a.$$
Hence, this is an example of the commutative property of multiplication.

5. The identity
$$(4)(11) = (11)(4)$$
has the form
$$a \cdot b = b \cdot a.$$
Hence, this is an example of the commutative property of multiplication.

7. The identity
$$16 \cdot (8 + (-15)) = 16 \cdot 8 + 16 \cdot (-15)$$
has the form
$$a \cdot (b + c) = a \cdot b + a \cdot c.$$
Hence, this is an example of the distributive property.

9. The identity
$$(17)\big[(20)(11)\big] = \big[(17)(20)\big](11)$$
has the form
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c.$$
Hence, this is an example of the associative property of multiplication.

11. The identity
$$-19 \cdot 1 = -19$$
has the form
$$a \cdot 1 = a.$$
Hence, this is an example of the multiplicative identity property of multiplication.

13. The identity
$$8 \cdot 1 = 8$$
has the form
$$a \cdot 1 = a.$$
Hence, this is an example of the multiplicative identity property of multiplication.

15. The identity
$$14 \cdot (-12 + 7) = 14 \cdot (-12) + 14 \cdot 7$$
has the form
$$a \cdot (b + c) = a \cdot b + a \cdot c.$$
Hence, this is an example of the distributive property.

17. When multiplying, like signs give a positive answer. Therefore,

$$4 \cdot 7 = 28.$$

19. When multiplying, unlike signs give a negative answer. Therefore,

$$3 \cdot (-3) = -9.$$

21. The property "multiplying by minus one" says that $(-1)a = -a$. Hence,

$$-1 \cdot 10 = -10.$$

23. The multiplicative property of zero says that $a \cdot 0 = 0$. Hence,

$$-1 \cdot 0 = 0.$$

25. The property "multiplying by minus one" says that $(-1)a = -a$. Hence,

$$
\begin{aligned}
-1 \cdot (-14) &= -(-14) &&\text{\color{red}{Multiplication property of -1: $(-1)a = -a$.}}\\
&= 14 &&\text{\color{red}{The opposite of -14 is 14.}}
\end{aligned}
$$

27. The property "multiplying by minus one" says that $(-1)a = -a$. Hence,

$$
\begin{aligned}
-1 \cdot (-19) &= -(-19) &&\text{\color{red}{Multiplication property of -1: $(-1)a = -a$.}}\\
&= 19 &&\text{\color{red}{The opposite of -19 is 19.}}
\end{aligned}
$$

29. The multiplicative property of zero says that $a \cdot 0 = 0$. Hence,

$$2 \cdot 0 = 0.$$

31. When multiplying, unlike signs give a negative answer. Therefore,

$$-3 \cdot 8 = -24.$$

33. When multiplying, like signs give a positive answer. Therefore,

$$7 \cdot 9 = 63.$$

35. The property "multiplying by minus one" says that $(-1)a = -a$. Hence,

$$-1 \cdot 5 = -5.$$

37. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$
\begin{aligned}
(-7)(-1)(3) &= (7)(3) && \text{Work left to right.} \\
&&& \text{Like signs: } (-7)(-1) = 7. \\
&= 21 && \text{Like signs: : } (7)(3) = 21.
\end{aligned}
$$

39. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$
\begin{aligned}
(-7)(9)(10)(-10) &= (-63)(10)(-10) && \text{Work left to right.} \\
&&& \text{Unlike signs: } (-7)(9) = -63. \\
&= (-630)(-10) && \text{Work left to right.} \\
&&& \text{Unlike signs: : } (-63)(10) = -630. \\
&= (6300) && \text{Like signs: : } (-630)(-10) = 6300.
\end{aligned}
$$

41. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$
\begin{aligned}
(6)(5)(8) &= (30)(8) && \text{Work left to right.} \\
&&& \text{Like signs: } (6)(5) = 30. \\
&= 240 && \text{Like signs: : } (30)(8) = 240.
\end{aligned}
$$

43. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$
\begin{aligned}
(-10)(4)(-3)(8) &= (-40)(-3)(8) && \text{Work left to right.} \\
&&& \text{Unlike signs: } (-10)(4) = -40. \\
&= (120)(8) && \text{Work left to right.} \\
&&& \text{Like signs: : } (-40)(-3) = 120. \\
&= (960) && \text{Like signs: : } (120)(8) = 960.
\end{aligned}
$$

45. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$(6)(-3)(-8) = (-18)(-8) \qquad \text{Work left to right.}$$
$$\text{Unlike signs: } (6)(-3) = -18.$$
$$= 144 \qquad \text{Like signs: : } (-18)(-8) = 144.$$

47. Order of operations demands that we work from left to right. *Remember, multiplying like signs gives a positive result, multiplying unlike signs gives a negative result.*

$$(2)(1)(3)(4) = (2)(3)(4) \qquad \text{Work left to right.}$$
$$\text{Like signs: } (2)(1) = 2.$$
$$= (6)(4) \qquad \text{Work left to right.}$$
$$\text{Like signs: : } (2)(3) = 6.$$
$$= (24) \qquad \text{Like signs: : } (6)(4) = 24.$$

49. In the expression $(-4)^4$, the exponent 4 tells us to write the base -4 four times as a factor. Thus,

$$(-4)^4 = (-4)(-4)(-4)(-4).$$

Now, the product of an even number of negative factors is positive.

$$(-4)^4 = 256$$

51. In the expression $(-5)^4$, the exponent 4 tells us to write the base -5 four times as a factor. Thus,

$$(-5)^4 = (-5)(-5)(-5)(-5).$$

Now, the product of an even number of negative factors is positive.

$$(-5)^4 = 625$$

53. In the expression $(-5)^2$, the exponent 2 tells us to write the base -5 two times as a factor. Thus,

$$(-5)^2 = (-5)(-5).$$

Now, the product of an even number of negative factors is positive.

$$(-5)^2 = 25$$

55. In the expression $(-6)^2$, the exponent 2 tells us to write the base -6 two times as a factor. Thus,

$$(-6)^2 = (-6)(-6).$$

Now, the product of an even number of negative factors is positive.

$$(-6)^2 = 36$$

57. In the expression $(-4)^5$, the exponent 5 tells us to write the base -4 five times as a factor. Thus,

$$(-4)^5 = (-4)(-4)(-4)(-4)(-4).$$

Now, the product of an odd number of negative factors is negative.

$$(-4)^5 = -1024$$

59. In the expression $(-5)^3$, the exponent 3 tells us to write the base -5 three times as a factor. Thus,

$$(-5)^3 = (-5)(-5)(-5).$$

Now, the product of an odd number of negative factors is negative.

$$(-5)^3 = -125$$

61. When dividing, like signs give a positive answer. Hence,

$$-16 \div (-8) = 2.$$

63. When dividing, unlike signs give a negative answer. Hence,

$$\frac{-8}{1} = -8.$$

65. Division by 0 is undefined. Thus,

$$\frac{-1}{0}$$

is undefined.

67. When dividing, unlike signs give a negative answer. Hence,
$$-3 \div 3 = -1.$$

69. When dividing, unlike signs give a negative answer. Hence,
$$\frac{56}{-28} = -2.$$

71. When zero is divided by a nonzero number, the answer is zero. That is,
$$0 \div 15 = 0.$$

73. When dividing, like signs give a positive answer. Hence,
$$\frac{63}{21} = 3.$$

75. When dividing, like signs give a positive answer. Hence,
$$\frac{78}{13} = 6.$$

77. When zero is divided by a nonzero number, the answer is zero. That is,
$$0 \div 5 = 0.$$

79. Division by 0 is undefined. Thus,
$$\frac{17}{0}$$
is undefined.

81. When dividing, unlike signs give a negative answer. Hence,
$$-45 \div 15 = -3.$$

83. When dividing, like signs give a positive answer. Hence,
$$12 \div 3 = 4.$$

85. The initial depth of the first diver is 25 feet below sea level, or -25. To find the final depth of the second diver, multiply the depth of the first diver by 5.
$$-25(5) = -125$$
The second diver's final depth is -125 feet, or, 125 feet below sea level.

2.5 Order of Operations with Integers

1. Order of operations demands that we do divisions before additions.

$$
\begin{aligned}
7 - \frac{-14}{2} &= 7 - (-7) && \text{Divide: } -14/2 = -7. \\
&= 7 + 7 && \text{Subtract by adding the opposite.} \\
&= 14 && \text{Add: } 7 + 7 = 14.
\end{aligned}
$$

3. Order of operations demands that we do divisions before additions.

$$
\begin{aligned}
-7 - \frac{-18}{9} &= -7 - (-2) && \text{Divide: } -18/9 = -2. \\
&= -7 + 2 && \text{Subtract by adding the opposite.} \\
&= -5 && \text{Add: } -7 + 2 = -5.
\end{aligned}
$$

5. Order of operations demands that we apply the exponent first. The exponent 4 tells us to write the base 5 as a factor 4 times. That is,

$$
-5^4 = -(5)(5)(5)(5)
$$

Take the product of the factors, then negate.

$$
-5^4 = -625
$$

7.

$$
\begin{aligned}
9 - 1(-7) &= 9 - (-7) && \text{Multiply first: } 1(-7) = -7. \\
&= 9 + 7 && \text{Add the opposite.} \\
&= 16 && \text{Add: } 9 + 7 = 16.
\end{aligned}
$$

9. Order of operations demands that we apply the exponent first. The exponent 3 tells us to write the base 6 as a factor 3 times. That is,

$$
-6^3 = -(6)(6)(6)
$$

Take the product of the factors, then negate.

$$
-6^3 = -216
$$

11. Order of operations demands that multiplication is applied first, then addition.

$$3 + 9(4) = 3 + 36 \qquad \text{Multiply first: } 9(4) = 36.$$
$$= 39 \qquad \text{Add: } 3 + 36 = 39.$$

13. Divisions and multiplications first, in the order that they appear, as you work from left to right.

$$10 - 72 \div 6 \cdot 3 + 8 = 10 - 12 \cdot 3 + 8 \qquad \text{Divide: } 72 \div 6 = 12.$$
$$= 10 - 36 + 8 \qquad \text{Multiply: } 12 \cdot 3 = 36.$$
$$= 10 + (-36) + 8 \qquad \text{Subtract by adding the opposite.}$$

Additions are done in order, working left to right.

$$= -26 + 8 \qquad \text{Add: } 10 + (-36) = -26.$$
$$= -18 \qquad \text{Add: } -26 + 8 = -18.$$

15. Order of operations demands that we do divisions before additions.

$$6 + \frac{14}{2} = 6 + 7 \qquad \text{Divide: } 14/2 = 7.$$
$$= 13 \qquad \text{Add: } 6 + 7 = 13.$$

17. Order of operations demands that we apply the exponent first. The exponent 4 tells us to write the base 3 as a factor 4 times. That is,

$$-3^4 = -(3)(3)(3)(3)$$

Take the product of the factors, then negate.

$$-3^4 = -81$$

19. Divisions and multiplications first, in the order that they appear, as you work from left to right.

$$3 - 24 \div 4 \cdot 3 + 4 = 3 - 6 \cdot 3 + 4 \qquad \text{Divide: } 24 \div 4 = 6.$$
$$= 3 - 18 + 4 \qquad \text{Multiply: } 6 \cdot 3 = 18.$$
$$= 3 + (-18) + 4 \qquad \text{Subtract by adding the opposite.}$$

Additions are done in order, working left to right.

$$= -15 + 4 \qquad \text{Add: } 3 + (-18) = -15.$$
$$= -11 \qquad \text{Add: } -15 + 4 = -11.$$

21. Division and multiplication must be done in the order that they appear, working from left to right.

$$64 \div 4 \cdot 4 = 16 \cdot 4 \qquad \text{Divide: } 64 \div 4 = 16.$$
$$= 64 \qquad \text{Multiply: } 16 \cdot 4 = 64.$$

23.

$$-2 - 3(-5) = -2 - (-15) \qquad \text{Multiply first: } 3(-5) = -15.$$
$$= -2 + 15 \qquad \text{Add the opposite.}$$
$$= 13 \qquad \text{Add: } -2 + 15 = 13.$$

25. Division and multiplication must be done in the order that they appear, working from left to right.

$$15 \div 1 \cdot 3 = 15 \cdot 3 \qquad \text{Divide: } 15 \div 1 = 15.$$
$$= 45 \qquad \text{Multiply: } 15 \cdot 3 = 45.$$

27. Divisions and multiplications first, in the order that they appear, as you work from left to right.

$$8 + 12 \div 6 \cdot 1 - 5 = 8 + 2 \cdot 1 - 5 \qquad \text{Divide: } 12 \div 6 = 2.$$
$$= 8 + 2 - 5 \qquad \text{Multiply: } 2 \cdot 1 = 2.$$
$$= 8 + 2 + (-5) \qquad \text{Subtract by adding the opposite.}$$

Additions are done in order, working left to right.

$$= 10 + (-5) \qquad \text{Add: } 8 + 2 = 10.$$
$$= 5 \qquad \text{Add: } 10 + (-5) = 5.$$

29. Division and multiplication must be done in the order that they appear, working from left to right.

$$32 \div 4 \cdot 4 = 8 \cdot 4 \qquad \text{Divide: } 32 \div 4 = 8.$$
$$= 32 \qquad \text{Multiply: } 8 \cdot 4 = 32.$$

31. Order of operations demands that we do divisions before additions.

$$-11 + \frac{16}{16} = -11 + 1 \qquad \text{Divide: } 16/16 = 1.$$
$$= -10 \qquad \text{Add: } -11 + 1 = -10.$$

33. Order of operations demands that we apply the exponent first. The exponent 2 tells us to write the base 5 as a factor 2 times. That is,

$$-5^2 = -(5)(5)$$

Take the product of the factors, then negate.

$$-5^2 = -25$$

35. Order of operations demands that multiplication is applied first, then addition.

$$
\begin{aligned}
10 + 12(-5) &= 10 + (-60) &&\text{Multiply first: } 12(-5) = -60.\\
&= -50 &&\text{Add: } 10 + (-60) = -50.
\end{aligned}
$$

37. Divisions and multiplications first, in the order that they appear, as you work from left to right.

$$
\begin{aligned}
2 + 6 \div 1 \cdot 6 - 1 &= 2 + 6 \cdot 6 - 1 &&\text{Divide: } 6 \div 1 = 6.\\
&= 2 + 36 - 1 &&\text{Multiply: } 6 \cdot 6 = 36.\\
&= 2 + 36 + (-1) &&\text{Subtract by adding the opposite.}
\end{aligned}
$$

Additions are done in order, working left to right.

$$
\begin{aligned}
&= 38 + (-1) &&\text{Add: } 2 + 36 = 38.\\
&= 37 &&\text{Add: } 38 + (-1) = 37.
\end{aligned}
$$

39. Division and multiplication must be done in the order that they appear, working from left to right.

$$
\begin{aligned}
40 \div 5 \cdot 4 &= 8 \cdot 4 &&\text{Divide: } 40 \div 5 = 8.\\
&= 32 &&\text{Multiply: } 8 \cdot 4 = 32.
\end{aligned}
$$

41. Absolute value bars are like grouping symbols, only stronger. We must simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
-11 + |-1 - (-6)^2| &= -11 + |-1 - 36| &&\text{Inside the abolute value bars,}\\
&&&\text{Apply exponent: } (-6)^2 = 36.\\
&= -11 + |-1 + (-36)| &&\text{Subtract: Add the opposite.}\\
&= -11 + |-37| &&\text{Add: } -1 + (-36) = -37.\\
&= -11 + 37 &&\text{Take absolute value: } |-37| = 37.\\
&= 26 &&\text{Add: } -11 + 37 = 26.
\end{aligned}
$$

43. Absolute values act like grouping symbols, only stronger. Therefore, we must simplify the expression inside the absolute value first.

$$|0(-4)| - 4(-4) = |0| - 4(-4) \qquad \text{Inside absolute value bars,}$$

Multiply: $0(-4) = 0$.

$$= 0 - 4(-4) \qquad \text{Take absolute value: } |0| = 0.$$
$$= 0 - (-16) \qquad \text{Multiply: } 4(-4) = -16.$$
$$= 0 + 16 \qquad \text{Subtract: Add the opposite.}$$
$$= 16 \qquad \text{Add: } 0 + 16 = 16.$$

45. The parenthetical expression must be simplified first. Inside the parentheses, order of operations demands that we multiply first.

$$(2 + 3 \cdot 4) - 8 = (2 + 12) - 8 \qquad \text{Multiply: } 3 \cdot 4 = 12.$$
$$= 14 - 8 \qquad \text{Add: } 2 + 12 = 14.$$
$$= 6 \qquad \text{Subtract: } 14 - 8 = 6.$$

47. The parenthetical expression must be simplified first. Inside the parentheses, order of operations demands that we multiply first.

$$(8 - 1 \cdot 12) + 4 = (8 - 12) + 4 \qquad \text{Multiply: } 1 \cdot 12 = 12.$$
$$= -4 + 4 \qquad \text{Subtract: } 8 - 12 = -4.$$
$$= 0 \qquad \text{Add: } -4 + 4 = 0.$$

49. The parenthetical expression must be simplified first. Inside the parentheses, order of operations demands that we multiply first.

$$(6 + 10 \cdot 4) - 6 = (6 + 40) - 6 \qquad \text{Multiply: } 10 \cdot 4 = 40.$$
$$= 46 - 6 \qquad \text{Add: } 6 + 40 = 46.$$
$$= 40 \qquad \text{Subtract: } 46 - 6 = 40.$$

51.

$$10 + (6 - 4)^3 - 3 = 10 + 2^3 - 3 \qquad \text{Subtract: } 6 - 4 = 2.$$
$$= 10 + 8 - 3 \qquad \text{Apply the exponent: } 2^3 = 8.$$
$$= 10 + 8 + (-3) \qquad \text{Subtract: Add the opposite.}$$
$$= 18 + (-3) \qquad \text{Add: } 10 + 8 = 18.$$
$$= 15 \qquad \text{Add: } 18 + (-3) = 15.$$

53. We must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$
\begin{aligned}
(6-8)^2 - (4-7)^2 &= (-2)^2 - (-3)^3 && \text{Subtract: } 6-8=-2 \text{ and } 4-7=-3. \\
&= 4 - (-27) && \text{Square: } (-2)^2 = 4. \\
& && \text{Cube: } (-3)^3 = -27. \\
&= 4 + 27 && \text{Subtract: Add the opposite.} \\
&= 31 && \text{Add: } 4+27 = 31.
\end{aligned}
$$

55. Absolute values act like grouping symbols, only stronger. Therefore, we must simplify the expression inside the absolute value first.

$$
\begin{aligned}
|0(-10)| + 4(-4) &= |0| + 4(-4) && \text{Inside absolute value bars,} \\
& && \text{Multiply: } 0(-10) = 0. \\
&= 0 + 4(-4) && \text{Take absolute value: } |0| = 0. \\
&= 0 + (-16) && \text{Multiply: } 4(-4) = -16. \\
&= -16 && \text{Add: } 0 + (-16) = -16.
\end{aligned}
$$

57. Absolute values act like grouping symbols, only stronger. Therefore, we must simplify the expression inside the absolute value first.

$$
\begin{aligned}
|8(-1)| - 8(-7) &= |-8| - 8(-7) && \text{Inside absolute value bars,} \\
& && \text{Multiply: } 8(-1) = -8. \\
&= 8 - 8(-7) && \text{Take absolute value: } |-8| = 8. \\
&= 8 - (-56) && \text{Multiply: } 8(-7) = -56. \\
&= 8 + 56 && \text{Subtract: Add the opposite.} \\
&= 64 && \text{Add: } 8 + 56 = 64.
\end{aligned}
$$

59.

$$
\begin{aligned}
3 + (3-8)^2 - 7 &= 3 + (-5)^2 - 7 && \text{Subtract: } 3-8 = -5. \\
&= 3 + 25 - 7 && \text{Apply the exponent: } (-5)^2 = 25. \\
&= 3 + 25 + (-7) && \text{Subtract: Add the opposite.} \\
&= 28 + (-7) && \text{Add: } 3 + 25 = 28. \\
&= 21 && \text{Add: } 28 + (-7) = 21.
\end{aligned}
$$

61. We must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$
\begin{aligned}
(4-2)^2 - (7-2)^2 &= 2^2 - 5^3 && \text{Subtract: } 4-2=2 \text{ and } 7-2=5.\\
&= 4 - 125 && \text{Square: } 2^2 = 4.\\
&&& \text{Cube: } 5^3 = 125.\\
&= 4 + (-125) && \text{Subtract: Add the opposite.}\\
&= -121 && \text{Add: } 4 + (-125) = -121.
\end{aligned}
$$

63. Absolute value bars are like grouping symbols, only stronger. We must simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
8 - |-25 - (-4)^2| &= 8 - |-25 - 16| && \text{Inside the abolute value bars,}\\
&&& \text{Apply exponent: } (-4)^2 = 16.\\
&= 8 - |-25 + (-16)| && \text{Subtract: Add the opposite.}\\
&= 8 - |-41| && \text{Add: } -25 + (-16) = -41.\\
&= 8 - 41 && \text{Take absolute value: } |-41| = 41.\\
&= 8 + (-41) && \text{Subtract: Add the opposite.}\\
&= -33 && \text{Add: } 8 + (-41) = -33.
\end{aligned}
$$

65. Absolute value bars are like grouping symbols, only stronger. We must simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
-4 - |30 - (-5)^2| &= -4 - |30 - 25| && \text{Inside the abolute value bars,}\\
&&& \text{Apply exponent: } (-5)^2 = 25.\\
&= -4 - |30 + (-25)| && \text{Subtract: Add the opposite.}\\
&= -4 - |5| && \text{Add: } 30 + (-25) = 5.\\
&= -4 - 5 && \text{Take absolute value: } |5| = 5.\\
&= -4 + (-5) && \text{Subtract: Add the opposite.}\\
&= -9 && \text{Add: } -4 + (-5) = -9.
\end{aligned}
$$

67. We must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$
\begin{aligned}
(8-7)^2 - (2-6)^2 &= 1^2 - (-4)^3 && \text{Subtract: } 8-7=1 \text{ and } 2-6=-4.\\
&= 1 - (-64) && \text{Square: } 1^2 = 1.\\
&&& \text{Cube: } (-4)^3 = -64.\\
&= 1 + 64 && \text{Subtract: Add the opposite.}\\
&= 65 && \text{Add: } 1 + 64 = 65.
\end{aligned}
$$

69. Simplify the expression inside the parentheses first, then apply the exponent, then add and subtract, moving left to right.

$$
\begin{aligned}
4 - (3-6)^3 + 4 &= 4 - (-3)^3 + 4 && \text{Subtract: } 3 - 6 = -3. \\
&= 4 - (-27) + 4 && \text{Apply the exponent: } (-3)^3 = -27. \\
&= 4 + 27 + 4 && \text{Subtract: Add the opposite.} \\
&= 31 + 4 && \text{Add: } 4 + 27 = 31. \\
&= 35 && \text{Add: } 31 + 4 = 35.
\end{aligned}
$$

71. Absolute value bars are like grouping symbols, only stronger. We must simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
-3 + |-22 - 5^2| &= -3 + |-22 - 25| && \text{Inside the abolute value bars,} \\
& && \text{Apply exponent: } 5^2 = 25. \\
&= -3 + |-22 + (-25)| && \text{Subtract: Add the opposite.} \\
&= -3 + |-47| && \text{Add: } -22 + (-25) = -47. \\
&= -3 + 47 && \text{Take absolute value: } |-47| = 47. \\
&= 44 && \text{Add: } -3 + 47 = 44.
\end{aligned}
$$

73. The parenthetical expression must be simplified first. Inside the parentheses, order of operations demands that we multiply first.

$$
\begin{aligned}
(3 - 4 \cdot 1) + 6 &= (3 - 4) + 6 && \text{Multiply: } 4 \cdot 1 = 4. \\
&= -1 + 6 && \text{Subtract: } 3 - 4 = -1. \\
&= 5 && \text{Add: } -1 + 6 = 5.
\end{aligned}
$$

75. Simplify the expression inside the parentheses first, then apply the exponent, then add and subtract, moving left to right.

$$
\begin{aligned}
1 - (1-5)^2 + 11 &= 1 - (-4)^2 + 11 && \text{Subtract: } 1 - 5 = -4. \\
&= 1 - 16 + 11 && \text{Apply the exponent: } (-4)^2 = 16. \\
&= 1 + (-16) + 11 && \text{Subtract: Add the opposite.} \\
&= -15 + 11 && \text{Add: } 1 + (-16) = -15. \\
&= -4 && \text{Add: } -15 + 11 = -4.
\end{aligned}
$$

77. We must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$(2-6)^2 - (8-6)^2 = (-4)^2 - 2^3 \qquad \text{Subtract: } 2-6 = -4 \text{ and } 8-6 = 2.$$
$$= 16 - 8 \qquad \text{Square: } (-4)^2 = 16.$$
$$\text{Cube: } 2^3 = 8.$$
$$= 16 + (-8) \qquad \text{Subtract: Add the opposite.}$$
$$= 8 \qquad \text{Add: } 16 + (-8) = 8.$$

79. Absolute values act like grouping symbols, only stronger. Therefore, we must simplify the expression inside the absolute value first.

$$|9(-3)| + 12(-2) = |-27| + 12(-2) \qquad \text{Inside absolute value bars,}$$
$$\text{Multiply: } 9(-3) = -27.$$
$$= 27 + 12(-2) \qquad \text{Take absolute value: } |-27| = 27.$$
$$= 27 + (-24) \qquad \text{Multiply: } 12(-2) = -24.$$
$$= 3 \qquad \text{Add: } 27 + (-24) = 3.$$

81. We must simplify numerator first. This requires that we first multiply.

$$\frac{4(-10) - 5}{-9} = \frac{-40 - 5}{-9} \qquad \text{Multiply: } 4(-10) = -40.$$
$$= \frac{-40 + (-5)}{-9} \qquad \text{Subtract: Add the opposite.}$$
$$= \frac{-45}{-9} \qquad \text{Add: } -40 + (-5) = -45.$$
$$= 5 \qquad \text{Divide: } -45/(-9) = 5.$$

83. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{10^2 - 4^2}{2 \cdot 6 - 10} = \frac{100 - 16}{12 - 10} \qquad \text{Numerator: } 10^2 = 100, \ 4^2 = 16.$$
$$\text{Denominator: } 2 \cdot 6 = 12.$$
$$= \frac{84}{2} \qquad \text{Numerator: } 100 - 16 = 84.$$
$$\text{Denominator: } 12 - 10 = 2.$$
$$= 42 \qquad \text{Divide: } 84/2 = 42.$$

85. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{3^2 + 6^2}{5 - 1 \cdot 8} = \frac{9 + 36}{5 - 8}$$

Numerator: $3^2 = 9$, $6^2 = 36$.

Denominator: $1 \cdot 8 = 8$.

$$= \frac{45}{-3}$$

Numerator: $9 + 36 = 45$.

Denominator: $5 - 8 = -3$.

$$= -15$$

Divide: $45/(-3) = -15$.

87. With a fractional expression, we must simplify numerator and denominator first, then divide.

$$\frac{-8 - 4}{7 - 13} = \frac{-8 + (-4)}{7 + (-13)}$$

In numerator and denominator,

add the opposite.

$$= \frac{-12}{-6}$$

Numerator: $-8 + (-4) = -12$.

Denominator: $7 + (-13) = -6$.

$$= 2$$

Divide: $-12/(-6) = 2$.

89. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{2^2 + 6^2}{11 - 4 \cdot 4} = \frac{4 + 36}{11 - 16}$$

Numerator: $2^2 = 4$, $6^2 = 36$.

Denominator: $4 \cdot 4 = 16$.

$$= \frac{40}{-5}$$

Numerator: $4 + 36 = 40$.

Denominator: $11 - 16 = -5$.

$$= -8$$

Divide: $40/(-5) = -8$.

91. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{1^2 - 5^2}{9 \cdot 1 - 5} = \frac{1 - 25}{9 - 5}$$

Numerator: $1^2 = 1$, $5^2 = 25$.

Denominator: $9 \cdot 1 = 9$.

$$= \frac{-24}{4}$$

Numerator: $1 - 25 = -24$.

Denominator: $9 - 5 = 4$.

$$= -6$$

Divide: $-24/4 = -6$.

93. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{4^2 - 8^2}{6 \cdot 3 - 2} = \frac{16 - 64}{18 - 2}$$ Numerator: $4^2 = 16$, $8^2 = 64$.

Denominator: $6 \cdot 3 = 18$.

$$= \frac{-48}{16}$$ Numerator: $16 - 64 = -48$.

Denominator: $18 - 2 = 16$.

$$= -3$$ Divide: $-48/16 = -3$.

95. First, simplify numerator and denominator. In the numerator, exponents first, then subtract. In the denominator, multiply, then subtract.

$$\frac{10^2 + 2^2}{7 - 2 \cdot 10} = \frac{100 + 4}{7 - 20}$$ Numerator: $10^2 = 100$, $2^2 = 4$.

Denominator: $2 \cdot 10 = 20$.

$$= \frac{104}{-13}$$ Numerator: $100 + 4 = 104$.

Denominator: $7 - 20 = -13$.

$$= -8$$ Divide: $104/(-13) = -8$.

97. With a fractional expression, we must simplify numerator and denominator first, then divide.

$$\frac{16 - (-2)}{19 - 1} = \frac{16 + 2}{19 + (-1)}$$ In numerator and denominator,

add the opposite.

$$= \frac{18}{18}$$ Numerator: $16 + 2 = 18$.

Denominator: $19 + (-1) = 18$.

$$= 1$$ Divide: $18/18 = 1$.

99. With a fractional expression, we must simplify numerator and denominator first, then divide.

$$\frac{15 - (-15)}{13 - (-17)} = \frac{15 + 15}{13 + 17}$$ In numerator and denominator,

add the opposite.

$$= \frac{30}{30}$$ Numerator: $15 + 15 = 30$.

Denominator: $13 + 17 = 30$.

$$= 1$$ Divide: $30/30 = 1$.

101. We must simplify numerator first. This requires that we first multiply.

$$\frac{4 \cdot 5 - (-19)}{3} = \frac{20 - (-19)}{3} \qquad \text{Multiply: } 4 \cdot 5 = 20.$$

$$= \frac{20 + 19}{3} \qquad \text{Subtract: Add the opposite.}$$

$$= \frac{39}{3} \qquad \text{Add: } 20 + 19 = 39.$$

$$= 13 \qquad \text{Divide: } 39/3 = 13.$$

103. We must simplify numerator first. This requires that we first multiply.

$$\frac{-6 \cdot 9 - (-4)}{2} = \frac{-54 - (-4)}{2} \qquad \text{Multiply: } -6 \cdot 9 = -54.$$

$$= \frac{-54 + 4}{2} \qquad \text{Subtract: Add the opposite.}$$

$$= \frac{-50}{2} \qquad \text{Add: } -54 + 4 = -50.$$

$$= -25 \qquad \text{Divide: } -50/2 = -25.$$

2.6   Solving Equations Involving Integers

1. To see if -11 is a solution of $2x + 3 = -19$, we substitute -11 for x in the equation and check to see if this results in a true or false statement.

$$2x + 3 = -19 \qquad \text{Original equation.}$$
$$2(-11) + 3 = -19 \qquad \text{Substitute } x = -11.$$
$$-22 + 3 = -19 \qquad \text{On the left, multiply: } 2(-11) = -22.$$
$$-19 = -19 \qquad \text{On the left, add: } -22 + 3 = -19.$$

This last statement is a true statement. Therefore, -11 is a solution of the equation $2x + 3 = -19$.

3. To see if 6 is a solution of $3x + 1 = 19$, we substitute 6 for x in the equation and check to see if this results in a true or false statement.

$$3x + 1 = 19 \qquad \text{Original equation.}$$
$$3(6) + 1 = 19 \qquad \text{Substitute } x = 6.$$
$$18 + 1 = 19 \qquad \text{On the left, multiply: } 3(6) = 18.$$
$$19 = 19 \qquad \text{On the left, add: } 18 + 1 = 19.$$

This last statement is a true statement. Therefore, 6 is a solution of the equation $3x + 1 = 19$.

5. To see if 12 is a solution of $4x + 5 = -8$, we substitute 12 for x in the equation and check to see if this results in a true or false statement.

$$
\begin{aligned}
4x + 5 &= -8 &&\text{Original equation.} \\
4(12) + 5 &= -8 &&\text{Substitute } x = 12. \\
48 + 5 &= -8 &&\text{On the left, multiply: } 4(12) = 48. \\
53 &= -8 &&\text{On the left, add: } 48 + 5 = 53.
\end{aligned}
$$

This last statement is a false statement. Therefore, 12 is **not** a solution of the equation $4x + 5 = -8$.

7. To see if 15 is a solution of $2x + 6 = -9$, we substitute 15 for x in the equation and check to see if this results in a true or false statement.

$$
\begin{aligned}
2x + 6 &= -9 &&\text{Original equation.} \\
2(15) + 6 &= -9 &&\text{Substitute } x = 15. \\
30 + 6 &= -9 &&\text{On the left, multiply: } 2(15) = 30. \\
36 &= -9 &&\text{On the left, add: } 30 + 6 = 36.
\end{aligned}
$$

This last statement is a false statement. Therefore, 15 is **not** a solution of the equation $2x + 6 = -9$.

9. To see if -15 is a solution of $-3x + 6 = -17$, we substitute -15 for x in the equation and check to see if this results in a true or false statement.

$$
\begin{aligned}
-3x + 6 &= -17 &&\text{Original equation.} \\
-3(-15) + 6 &= -17 &&\text{Substitute } x = -15. \\
45 + 6 &= -17 &&\text{On the left, multiply: } -3(-15) = 45. \\
51 &= -17 &&\text{On the left, add: } 45 + 6 = 51.
\end{aligned}
$$

This last statement is a false statement. Therefore, -15 is **not** a solution of the equation $-3x + 6 = -17$.

11. To see if -6 is a solution of $-2x + 3 = 15$, we substitute -6 for x in the equation and check to see if this results in a true or false statement.

$$
\begin{aligned}
-2x + 3 &= 15 &&\text{Original equation.} \\
-2(-6) + 3 &= 15 &&\text{Substitute } x = -6. \\
12 + 3 &= 15 &&\text{On the left, multiply: } -2(-6) = 12. \\
15 &= 15 &&\text{On the left, add: } 12 + 3 = 15.
\end{aligned}
$$

This last statement is a true statement. Therefore, -6 is a solution of the equation $-2x + 3 = 15$.

13. To undo the effect of subtracting 13, add -13 from both sides of the equation.

$$
\begin{aligned}
x - 13 &= 11 && \text{Original equation.} \\
x - 13 + 13 &= 11 + 13 && \text{Add 13 to both sides.} \\
x &= 24 && \text{On the left, simplify.} \\
& && \text{On the right: } 11 + 13 = 24.
\end{aligned}
$$

15. To undo the effect of subtracting 3, add -3 from both sides of the equation.

$$
\begin{aligned}
x - 3 &= 6 && \text{Original equation.} \\
x - 3 + 3 &= 6 + 3 && \text{Add 3 to both sides.} \\
x &= 9 && \text{On the left, simplify.} \\
& && \text{On the right: } 6 + 3 = 9.
\end{aligned}
$$

17. To undo the effect of adding 10, subtract 10 from both sides of the equation.

$$
\begin{aligned}
x + 10 &= 17 && \text{Original equation.} \\
x + 10 - 10 &= 17 - 10 && \text{Subtract 10 from both sides.} \\
x &= 17 + (-10) && \text{On the left, simplify.} \\
& && \text{On the right, add the opposite.} \\
x &= 7 && \text{Add: } 17 + (-10) = 7.
\end{aligned}
$$

19. To undo the effect of subtracting 6, add -6 from both sides of the equation.

$$
\begin{aligned}
x - 6 &= 1 && \text{Original equation.} \\
x - 6 + 6 &= 1 + 6 && \text{Add 6 to both sides.} \\
x &= 7 && \text{On the left, simplify.} \\
& && \text{On the right: } 1 + 6 = 7.
\end{aligned}
$$

21. To undo the effect of subtracting 15, add -15 from both sides of the equation.

$$
\begin{aligned}
x - 15 &= -12 && \text{Original equation.} \\
x - 15 + 15 &= -12 + 15 && \text{Add 15 to both sides.} \\
x &= 3 && \text{On the left, simplify.} \\
& && \text{On the right: } -12 + 15 = 3.
\end{aligned}
$$

23. To undo the effect of adding 11, subtract 11 from both sides of the equation.

$$x + 11 = -19 \qquad \text{Original equation.}$$
$$x + 11 - 11 = -19 - 11 \qquad \text{Subtract 11 from both sides.}$$
$$x = -19 + (-11) \qquad \text{On the left, simplify.}$$
$$\text{On the right, add the opposite.}$$
$$x = -30 \qquad \text{Add: } -19 + (-11) = -30.$$

25. To undo the effect of adding 2, subtract 2 from both sides of the equation.

$$x + 2 = 1 \qquad \text{Original equation.}$$
$$x + 2 - 2 = 1 - 2 \qquad \text{Subtract 2 from both sides.}$$
$$x = 1 + (-2) \qquad \text{On the left, simplify.}$$
$$\text{On the right, add the opposite.}$$
$$x = -1 \qquad \text{Add: } 1 + (-2) = -1.$$

27. To undo the effect of adding 5, subtract 5 from both sides of the equation.

$$x + 5 = -5 \qquad \text{Original equation.}$$
$$x + 5 - 5 = -5 - 5 \qquad \text{Subtract 5 from both sides.}$$
$$x = -5 + (-5) \qquad \text{On the left, simplify.}$$
$$\text{On the right, add the opposite.}$$
$$x = -10 \qquad \text{Add: } -5 + (-5) = -10.$$

29. To undo the effect of multiplying by -1, divide both sides of the equation by -1.

$$-x = -20 \qquad \text{Original equation.}$$
$$\frac{-x}{-1} = \frac{-20}{-1} \qquad \text{Divide boths sides by } -1.$$
$$x = 20 \qquad \text{On the left, simplify.}$$
$$\text{On the right, divide: } -20/(-1) = 20.$$

31. To undo the effect of dividing by -7, multiply both sides of the equation by -7.

$$\frac{x}{-7} = 10 \qquad \text{Original equation.}$$
$$-7\left(\frac{x}{-7}\right) = -7\,(10) \qquad \text{Multiply both sides by } -7.$$
$$x = -70 \qquad \text{On the left, simplify.}$$
$$\text{On the right, multiply: } -7(10) = -70.$$

33. To undo the effect of dividing by -10, multiply both sides of the equation by -10.

$$\frac{x}{-10} = 12 \qquad \text{Original equation.}$$

$$-10\left(\frac{x}{-10}\right) = -10\,(12) \qquad \text{Multiply both sides by } -10.$$

$$x = -120 \qquad \begin{array}{l}\text{On the left, simplify.} \\ \text{On the right, multiply: } -10(12) = -120.\end{array}$$

35. To undo the effect of dividing by 9, multiply both sides of the equation by 9.

$$\frac{x}{9} = -16 \qquad \text{Original equation.}$$

$$9\left(\frac{x}{9}\right) = 9\,(-16) \qquad \text{Multiply both sides by } 9.$$

$$x = -144 \qquad \begin{array}{l}\text{On the left, simplify.} \\ \text{On the right, multiply: } 9(-16) = -144.\end{array}$$

37. To undo the effect of multiplying by -10, divide both sides of the equation by -10.

$$-10x = 20 \qquad \text{Original equation.}$$

$$\frac{-10x}{-10} = \frac{20}{-10} \qquad \text{Divide boths sides by } -10.$$

$$x = -2 \qquad \begin{array}{l}\text{On the left, simplify.} \\ \text{On the right, divide: } 20/(-10) = -2.\end{array}$$

39. To undo the effect of multiplying by 14, divide both sides of the equation by 14.

$$14x = 84 \qquad \text{Original equation.}$$

$$\frac{14x}{14} = \frac{84}{14} \qquad \text{Divide boths sides by } 14.$$

$$x = 6 \qquad \begin{array}{l}\text{On the left, simplify.} \\ \text{On the right, divide: } 84/14 = 6.\end{array}$$

41. To undo the effect of multiplying by -2, divide both sides of the equation by -2.

$$-2x = 28 \qquad \text{Original equation.}$$

$$\frac{-2x}{-2} = \frac{28}{-2} \qquad \text{Divide boths sides by } -2.$$

$$x = -14 \qquad \begin{array}{l}\text{On the left, simplify.} \\ \text{On the right, divide: } 28/(-2) = -14.\end{array}$$

43. To undo the effect of dividing by -10, multiply both sides of the equation by -10.

$$\frac{x}{-10} = 15 \qquad \text{Original equation.}$$

$$-10\left(\frac{x}{-10}\right) = -10\,(15) \qquad \text{Multiply both sides by } -10.$$

$$x = -150 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, multiply: } -10(15) = -150.\end{array}$$

45. On the left, order of operations demands that we first multiply by -4, then subtract 4. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 4 to both sides of the equation, then divide both sides of the resulting equation by -4.

$$-4x - 4 = 16 \qquad \text{Original equation.}$$

$$-4x - 4 + 4 = 16 + 4 \qquad \text{Subtract } -4 \text{ from both sides.}$$

$$-4x = 20 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, add: } 16 + 4 = 20.\end{array}$$

$$\frac{-4x}{-4} = \frac{20}{-4} \qquad \text{Divide both sides by } -4.$$

$$x = -5 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, } 20/(-4) = -5.\end{array}$$

47. On the left, order of operations demands that we first multiply by 4, then subtract 4. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 4 to both sides of the equation, then divide both sides of the resulting equation by 4.

$$4x - 4 = 76 \qquad \text{Original equation.}$$

$$4x - 4 + 4 = 76 + 4 \qquad \text{Subtract } -4 \text{ from both sides.}$$

$$4x = 80 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, add: } 76 + 4 = 80.\end{array}$$

$$\frac{4x}{4} = \frac{80}{4} \qquad \text{Divide both sides by 4.}$$

$$x = 20 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, } 80/4 = 20.\end{array}$$

49. On the left, order of operations demands that we first multiply by 5, then subtract 14. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 14 to both sides of the

equation, then divide both sides of the resulting equation by 5.

$$5x - 14 = -79 \qquad \text{Original equation.}$$

$$5x - 14 + 14 = -79 + 14 \qquad \text{Subtract } -14 \text{ from both sides.}$$

$$5x = -65 \qquad \text{On the left, simplify.}$$
On the right, add: $-79 + 14 = -65$.

$$\frac{5x}{5} = \frac{-65}{5} \qquad \text{Divide both sides by 5.}$$

$$x = -13 \qquad \text{On the left, simplify.}$$
On the right, $-65/5 = -13$.

51. On the left, order of operations demands that we first multiply by -10, then subtract 16. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 16 to both sides of the equation, then divide both sides of the resulting equation by -10.

$$-10x - 16 = 24 \qquad \text{Original equation.}$$

$$-10x - 16 + 16 = 24 + 16 \qquad \text{Subtract } -16 \text{ from both sides.}$$

$$-10x = 40 \qquad \text{On the left, simplify.}$$
On the right, add: $24 + 16 = 40$.

$$\frac{-10x}{-10} = \frac{40}{-10} \qquad \text{Divide both sides by } -10.$$

$$x = -4 \qquad \text{On the left, simplify.}$$
On the right, $40/(-10) = -4$.

53. On the left, order of operations demands that we first multiply by 9, then add 5. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 5 from both sides of the equation, then divide both sides of the resulting equation by 9.

$$9x + 5 = -85 \qquad \text{Original equation.}$$

$$9x + 5 - 5 = -85 - 5 \qquad \text{Subtract 5 from both sides.}$$

$$9x = -85 + (-5) \qquad \text{On the left, simplify.}$$
On the right, add the opposite.

$$9x = -90 \qquad \text{Add: } -85 + (-5) = -90.$$

$$\frac{9x}{9} = \frac{-90}{9} \qquad \text{Divide both sides by 9.}$$

$$x = -10 \qquad \text{On the left, simplify.}$$
On the right, $-90/9 = -10$.

55. On the left, order of operations demands that we first multiply by 7, then add 15. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 15 from both sides of the equation, then divide both sides of the resulting equation by 7.

$$7x + 15 = -55 \qquad \text{Original equation.}$$
$$7x + 15 - 15 = -55 - 15 \qquad \text{Subtract 15 from both sides.}$$
$$7x = -55 + (-15) \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, add the opposite.}\end{array}$$
$$7x = -70 \qquad \text{Add: } -55 + (-15) = -70.$$
$$\frac{7x}{7} = \frac{-70}{7} \qquad \text{Divide both sides by 7.}$$
$$x = -10 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, } -70/7 = -10.\end{array}$$

57. On the left, order of operations demands that we first multiply by -1, then add 8. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 8 from both sides of the equation, then divide both sides of the resulting equation by -1.

$$-x + 8 = 13 \qquad \text{Original equation.}$$
$$-x + 8 - 8 = 13 - 8 \qquad \text{Subtract 8 from both sides.}$$
$$-1x = 13 + (-8) \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, add the opposite.}\end{array}$$
$$-1x = 5 \qquad \text{Add: } 13 + (-8) = 5.$$
$$\frac{-1x}{-1} = \frac{5}{-1} \qquad \text{Divide both sides by } -1.$$
$$x = -5 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, } 5/(-1) = -5.\end{array}$$

59. On the left, order of operations demands that we first multiply by 12, then subtract 15. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 15 to both sides of the equation, then divide both sides of the resulting equation by 12.

$$12x - 15 = -3 \qquad \text{Original equation.}$$
$$12x - 15 + 15 = -3 + 15 \qquad \text{Subtract } -15 \text{ from both sides.}$$
$$12x = 12 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, add: } -3 + 15 = 12.\end{array}$$
$$\frac{12x}{12} = \frac{12}{12} \qquad \text{Divide both sides by 12.}$$
$$x = 1 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, } 12/12 = 1.\end{array}$$

61. On the left, order of operations demands that we first multiply by 4, then subtract 12. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 12 to both sides of the equation, then divide both sides of the resulting equation by 4.

$$4x - 12 = -56 \qquad \text{Original equation.}$$
$$4x - 12 + 12 = -56 + 12 \qquad \text{Subtract } -12 \text{ from both sides.}$$
$$4x = -44 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, add: } -56 + 12 = -44.\end{array}$$
$$\frac{4x}{4} = \frac{-44}{4} \qquad \text{Divide both sides by 4.}$$
$$x = -11 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, } -44/4 = -11.\end{array}$$

63. On the left, order of operations demands that we first multiply by 19, then add 18. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 18 from both sides of the equation, then divide both sides of the resulting equation by 19.

$$19x + 18 = 113 \qquad \text{Original equation.}$$
$$19x + 18 - 18 = 113 - 18 \qquad \text{Subtract 18 from both sides.}$$
$$19x = 113 + (-18) \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, add the opposite.}\end{array}$$
$$19x = 95 \qquad \text{Add: } 113 + (-18) = 95.$$
$$\frac{19x}{19} = \frac{95}{19} \qquad \text{Divide both sides by 19.}$$
$$x = 5 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, } 95/19 = 5.\end{array}$$

65. On the left, order of operations demands that we first multiply by -14, then add 12. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 12 from both sides of the equation, then divide both sides of the resulting equation by -14.

$$-14x + 12 = -2 \qquad \text{Original equation.}$$
$$-14x + 12 - 12 = -2 - 12 \qquad \text{Subtract 12 from both sides.}$$
$$-14x = -2 + (-12) \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, add the opposite.}\end{array}$$
$$-14x = -14 \qquad \text{Add: } -2 + (-12) = -14.$$
$$\frac{-14x}{-14} = \frac{-14}{-14} \qquad \text{Divide both sides by } -14.$$
$$x = 1 \qquad \begin{array}{l}\text{On the left, simplify.}\\ \text{On the right, } -14/(-14) = 1.\end{array}$$

67. On the left, order of operations demands that we first multiply by 14, then add 16. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 16 from both sides of the equation, then divide both sides of the resulting equation by 14.

$$14x + 16 = 44 \qquad \text{Original equation.}$$
$$14x + 16 - 16 = 44 - 16 \qquad \text{Subtract 16 from both sides.}$$
$$14x = 44 + (-16) \qquad \text{On the left, simplify.}$$
$$\text{On the right, add the opposite.}$$
$$14x = 28 \qquad \text{Add: } 44 + (-16) = 28.$$
$$\frac{14x}{14} = \frac{28}{14} \qquad \text{Divide both sides by 14.}$$
$$x = 2 \qquad \text{On the left, simplify.}$$
$$\text{On the right, } 28/14 = 2.$$

69. In our solution, we address each step of the *Requirements for Word Problem Solutions*.

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "Two less than eight times certain number is -74" becomes:

eight times a certain number	less	Two	is	-74
$8x$	$-$	2	$=$	-74

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 8, then subtract 2. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) add 2 from both sides of the equation, then (2) divide both sides of the resulting equation by 8.

$$8x - 2 = -74 \qquad \text{Original equation.}$$
$$8x - 2 + 2 = -74 + 2 \qquad \text{Add 2 to both sides.}$$
$$8x = -72 \qquad \text{Simplify both sides.}$$
$$\frac{8x}{8} = \frac{-72}{8} \qquad \text{Divide both sides by 8.}$$
$$x = -9 \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The unknown number is -9.

5. *Look Back.* Does the answer satisfy the problem constraints? Two less than 8 times -9 is 2 less than -72, or -74. So the solution is correct.

71. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "Eight more than two times certain number is 0" becomes:

Eight	more than	two times a certain number	is	0
8	+	$2x$	=	0

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 2, then add 8. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract 8 from both sides of the equation, then (2) divide both sides of the resulting equation by 2.

$$
\begin{aligned}
8 + 2x &= 0 && \text{Original equation.} \\
8 + 2x - 8 &= 0 - 8 && \text{Subtract 8 from both sides.} \\
2x &= -8 && \text{Simplify both sides.} \\
\frac{2x}{2} &= \frac{-8}{2} && \text{Divide both sides by 2.} \\
x &= -4 && \text{Simplify both sides.}
\end{aligned}
$$

4. *Answer the Question.* The unknown number is -4.

5. *Look Back.* Does the answer satisfy the problem constraints? Eight more than 2 times -4 is 8 more than -8, or 0. So the solution is correct.

73. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "The number -6 is 2 more than an unknown number" becomes:

-6	is	2	more than	unknown number
-6	=	2	+	x

3. *Solve the Equation.* To "undo" adding 2, subtract 2 from both sides of the equation.

$$-6 = 2 + x \qquad \text{Original equation.}$$
$$-6 - 2 = 2 + x - 2 \qquad \text{Subtract 2 from both sides.}$$
$$-8 = x \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The unknown number is -8.

5. *Look Back.* Does the answer satisfy the problem constraints? Well, 2 more than -8 is -6, so the answer is correct.

75. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "Three more than eight times certain number is -29" becomes:

Three	more than	eight times a certain number	is	-29
3	$+$	$8x$	$=$	-29

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 8, then add 3. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract 3 from both sides of the equation, then (2) divide both sides of the resulting equation by 8.

$$3 + 8x = -29 \qquad \text{Original equation.}$$
$$3 + 8x - 3 = -29 - 3 \qquad \text{Subtract 3 from both sides.}$$
$$8x = -32 \qquad \text{Simplify both sides.}$$
$$\frac{8x}{8} = \frac{-32}{8} \qquad \text{Divide both sides by 8.}$$
$$x = -4 \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The unknown number is -4.

5. *Look Back.* Does the answer satisfy the problem constraints? Three more than 8 times -4 is 3 more than -32, or -29. So the solution is correct.

77. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the score on Alan's fourth exam. Let x represent Alan's score on his fourth exam.

2. *Set up an Equation.* To find the average of four exam scores, sum the four scores, then divide by 4.

Sum of four exam scores	divided by	4	equals	Average score
$(79 + 61 + 54 + x)$	$\div$	4	$=$	71

This last result can be simplified by summing the three known exam scores.
$$\frac{194 + x}{4} = 71$$

3. *Solve the Equation.* To "undo" the effect of dividing by 4, multiply both sides of the equation by 4.

$$\frac{194 + x}{4} = 71 \qquad \text{Original equation.}$$
$$4\left(\frac{194 + x}{4}\right) = 4(71) \qquad \text{Multiply both sides by 4.}$$
$$x + 194 = 284 \qquad \text{Simplify both sides.}$$

To "undo" the effect of adding 194, subtract 194 from both sides of the equation.

$$x + 194 - 194 = 284 - 194 \qquad \text{Subtract 194 from both sides.}$$
$$x = 90 \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The fourth exam score is 90.

5. *Look Back.* Add the four exam scores, 79, 61, 54, and 90, to get 284. Divide this sum by 4 to get 71, which is the required average. Hence, our solution is correct.

79. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* One interpretation of "the quotient of -2 and x is 5 is:

-2	divided by	x	equals	5
-2	$\div$	x	$=$	5

This can be written as follows:

$$\frac{-2}{x} = 5$$

3. *Solve the Equation.* To "undo" dividing by x, multiply both sides of the equation by x.

$$\frac{-2}{x} = 5 \qquad \text{Original equation.}$$

$$x\left(\frac{-2}{x}\right) = x(5) \qquad \text{Multiply both sides by } x.$$

$$-2 = 5x \qquad \text{Simplify both sides.}$$

$$\frac{-2}{5} = \frac{5x}{5} \qquad \text{Divide both sides by 5.}$$

$$-\frac{2}{5} = x \qquad \text{Simplify.}$$

However, $-2/5$ is not an integer. This is the wrong interpretation of the "quotient." Another interpretation of the quotient of -2 and x is:

x	divided by	-2	equals	5
x	$\div$	-2	$=$	5

This can be written as follows:

$$\frac{x}{-2} = 5$$

4. *Solve the Equation.* To "undo" dividing by -2, multiply both sides of the equation by -2.

$$\frac{x}{-2} = 5 \qquad \text{Original equation.}$$

$$-2\left(\frac{x}{-2}\right) = -2(5) \qquad \text{Multiply both sides by } -2.$$

$$x = -10 \qquad \text{Simplify both sides.}$$

This result is an integer.

5. *Answer the Question.* The unknown number is -10.

6. *Look Back.* Does the answer satisfy the problem constraints? Well, the quotient of -10 and -2 is 5, so our answer is correct.

81. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* One interpretation of "the quotient of -8 and x is 9 is:

-8	divided by	x	equals	9
-8	$\div$	x	$=$	9

This can be written as follows:

$$\frac{-8}{x} = 9$$

3. *Solve the Equation.* To "undo" dividing by x, multiply both sides of the equation by x.

$$\frac{-8}{x} = 9 \qquad\qquad \text{Original equation.}$$

$$x\left(\frac{-8}{x}\right) = x(9) \qquad\qquad \text{Multiply both sides by } x.$$

$$-8 = 9x \qquad\qquad \text{Simplify both sides.}$$

$$\frac{-8}{9} = \frac{9x}{9} \qquad\qquad \text{Divide both sides by 9.}$$

$$-\frac{8}{9} = x \qquad\qquad \text{Simplify.}$$

However, $-8/9$ is not an integer. This is the wrong interpretation of the "quotient." Another interpretation of the quotient of -8 and x is:

x	divided by	-8	equals	9
x	$\div$	-8	$=$	9

This can be written as follows:

$$\frac{x}{-8} = 9$$

4. *Solve the Equation.* To "undo" dividing by -8, multiply both sides of the equation by -8.

$$\frac{x}{-8} = 9 \qquad \text{Original equation.}$$

$$-8\left(\frac{x}{-8}\right) = -8(9) \qquad \text{Multiply both sides by } -8.$$

$$x = -72 \qquad \text{Simplify both sides.}$$

This result is an integer.

5. *Answer the Question.* The unknown number is -72.

6. *Look Back.* Does the answer satisfy the problem constraints? Well, the quotient of -72 and -8 is 9, so our answer is correct.

83. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "The number -5 is 8 more than an unknown number" becomes:

-5	is	8	more than	unknown number
-5	$=$	8	$+$	x

3. *Solve the Equation.* To "undo" adding 8, subtract 8 from both sides of the equation.

$$-5 = 8 + x \qquad \text{Original equation.}$$

$$-5 - 8 = 8 + x - 8 \qquad \text{Subtract 8 from both sides.}$$

$$-13 = x \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The unknown number is -13.

5. *Look Back.* Does the answer satisfy the problem constraints? Well, 8 more than -13 is -5, so the answer is correct.

85. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the original balance in the student's account. Let B represent this original balance.

2. *Set up an Equation.* A positive integer represents a healthy balance, while a negative number represents an account that is overdrawn. After the student's deposit, the account is still overdrawn by $70. We will say that this balance is $-$70$. Thus,

Original Balance	plus	Student Deposit	equals	Current Balance
B	$+$	$260	$=$	$-$70

3. *Solve the Equation.* To "undo" the addition, subtract 260 from both sides of the equation.

$$\begin{aligned} B + 260 &= -70 && \text{Original equation.} \\ B + 260 - 260 &= -70 - 260 && \text{Subtract 260 from both sides.} \\ B &= -330 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The original balance was overdrawn to the tune of 330 dollars.

5. *Look Back.* If the original balance was overdrawn by $330, then we let $-$330$ represent this balance. The student makes a deposit of $260. Add this to the original balance to get $-$330 + $120 = -$70$, the correct current balance.

87. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the original balance in the student's account. Let B represent this original balance.

2. *Set up an Equation.* A positive integer represents a healthy balance, while a negative number represents an account that is overdrawn. After the student's deposit, the account is still overdrawn by $90. We will say that this balance is $-$90$. Thus,

Original Balance	plus	Student Deposit	equals	Current Balance
B	$+$	$360	$=$	$-$90

3. *Solve the Equation.* To "undo" the addition, subtract 360 from both sides of the equation.

$$\begin{aligned} B + 360 &= -90 && \text{Original equation.} \\ B + 360 - 360 &= -90 - 360 && \text{Subtract 360 from both sides.} \\ B &= -450 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The original balance was overdrawn to the tune of 450 dollars.

5. *Look Back.* If the original balance was overdrawn by $450, then we let $-$450$ represent this balance. The student makes a deposit of $360. Add this to the original balance to get $-$450 + $120 = -$90$, the correct current balance.

89. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "The number -10 is -5 times larger than an unknown number" becomes:

-10	is	-5	times	unknown number
-10	$=$	-5	$\cdot$	x

3. *Solve the Equation.* To "undo" multiplying by -5, divide both sides of the equation by -5.

$$-10 = -5x \qquad \text{Original equation.}$$
$$\frac{-10}{-5} = \frac{-5x}{-5} \qquad \text{Divide both sides by } -5.$$
$$2 = x \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The unknown number is 2.

5. *Look Back.* Does the answer satisfy the problem constraints? Well, -5 times 2 is -10, so the answer is correct.

91. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "The number -15 is -5 times larger than an unknown number" becomes:

-15	is	-5	times	unknown number
-15	$=$	-5	$\cdot$	x

3. *Solve the Equation.* To "undo" multiplying by -5, divide both sides of the equation by -5.

$$-15 = -5x \qquad \text{Original equation.}$$

$$\frac{-15}{-5} = \frac{-5x}{-5} \qquad \text{Divide both sides by } -5.$$

$$3 = x \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The unknown number is 3.

5. *Look Back.* Does the answer satisfy the problem constraints? Well, -5 times 3 is -15, so the answer is correct.

93. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x represent the unknown number.

2. *Set up an Equation.* "Two less than nine times certain number is 7" becomes:

nine times a certain number	less	Two	is	7
$9x$	$-$	2	$=$	7

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 9, then subtract 2. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) add 2 from both sides of the equation, then (2) divide both sides of the resulting equation by 9.

$$9x - 2 = 7 \qquad \text{Original equation.}$$

$$9x - 2 + 2 = 7 + 2 \qquad \text{Add 2 to both sides.}$$

$$9x = 9 \qquad \text{Simplify both sides.}$$

$$\frac{9x}{9} = \frac{9}{9} \qquad \text{Divide both sides by 9.}$$

$$x = 1 \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The unknown number is 1.

5. *Look Back.* Does the answer satisfy the problem constraints? Two less than 9 times 1 is 2 less than 9, or 7. So the solution is correct.

95. In our solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, the unknown is the score on Mark's fourth exam. Let x represent Mark's score on his fourth exam.

2. *Set up an Equation.* To find the average of four exam scores, sum the four scores, then divide by 4.

Sum of four exam scores	divided by	4	equals	Average score
$(79 + 84 + 71 + x)$	$\div$	4	$=$	74

   This last result can be simplified by summing the three known exam scores.
$$\frac{234 + x}{4} = 74$$

3. *Solve the Equation.* To "undo" the effect of dividing by 4, multiply both sides of the equation by 4.

$$\frac{234 + x}{4} = 74 \qquad \text{Original equation.}$$
$$4\left(\frac{234 + x}{4}\right) = 4(74) \qquad \text{Multiply both sides by 4.}$$
$$x + 234 = 296 \qquad \text{Simplify both sides.}$$

   To "undo" the effect of adding 234, subtract 234 from both sides of the equation.

$$x + 234 - 234 = 296 - 234 \qquad \text{Subtract 234 from both sides.}$$
$$x = 62 \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The fourth exam score is 62.

5. *Look Back.* Add the four exam scores, 79, 84, 71, and 62, to get 296. Divide this sum by 4 to get 74, which is the required average. Hence, our solution is correct.

Chapter *3*

The Fundamentals of Algebra

3.1 Mathematical Expressions

1. The word "times" corresponds to multiplication. Therefore, the width is multiplied by 8. In terms of the variable n, the corresponding mathematical expression is $8n$.

3. The word "sum" corresponds to addition, and the word "times" corresponds to multiplication. Therefore, the numbers n and 3 are first added, and then the result is multiplied by 6. In terms of the variable n, the corresponding mathematical expression is $6(n + 3)$.

5. When a quantity is quadrupled, it is multiplied by 4. Therefore, in terms of the variable b, the corresponding mathematical expression is $4b$.

7. To decrease a quantity is by 33, we must subtract 33. Therefore, in terms of the variable y, the corresponding mathematical expression is $y - 33$.

9. The word "times" corresponds to multiplication. Therefore, the width is multiplied by 10. In terms of the variable n, the corresponding mathematical expression is $10n$.

11. The word "sum" corresponds to addition, and the word "times" corresponds to multiplication. Therefore, the numbers z and 2 are first added, and then the result is multiplied by 9. In terms of the variable z, the corresponding mathematical expression is $9(z + 2)$.

13. When a quantity is doubled, it is multiplied by 2. Therefore, in terms of the variable y, the corresponding mathematical expression is $2y$.

15. The phrase "more than" corresponds to addition, and the word "times" corresponds to multiplication. Therefore, the number p is multiplied by 15, and then 13 is added. In terms of the variable p, the corresponding mathematical expression is $15p + 13$.

17. The phrase "less than" corresponds to subtraction, and the word "times" corresponds to multiplication. Therefore, the number x is multiplied by 11, and then 4 is subtracted. In terms of the variable x, the corresponding mathematical expression is $11x - 4$.

19. To decrease a quantity is by 10, we must subtract 10. Therefore, in terms of the variable u, the corresponding mathematical expression is $u - 10$.

21.

i) Since n is representing a whole number, adding one to n would give you the next whole number after n. For instance, suppose $n = 3$. Then $n + 1$ would be $3 + 1 = 4$.

ii) Since n is representing a whole number, adding two to n would give two *more than n*. For instance, suppose $n = 3$. Then $n + 2$ would represent $3 + 2 = 5$.

iii) Since n represents a whole number, subtracting one from n would represent *one less than n*. For instance, suppose $n = 3$. Then $n - 1$ would represent $3 - 1 = 2$.

23. Since $2n + 1$ is an odd whole number, the next odd number would be *two more than* $2n + 1 + 2$, represented by $2n + 3$.

25. The statement says "Steve sells twice as much product as Mike". The amount of product that Steve sells in given in terms of what Mike sells. So let Mike's sales be represented by the variable p. Once we have an expression of Mike's sales, we can write Steve's sales as twice as much as Mike's, or, $2p$.

3.2 Evaluating Algebraic Expressions

1. First replace all occurrences of the variables in the expression with open parentheses:

$$-3x^2 - 6x + 3 = -3(\ \ )^2 - 6(\ \ ) + 3$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
-3x^2 - 6x + 3 &= -3(7)^2 - 6(7) + 3 && \text{Substitute 7 for } x. \\
&= -3(49) - 6(7) + 3 && \text{Evaluate exponents first.} \\
&= -147 - 42 + 3 && \text{Perform multiplications, left to right.} \\
&= -186 && \text{Perform additions and subtractions, left to right.}
\end{aligned}
$$

3. First replace all occurrences of the variables in the expression with open parentheses:

$$-6x - 6 = -6(\ \ ) - 6$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
-6x - 6 &= -6(3) - 6 && \text{Substitute 3 for } x. \\
&= -18 - 6 && \text{Multiply first: } -6(3) = -18 \\
&= -24 && \text{Subtract.}
\end{aligned}
$$

5. First replace all occurrences of the variables in the expression with open parentheses:

$$5x^2 + 2x + 4 = 5(\ \ )^2 + 2(\ \ ) + 4$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
5x^2 + 2x + 4 &= 5(-1)^2 + 2(-1) + 4 && \text{Substitute } -1 \text{ for } x. \\
&= 5(1) + 2(-1) + 4 && \text{Evaluate exponents first.} \\
&= 5 - 2 + 4 && \text{Perform multiplications, left to right.} \\
&= 7 && \text{Perform additions and subtractions, left to right.}
\end{aligned}
$$

7. First replace all occurrences of the variables in the expression with open parentheses:

$$-9x - 5 = -9(\ \ ) - 5$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
-9x - 5 &= -9(-2) - 5 && \text{Substitute } -2 \text{ for } x. \\
&= 18 - 5 && \text{Multiply first: } -9(-2) = 18 \\
&= 13 && \text{Subtract.}
\end{aligned}
$$

9. First replace all occurrences of the variables in the expression with open parentheses:

$$4x^2 + 2x + 6 = 4(\quad)^2 + 2(\quad) + 6$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
4x^2 + 2x + 6 &= 4(-6)^2 + 2(-6) + 6 && \text{Substitute } -6 \text{ for } x. \\
&= 4(36) + 2(-6) + 6 && \text{Evaluate exponents first.} \\
&= 144 - 12 + 6 && \text{Perform multiplications, left to right.} \\
&= 138 && \text{Perform additions and subtractions, left to right.}
\end{aligned}
$$

11. First replace all occurrences of the variables in the expression with open parentheses:

$$12x + 10 = 12(\quad) + 10$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
12x + 10 &= 12(-12) + 10 && \text{Substitute } -12 \text{ for } x. \\
&= -144 + 10 && \text{Multiply first: } 12(-12) = -144 \\
&= -134 && \text{Add.}
\end{aligned}
$$

13. First replace all occurrences of the variables in the expression with open parentheses:

$$|x| - |y| = |(\quad)| - |(\quad)|$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
|x| - |y| &= |(-5)| - |(4)| && \text{Substitute } -5 \text{ for } x \text{ and } 4 \text{ for } y. \\
&= 5 - 4 && \text{Compute absolute values first.} \\
&= 1 && \text{Subtract.}
\end{aligned}
$$

15. First replace all occurrences of the variables in the expression with open parentheses:

$$-5x^2 + 2y^2 = -5(\quad)^2 + 2(\quad)^2$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
-5x^2 + 2y^2 &= -5(4)^2 + 2(2)^2 && \text{Substitute } 4 \text{ for } x \text{ and } 2 \text{ for } y. \\
&= -5(16) + 2(4) && \text{Evaluate exponents first.} \\
&= -80 + 8 && \text{Perform multiplications, left to right.} \\
&= -72 && \text{Perform additions and subtractions, left to right.}
\end{aligned}
$$

17. First replace all occurrences of the variables in the expression with open parentheses:

$$|x| - |y| = |(\ \ )| - |(\ \ )|$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
|x| - |y| &= |(0)| - |(2)| && \text{\color{red}Substitute 0 for } x \text{ \color{red}and 2 for } y. \\
&= -2 && \text{\color{red}Compute absolute values first.} \\
&= -2 && \text{\color{red}Subtract.}
\end{aligned}
$$

19. First replace all occurrences of the variables in the expression with open parentheses:

$$|x - y| = |(\ \ ) - (\ \ )|$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
|x - y| &= |(4) - (5)| && \text{\color{red}Substitute 4 for } x \text{ \color{red}and 5 for } y. \\
&= |-1| && \text{\color{red}Subtract.} \\
&= 1 && \text{\color{red}Compute the absolute value.}
\end{aligned}
$$

21. First replace all occurrences of the variables in the expression with open parentheses:

$$5x^2 - 4xy + 3y^2 = 5(\ \ )^2 - 4(\ \ )(\ \ ) + 3(\ \ )^2$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
5x^2 - 4xy + 3y^2 &= 5(1)^2 - 4(1)(-4) + 3(-4)^2 && \text{\color{red}Substitute 1 for } x \text{ \color{red}and } -4 \text{ \color{red}for } y. \\
&= 5(1) - 4(1)(-4) + 3(16) && \text{\color{red}Evaluate exponents first.} \\
&= 5 + 16 + 48 && \text{\color{red}Perform multiplications, left to right.} \\
&= 69 && \text{\color{red}Perform additions and subtractions, left to right.}
\end{aligned}
$$

23. First replace all occurrences of the variables in the expression with open parentheses:

$$|x - y| = |(\ \ ) - (\ \ )|$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
|x - y| &= |(4) - (4)| && \text{\color{red}Substitute 4 for } x \text{ \color{red}and 4 for } y. \\
&= |0| && \text{\color{red}Subtract.} \\
&= 0 && \text{\color{red}Compute the absolute value.}
\end{aligned}
$$

25. First replace all occurrences of the variables in the expression with open parentheses:

$$-5x^2 - 3xy + 5y^2 = -5(\ \ )^2 - 3(\ \ )(\ \ ) + 5(\ \ )^2$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
-5x^2 - 3xy + 5y^2 &= -5(-1)^2 - 3(-1)(-2) + 5(-2)^2 &&\text{Substitute } -1 \text{ for } x \text{ and } -2 \text{ for } y. \\
&= -5(1) - 3(-1)(-2) + 5(4) &&\text{Evaluate exponents first.} \\
&= -5 - 6 + 20 &&\text{Perform multiplications, left to right.} \\
&= 9 &&\text{Perform additions and subtractions, left to right}
\end{aligned}
$$

27. First replace all occurrences of the variables in the expression with open parentheses:

$$5x^2 + 4y^2 = 5(\ \ )^2 + 4(\ \ )^2$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
5x^2 + 4y^2 &= 5(-2)^2 + 4(-2)^2 &&\text{Substitute } -2 \text{ for } x \text{ and } -2 \text{ for } y. \\
&= 5(4) + 4(4) &&\text{Evaluate exponents first.} \\
&= 20 + 16 &&\text{Perform multiplications, left to right.} \\
&= 36 &&\text{Perform additions and subtractions, left to right.}
\end{aligned}
$$

29. First replace all occurrences of the variables in the expression with open parentheses:

$$\frac{9 + 9x}{-x} = \frac{9 + 9(\ \ )}{-(\ \ )}$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
\frac{9 + 9x}{-x} &= \frac{9 + 9(-3)}{-(-3)} &&\text{Substitute } -3 \text{ for } x. \\
&= \frac{-18}{3} &&\text{Evaluate numerator and denominator.} \\
&= -6 &&\text{Divide.}
\end{aligned}
$$

31. First replace all occurrences of the variables in the expression with open parentheses:

$$\frac{-8x + 9}{-9 + x} = \frac{-8(\ \ ) + 9}{-9 + (\ \ )}$$

Then replace each variable with its given value, and evaluate the expression:

$$
\begin{aligned}
\frac{-8x + 9}{-9 + x} &= \frac{-8(10) + 9}{-9 + (10)} &&\text{Substitute } 10 \text{ for } x. \\
&= \frac{-71}{1} &&\text{Evaluate numerator and denominator.} \\
&= -71 &&\text{Divide.}
\end{aligned}
$$

33. First replace all occurrences of the variables in the expression with open parentheses:

$$\frac{-4 + 9x}{7x} = \frac{-4 + 9()}{7()}$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned} \frac{-4 + 9x}{7x} &= \frac{-4 + 9(2)}{7(2)} && \text{Substitute 2 for } x. \\ &= \frac{14}{14} && \text{Evaluate numerator and denominator.} \\ &= 1 && \text{Divide.} \end{aligned}$$

35. First replace all occurrences of the variables in the expression with open parentheses:

$$\frac{-12 - 7x}{x} = \frac{-12 - 7()}{()}$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned} \frac{-12 - 7x}{x} &= \frac{-12 - 7(-1)}{(-1)} && \text{Substitute } -1 \text{ for } x. \\ &= \frac{-5}{-1} && \text{Evaluate numerator and denominator.} \\ &= 5 && \text{Divide.} \end{aligned}$$

37. First replace all occurrences of the variables in the expression with open parentheses:

$$\frac{6x - 10}{5 + x} = \frac{6() - 10}{5 + ()}$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned} \frac{6x - 10}{5 + x} &= \frac{6(-6) - 10}{5 + (-6)} && \text{Substitute } -6 \text{ for } x. \\ &= \frac{-46}{-1} && \text{Evaluate numerator and denominator.} \\ &= 46 && \text{Divide.} \end{aligned}$$

39. First replace all occurrences of the variables in the expression with open parentheses:

$$\frac{10x + 11}{5 + x} = \frac{10() + 11}{5 + ()}$$

Then replace each variable with its given value, and evaluate the expression:

$$\frac{10x+11}{5+x} = \frac{10(-4)+11}{5+(-4)}$$ Substitute -4 for x.

$$= \frac{-29}{1}$$ Evaluate numerator and denominator.

$$= -29$$ Divide.

41. First replace all occurrences of the variables in the expression with open parentheses:

$$16t^2 = 16(\ \ )^2$$

Then replace each variable with its given value, and evaluate the expression:

$$d = 16(4)^2 \text{ feet}$$ Substitute 4 for t.

$$= 16(16) \text{ feet}$$ Evaluate exponents first.

$$= 256 \text{ feet}$$ Multiply.

43. First replace all occurrences of the variables in the expression with open parentheses:

$$\frac{5(F-32)}{9} = \frac{5((\ \ )-32)}{9}$$

Then replace each variable with its given value, and evaluate the expression:

$$C = \frac{5((230)-32)}{9} \text{ degrees}$$ Substitute 230 for F.

$$= \frac{5(198)}{9} \text{ degrees}$$ Subtract inside the parentheses in the numerator.

$$= \frac{990}{9} \text{ degrees}$$ Multiply in the numerator.

$$= 110 \text{ degrees}$$ Divide.

45. Prepare the formula for a substitution for K.

$$F = \frac{9\left((\ \ )-273\right)}{5} + 32$$

Now, substitute 28 for K and simplify.

$$F = \frac{9\left((28) - 273\right)}{5} + 32 \qquad \text{Substitute 28 for } K.$$

$$= \frac{9(-245)}{5} + 32 \qquad \text{Parentheses: } 28 - 273 = -245.$$

$$= \frac{-2205}{5} + 32 \qquad \text{Multiply: } 9(-245) = -2205.$$

$$= -441 + 32 \qquad \text{Divide: } -2205/5 = -441.$$

$$= -409 \qquad \text{Add: } -441 + 32 = -409.$$

Hence, the Fahrenheit temperature is $F = -409°\,\text{F}$.

47. If we substitute the initial velocity 272 for v_0 and the acceleration 32 for g, the formula
$$v = v_0 - gt$$
becomes
$$v = 272 - 32t.$$

To find the velocity at $t = 6$ seconds, substitute 6 for t and simplify.

$$v = 272 - 32t \qquad \text{Velocity equation.}$$
$$v = 272 - 32(6) \qquad \text{Substitute 6 for } t.$$
$$v = 272 - 192 \qquad \text{Multiply: } 32(6) = 192.$$
$$v = 80 \qquad \text{Add: } 272 - 192 = 80.$$

Thus, the velocity at 6 seconds is 80 feet per second.

49.

  i) Let $n = 1$. Then, $2 \cdot 1 = 2$

  ii) Let $n = 2$. Then, $2 \cdot 2 = 4$

  iii) Let $n = 3$. Then, $2 \cdot 3 = 6$

  iv) Let $n = -4$. Then, $2 \cdot -4 = -8$

  v) Let $n = -5$. Then, $2 \cdot -5 = -10$

  vi) Any number multiplied by 2 will always result in an even number as the result will always have 2 as a factor. Any number with 2 as a factor must necessarily be even.

3.3   Simplifying Algebraic Expressions

1. Use the associative property to regroup, then simplify.

$$10(-4x) = (10 \cdot (-4))x \qquad \text{Apply the associative property.}$$
$$= -40x \qquad \text{Simplify.}$$

3. Use the commutative and associative properties to reorder and regroup, then simplify.

$$(-10x)(-3) = ((-10) \cdot (-3))x \qquad \text{Change the order and regroup.}$$
$$= 30x \qquad \text{Simplify.}$$

5. Use the associative property to regroup, then simplify.

$$-5(3x) = ((-5) \cdot 3)x \qquad \text{Apply the associative property.}$$
$$= -15x \qquad \text{Simplify.}$$

7. Use the commutative and associative properties to reorder and regroup, then simplify.

$$(-4x)10 = ((-4) \cdot 10)x \qquad \text{Change the order and regroup.}$$
$$= -40x \qquad \text{Simplify.}$$

9. Use the commutative and associative properties to reorder and regroup, then simplify.

$$(5x)3 = (5 \cdot 3)x \qquad \text{Change the order and regroup.}$$
$$= 15x \qquad \text{Simplify.}$$

11. Use the commutative and associative properties to reorder and regroup, then simplify.

$$(5x)10 = (5 \cdot 10)x \qquad \text{Change the order and regroup.}$$
$$= 50x \qquad \text{Simplify.}$$

13. Use the associative property to regroup, then simplify.

$$-9(-7x) = ((-9) \cdot (-7))x \qquad \text{Apply the associative property.}$$
$$= 63x \qquad \text{Simplify.}$$

15. Use the associative property to regroup, then simplify.

$$6(2x) = (6 \cdot 2)x \qquad \text{Apply the associative property.}$$
$$= 12x \qquad \text{Simplify.}$$

17. Use the associative property to regroup, then simplify.

$$-8(-9x) = ((-8) \cdot (-9))x \qquad \text{Apply the associative property.}$$
$$= 72x \qquad \text{Simplify.}$$

19. Use the commutative and associative properties to reorder and regroup, then simplify.

$$(6x)7 = (6 \cdot 7)x \qquad \text{Change the order and regroup.}$$
$$= 42x \qquad \text{Simplify.}$$

21. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$8(7x + 8) = 8(7x) + 8(8) \qquad \text{Apply the distributive property.}$$
$$= 56x + 64 \qquad \text{Simplify.}$$

23. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$9(-2 + 10x) = 9(-2) + 9(10x) \qquad \text{Apply the distributive property.}$$
$$= -18 + 90x \qquad \text{Simplify.}$$

25. To negate a sum, simply negate each term of the sum:

$$-(-2x + 10y - 6) = 2x - 10y + 6$$

27. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$2(10 + x) = 2(10) + 2(x) \qquad \text{Apply the distributive property.}$$
$$= 20 + 2x \qquad \text{Simplify.}$$

29. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$3(3 + 4x) = 3(3) + 3(4x) \qquad \text{Apply the distributive property.}$$
$$= 9 + 12x \qquad \text{Simplify.}$$

31. To negate a sum, simply negate each term of the sum:

$$-(-5 - 7x + 2y) = 5 + 7x - 2y$$

33. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$4(-6x + 7) = 4(-6x) + 4(7) \qquad \text{Apply the distributive property.}$$
$$= -24x + 28 \qquad \text{Simplify.}$$

35. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$4(8x - 9) = 4(8x) - 4(9) \qquad \text{Apply the distributive property.}$$
$$= 32x - 36 \qquad \text{Simplify.}$$

37. To negate a sum, simply negate each term of the sum:

$$-(4 - 2x - 10y) = -4 + 2x + 10y$$

39. To negate a sum, simply negate each term of the sum:

$$-(-5x + 1 + 9y) = 5x - 1 - 9y$$

41. To negate a sum, simply negate each term of the sum:

$$-(6x + 2 - 10y) = -6x - 2 + 10y$$

43. To negate a sum, simply negate each term of the sum:

$$-(-3y - 4 + 4x) = 3y + 4 - 4x$$

3.4 Combining Like Terms

1. First use the distributive property to factor out the common variable part. Then simplify.

$$17xy^2 + 18xy^2 + 20xy^2 = (17 + 18 + 20)xy^2 \qquad \text{Distributive property.}$$
$$= 55xy^2 \qquad \text{Simplify.}$$

3. First use the distributive property to factor out the common variable part. Then simplify.

$$-8xy^2 - 3xy^2 - 10xy^2 = (-8 - 3 - 10)xy^2 \qquad \text{Distributive property.}$$
$$= -21xy^2 \qquad \text{Simplify.}$$

5. First use the distributive property to factor out the common variable part. Then simplify.

$$4xy - 20xy = (4 - 20)xy \qquad \text{Distributive property.}$$
$$= -16xy \qquad \text{Simplify.}$$

7. First use the distributive property to factor out the common variable part. Then simplify.

$$12r - 12r = (12 - 12)r \qquad \text{Distributive property.}$$
$$= 0 \qquad \text{Simplify.}$$

9. First use the distributive property to factor out the common variable part. Then simplify.

$$-11x - 13x + 8x = (-11 - 13 + 8)x \qquad \text{Distributive property.}$$
$$= -16x \qquad \text{Simplify.}$$

11. First use the distributive property to factor out the common variable part. Then simplify.

$$-5q + 7q = (-5 + 7)q \qquad \text{Distributive property.}$$
$$= 2q \qquad \text{Simplify.}$$

13. First use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} r - 13r - 7r &= (1 - 13 - 7)r && \text{Distributive property.} \\ &= -19r && \text{Simplify.} \end{aligned}$$

15. First use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} 3x^3 - 18x^3 &= (3 - 18)x^3 && \text{Distributive property.} \\ &= -15x^3 && \text{Simplify.} \end{aligned}$$

17. First rearrange the terms and use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} -8 + 17n + 10 + 8n &= -8 + 10 + 17n + 8n && \text{Rearrange terms.} \\ &= (-8 + 10) + (17 + 8)n && \text{Distributive property.} \\ &= 2 + 25n && \text{Simplify.} \end{aligned}$$

19. First rearrange the terms and use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} -2x^3 - 19x^2y - 15x^2y + 11x^3 &= -2x^3 + 11x^3 - 19x^2y - 15x^2y && \text{Rearrange terms.} \\ &= (-2 + 11)x^3 + (-19 - 15)x^2y && \text{Distributive property.} \\ &= 9x^3 - 34x^2y && \text{Simplify.} \end{aligned}$$

21. First rearrange the terms and use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} -14xy - 2x^3 - 2x^3 - 4xy &= -14xy - 4xy - 2x^3 - 2x^3 && \text{Rearrange terms.} \\ &= (-14 - 4)xy + (-2 - 2)x^3 && \text{Distributive property.} \\ &= -18xy - 4x^3 && \text{Simplify.} \end{aligned}$$

23. First rearrange the terms and use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} -13 + 16m + m + 16 &= -13 + 16 + 16m + m && \text{Rearrange terms.} \\ &= (-13 + 16) + (16 + 1)m && \text{Distributive property.} \\ &= 3 + 17m && \text{Simplify.} \end{aligned}$$

25. First rearrange the terms and use the distributive property to factor out the common variable part. Then simplify.

$$
\begin{aligned}
-14x^2y - 2xy^2 + 8x^2y + 18xy^2 &= -14x^2y + 8x^2y - 2xy^2 + 18xy^2 && \text{\color{red}{Rearrange terms.}} \\
&= (-14 + 8)x^2y + (-2 + 18)xy^2 && \text{\color{red}{Distributive property.}} \\
&= -6x^2y + 16xy^2 && \text{\color{red}{Simplify.}}
\end{aligned}
$$

27. First rearrange the terms and use the distributive property to factor out the common variable part. Then simplify.

$$
\begin{aligned}
-14x^3 + 16xy + 5x^3 + 8xy &= -14x^3 + 5x^3 + 16xy + 8xy && \text{\color{red}{Rearrange terms.}} \\
&= (-14 + 5)x^3 + (16 + 8)xy && \text{\color{red}{Distributive property.}} \\
&= -9x^3 + 24xy && \text{\color{red}{Simplify.}}
\end{aligned}
$$

29. First rearrange the terms and use the distributive property to factor out the common variable part. Then simplify.

$$
\begin{aligned}
9n + 10 + 7 + 15n &= 9n + 15n + 10 + 7 && \text{\color{red}{Rearrange terms.}} \\
&= (9 + 15)n + (10 + 7) && \text{\color{red}{Distributive property.}} \\
&= 24n + 17 && \text{\color{red}{Simplify.}}
\end{aligned}
$$

31. First rearrange the terms and use the distributive property to factor out the common variable part. Then simplify.

$$
\begin{aligned}
3y + 1 + 6y + 3 &= 3y + 6y + 1 + 3 && \text{\color{red}{Rearrange terms.}} \\
&= (3 + 6)y + (1 + 3) && \text{\color{red}{Distributive property.}} \\
&= 9y + 4 && \text{\color{red}{Simplify.}}
\end{aligned}
$$

33. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$
\begin{aligned}
-4(9x^2y + 8) + 6(10x^2y - 6) &= -36x^2y - 32 + 60x^2y - 36 && \text{\color{red}{Distribute.}} \\
&= -36x^2y + 60x^2y - 32 - 36 && \text{\color{red}{Rearrange terms.}} \\
&= 24x^2y - 68 && \text{\color{red}{Combine like terms.}}
\end{aligned}
$$

35. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$
\begin{aligned}
3(-4x^2 + 10y^2) + 10(4y^2 - x^2) &= -12x^2 + 30y^2 + 40y^2 - 10x^2 && \text{\color{red}{Distribute.}} \\
&= -12x^2 - 10x^2 + 30y^2 + 40y^2 && \text{\color{red}{Rearrange terms.}} \\
&= -22x^2 + 70y^2 && \text{\color{red}{Combine like terms.}}
\end{aligned}
$$

37. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$
\begin{aligned}
-s + 7 - (-1 - 3s) &= -s + 7 + 1 + 3s && \text{Distribute (negate the sum).} \\
&= -s + 3s + 7 + 1 && \text{Rearrange terms.} \\
&= 2s + 8 && \text{Combine like terms.}
\end{aligned}
$$

39. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$
\begin{aligned}
-10q - 10 - (-3q + 5) &= -10q - 10 + 3q - 5 && \text{Distribute (negate the sum).} \\
&= -10q + 3q - 10 - 5 && \text{Rearrange terms.} \\
&= -7q - 15 && \text{Combine like terms.}
\end{aligned}
$$

41. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$
\begin{aligned}
7(8y + 7) - 6(8 - 7y) &= 56y + 49 - 48 + 42y && \text{Distribute.} \\
&= 56y + 42y + 49 - 48 && \text{Rearrange terms.} \\
&= 98y + 1 && \text{Combine like terms.}
\end{aligned}
$$

43. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$
\begin{aligned}
7(10x^2 - 8xy^2) - 7(9xy^2 + 9x^2) &= 70x^2 - 56xy^2 - 63xy^2 - 63x^2 && \text{Distribute.} \\
&= 70x^2 - 63x^2 - 56xy^2 - 63xy^2 && \text{Rearrange terms.} \\
&= 7x^2 - 119xy^2 && \text{Combine like terms.}
\end{aligned}
$$

45. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$
\begin{aligned}
-2(6 + 4n) + 4(-n - 7) &= -12 - 8n - 4n - 28 && \text{Distribute.} \\
&= -12 - 28 - 8n - 4n && \text{Rearrange terms.} \\
&= -40 - 12n && \text{Combine like terms.}
\end{aligned}
$$

47. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$
\begin{aligned}
8 - (4 + 8y) &= 8 - 4 - 8y && \text{Distribute (negate the sum).} \\
&= 4 - 8y && \text{Combine like terms.}
\end{aligned}
$$

49. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$\begin{aligned} -8(-n+4) - 10(-4n+3) &= 8n - 32 + 40n - 30 &&\text{Distribute.} \\ &= 8n + 40n - 32 - 30 &&\text{Rearrange terms.} \\ &= 48n - 62 &&\text{Combine like terms.} \end{aligned}$$

51. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$\begin{aligned} -5 - (10p+5) &= -5 - 10p - 5 &&\text{Distribute (negate the sum).} \\ &= -5 - 5 - 10p &&\text{Rearrange terms.} \\ &= -10 - 10p &&\text{Combine like terms.} \end{aligned}$$

53. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$\begin{aligned} 7(1+7r) + 2(4-5r) &= 7 + 49r + 8 - 10r &&\text{Distribute.} \\ &= 7 + 8 + 49r - 10r &&\text{Rearrange terms.} \\ &= 15 + 39r &&\text{Combine like terms.} \end{aligned}$$

55. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$\begin{aligned} -2(-5 - 8x^2) - 6(6) &= 10 + 16x^2 - 36 &&\text{Distribute.} \\ &= 10 - 36 + 16x^2 &&\text{Rearrange terms.} \\ &= -26 + 16x^2 &&\text{Combine like terms.} \end{aligned}$$

57. The perimeter P is given by the formula

$$P = 2L + 2W$$

Since the length is 2 feet longer than 6 times its width, it follows that

$$L = 2 + 6W$$

Because $L = 2 + 6W$, we can substitute $2 + 6W$ for L into the perimeter equation to obtain

$$\begin{aligned} P &= 2L + 2W &&\text{Perimeter formula.} \\ &= 2(2 + 6W) + 2W &&\text{Substitute } L = 2 + 6W. \\ &= 4 + 12W + 2W &&\text{Distribute.} \\ &= 4 + 14W &&\text{Combine like terms.} \end{aligned}$$

59. The perimeter P is given by the formula

$$P = 2L + 2W$$

Since the width is 8 feet shorter than its length, it follows that

$$W = L - 8$$

Because $W = L - 8$, we can substitute $L - 8$ for W into the perimeter equation to obtain

$$
\begin{aligned}
P &= 2L + 2W &&\text{\textcolor{red}{Perimeter formula.}}\\
&= 2L + 2(L - 8) &&\text{\textcolor{red}{Substitute $W = 4L - 8$.}}\\
&= 2L + 2L - 16 &&\text{\textcolor{red}{Distribute.}}\\
&= 4L - 16 &&\text{\textcolor{red}{Combine like terms.}}
\end{aligned}
$$

61. The perimeter P is given by the formula

$$P = 2L + 2W$$

Since the length is 9 feet shorter than 4 times its width, it follows that

$$L = 4W - 9$$

Because $L = 4W - 9$, we can substitute $4W - 9$ for L into the perimeter equation to obtain

$$
\begin{aligned}
P &= 2L + 2W &&\text{\textcolor{red}{Perimeter formula.}}\\
&= 2(4W - 9) + 2W &&\text{\textcolor{red}{Substitute $L = 4W - 9$.}}\\
&= 8W - 18 + 2W &&\text{\textcolor{red}{Distribute.}}\\
&= 10W - 18 &&\text{\textcolor{red}{Combine like terms.}}
\end{aligned}
$$

3.5   Solving Equations Involving Integers II

1. Combine like terms on each side of the equation, if possible. Then isolate the term containing x on one side of the equation.

$$
\begin{aligned}
-9x + x &= -8 &&\text{\textcolor{red}{Original equation.}}\\
-8x &= -8 &&\text{\textcolor{red}{Combine like terms on the left side.}}\\
\frac{-8x}{-8} &= \frac{-8}{-8} &&\text{\textcolor{red}{Divide both sides by -8.}}\\
x &= 1 &&\text{\textcolor{red}{Simplify.}}
\end{aligned}
$$

3. Combine like terms on each side of the equation, if possible. Then isolate the term containing x on one side of the equation.

$$-4 = 3x - 4x \qquad \text{Original equation.}$$
$$-4 = -x \qquad \text{Combine like terms on the left side.}$$
$$\frac{-4}{-1} = \frac{-x}{-1} \qquad \text{Divide both sides by } -1.$$
$$4 = x \qquad \text{Simplify.}$$

5. First isolate the term containing x on one side of the equation.

$$27x + 51 = -84 \qquad \text{Original equation.}$$
$$27x + 51 - 51 = -84 - 51 \qquad \text{Subtract 51 from both sides.}$$
$$27x = -135 \qquad \text{Simplify}$$
$$\frac{27x}{27} = \frac{-135}{27} \qquad \text{Divide both sides by 27.}$$
$$x = -5 \qquad \text{Simplify.}$$

7. Combine like terms on each side of the equation, if possible. Then isolate the term containing x on one side of the equation.

$$9 = 5x + 9 - 6x \qquad \text{Original equation.}$$
$$9 = -x + 9 \qquad \text{Combine like terms on the left side.}$$
$$9 - 9 = -x + 9 - 9 \qquad \text{Subtract 9 from both sides.}$$
$$0 = -x \qquad \text{Simplify.}$$
$$\frac{0}{-1} = \frac{-x}{-1} \qquad \text{Divide both sides by } -1.$$
$$0 = x \qquad \text{Simplify.}$$

9. First isolate the term containing x on one side of the equation.

$$0 = -18x + 18 \qquad \text{Original equation.}$$
$$-18 = -18x + 18 - 18 \qquad \text{Subtract 18 from both sides.}$$
$$-18 = -18x \qquad \text{Simplify.}$$
$$\frac{-18}{-18} = \frac{-18x}{-18} \qquad \text{Divide both sides by } -18.$$
$$1 = x \qquad \text{Simplify.}$$

11. First isolate the term containing x on one side of the equation.

$$41 = 28x + 97 \qquad \text{Original equation.}$$
$$41 - 97 = 28x + 97 - 97 \qquad \text{Subtract 97 from both sides.}$$
$$-56 = 28x \qquad \text{Simplify}$$
$$\frac{-56}{28} = \frac{28x}{28} \qquad \text{Divide both sides by 28.}$$
$$-2 = x \qquad \text{Simplify.}$$

13. Combine like terms on each side of the equation, if possible. Then isolate the term containing x on one side of the equation.

$$8x - 8 - 9x = -3 \qquad \text{Original equation.}$$
$$-x - 8 = -3 \qquad \text{Combine like terms on the left side.}$$
$$-x - 8 + 8 = -3 + 8 \qquad \text{Add 8 to both sides.}$$
$$-x = 5 \qquad \text{Simplify.}$$
$$\frac{-x}{-1} = \frac{5}{-1} \qquad \text{Divide both sides by } -1.$$
$$x = -5 \qquad \text{Simplify.}$$

15. First isolate the term containing x on one side of the equation.

$$-85x + 85 = 0 \qquad \text{Original equation.}$$
$$-85x + 85 - 85 = -85 \qquad \text{Subtract 85 from both sides.}$$
$$-85x = -85 \qquad \text{Simplify.}$$
$$\frac{-85x}{-85} = \frac{-85}{-85} \qquad \text{Divide both sides by } -85.$$
$$x = 1 \qquad \text{Simplify.}$$

17. Isolate the terms containing x on one side of the equation.

$$-6x = -5x - 9 \qquad \text{Original equation.}$$
$$-6x + 5x = -5x - 9 + 5x \qquad \text{Add 5x to both sides.}$$
$$-x = -9 \qquad \text{Combine like terms.}$$
$$\frac{-x}{-1} = \frac{-9}{-1} \qquad \text{Divide both sides by } -1.$$
$$x = 9 \qquad \text{Simplify.}$$

19. Isolate the terms containing x on one side of the equation.

$$
\begin{aligned}
6x - 7 &= 5x && \text{Original equation.} \\
6x - 7 - 6x &= 5x - 6x && \text{Subtract 6x from both sides.} \\
-7 &= -x && \text{Combine like terms.} \\
\frac{-7}{-1} &= \frac{-x}{-1} && \text{Divide both sides by } -1. \\
7 &= x && \text{Simplify.}
\end{aligned}
$$

21. Isolate the terms containing x on one side of the equation.

$$
\begin{aligned}
4x - 3 &= 5x - 1 && \text{Original equation.} \\
4x - 3 - 5x &= 5x - 1 - 5x && \text{Subtract 5x from both sides.} \\
-x - 3 &= -1 && \text{Combine like terms.} \\
-x - 3 + 3 &= -1 + 3 && \text{Add 3 to both sides.} \\
-x &= 2 && \text{Simplify.} \\
\frac{-x}{-1} &= \frac{2}{-1} && \text{Divide both sides by } -1. \\
x &= -2 && \text{Simplify.}
\end{aligned}
$$

23. Isolate the terms containing x on one side of the equation.

$$
\begin{aligned}
-3x + 5 &= 3x - 1 && \text{Original equation.} \\
-3x + 5 - 3x &= 3x - 1 - 3x && \text{Subtract 3x from both sides.} \\
-6x + 5 &= -1 && \text{Combine like terms.} \\
-6x + 5 - 5 &= -1 - 5 && \text{Subtract 5 from both sides.} \\
-6x &= -6 && \text{Simplify.} \\
\frac{-6x}{-6} &= \frac{-6}{-6} && \text{Divide both sides by } -6. \\
x &= 1 && \text{Simplify.}
\end{aligned}
$$

25. Isolate the terms containing x on one side of the equation.

$$
\begin{aligned}
-5x &= -3x + 6 && \text{Original equation.} \\
-5x + 3x &= -3x + 6 + 3x && \text{Add 3x to both sides.} \\
-2x &= 6 && \text{Combine like terms.} \\
\frac{-2x}{-2} &= \frac{6}{-2} && \text{Divide both sides by } -2. \\
x &= -3 && \text{Simplify.}
\end{aligned}
$$

27. Isolate the terms containing x on one side of the equation.

$$2x - 2 = 4x \qquad \text{Original equation.}$$
$$2x - 2 - 2x = 4x - 2x \qquad \text{Subtract 2x from both sides.}$$
$$-2 = 2x \qquad \text{Combine like terms.}$$
$$\frac{-2}{2} = \frac{2x}{2} \qquad \text{Divide both sides by 2.}$$
$$-1 = x \qquad \text{Simplify.}$$

29. Isolate the terms containing x on one side of the equation.

$$-6x + 8 = -2x \qquad \text{Original equation.}$$
$$-6x + 8 + 6x = -2x + 6x \qquad \text{Add 6x to both sides.}$$
$$8 = 4x \qquad \text{Combine like terms.}$$
$$\frac{8}{4} = \frac{4x}{4} \qquad \text{Divide both sides by 4.}$$
$$2 = x \qquad \text{Simplify.}$$

31. Isolate the terms containing x on one side of the equation.

$$6x = 4x - 4 \qquad \text{Original equation.}$$
$$6x - 4x = 4x - 4 - 4x \qquad \text{Subtract 4x from both sides.}$$
$$2x = -4 \qquad \text{Combine like terms.}$$
$$\frac{2x}{2} = \frac{-4}{2} \qquad \text{Divide both sides by 2.}$$
$$x = -2 \qquad \text{Simplify.}$$

33. Isolate the terms containing x on one side of the equation.

$$-8x + 2 = -6x + 6 \qquad \text{Original equation.}$$
$$-8x + 2 + 6x = -6x + 6 + 6x \qquad \text{Add 6x to both sides.}$$
$$-2x + 2 = 6 \qquad \text{Combine like terms.}$$
$$-2x + 2 - 2 = 6 - 2 \qquad \text{Subtract 2 from both sides.}$$
$$-2x = 4 \qquad \text{Simplify.}$$
$$\frac{-2x}{-2} = \frac{4}{-2} \qquad \text{Divide both sides by } -2.$$
$$x = -2 \qquad \text{Simplify.}$$

35. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$1 - (x - 2) = -3$	Original equation.
$1 - x + 2 = -3$	Apply the distributive property.
$-x + 3 = -3$	Combine like terms on the left side.
$-x + 3 - 3 = -3 - 3$	Subtract 3 from both sides.
$-x = -6$	Simplify.
$\dfrac{-x}{-1} = \dfrac{-6}{-1}$	Divide both sides by -1.
$x = 6$	Simpify.

37. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$-7x + 6(x + 8) = -2$	Original equation.
$-7x + 6x + 48 = -2$	Apply the distributive property.
$-x + 48 = -2$	Combine like terms on the left side.
$-x + 48 - 48 = -2 - 48$	Subtract 48 from both sides.
$-x = -50$	Simplify.
$\dfrac{-x}{-1} = \dfrac{-50}{-1}$	Divide both sides by -1.
$x = 50$	Simplify.

39. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$8(-6x - 1) = -8$	Original equation.
$-48x - 8 = -8$	Apply the distributive property.
$-48x - 8 + 8 = -8 + 8$	Add 8 to both sides.
$-48x = 0$	Simplify.
$\dfrac{-48x}{-48} = \dfrac{0}{-48}$	Divide both sides by -48.
$= 0$	Simplify.

41. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$-7(-4x - 6) = -14 \qquad \text{Original equation.}$$
$$28x + 42 = -14 \qquad \text{Apply the distributive property.}$$
$$28x + 42 - 42 = -14 - 42 \qquad \text{Subtract 42 from both sides.}$$
$$28x = -56 \qquad \text{Simplify.}$$
$$\frac{28x}{28} = \frac{-56}{28} \qquad \text{Divide both sides by 28.}$$
$$= -2 \qquad \text{Simplify.}$$

43. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$2 - 9(x - 5) = -16 \qquad \text{Original equation.}$$
$$2 - 9x + 45 = -16 \qquad \text{Apply the distributive property.}$$
$$-9x + 47 = -16 \qquad \text{Combine like terms on the left side.}$$
$$-9x + 47 - 47 = -16 - 47 \qquad \text{Subtract 47 from both sides.}$$
$$-9x = -63 \qquad \text{Simplify.}$$
$$\frac{-9x}{-9} = \frac{-63}{-9} \qquad \text{Divide both sides by } -9.$$
$$x = 7 \qquad \text{Simpify.}$$

45. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$7x + 2(x + 9) = -9 \qquad \text{Original equation.}$$
$$7x + 2x + 18 = -9 \qquad \text{Apply the distributive property.}$$
$$9x + 18 = -9 \qquad \text{Combine like terms on the left side.}$$
$$9x + 18 - 18 = -9 - 18 \qquad \text{Subtract 18 from both sides.}$$
$$9x = -27 \qquad \text{Simplify.}$$
$$\frac{9x}{9} = \frac{-27}{9} \qquad \text{Divide both sides by 9.}$$
$$x = -3 \qquad \text{Simplify.}$$

47. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$2(-x+8) = 10 \qquad \text{Original equation.}$$
$$-2x + 16 = 10 \qquad \text{Apply the distributive property.}$$
$$-2x + 16 - 16 = 10 - 16 \qquad \text{Subtract 16 from both sides.}$$
$$-2x = -6 \qquad \text{Simplify.}$$
$$\frac{-2x}{-2} = \frac{-6}{-2} \qquad \text{Divide both sides by } -2.$$
$$= 3 \qquad \text{Simplify.}$$

49. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$8 + 2(x-5) = -4 \qquad \text{Original equation.}$$
$$8 + 2x - 10 = -4 \qquad \text{Apply the distributive property.}$$
$$2x - 2 = -4 \qquad \text{Combine like terms on the left side.}$$
$$2x - 2 + 2 = -4 + 2 \qquad \text{Add 2 to both sides.}$$
$$2x = -2 \qquad \text{Simplify.}$$
$$\frac{2x}{2} = \frac{-2}{2} \qquad \text{Divide both sides by 2.}$$
$$x = -1 \qquad \text{Simpify.}$$

51. First use the distributive property to expand the expression on the left side, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$9x - 2(x+5) = -10 \qquad \text{Original equation.}$$
$$9x - 2x - 10 = -10 \qquad \text{Apply the distributive property.}$$
$$7x - 10 = -10 \qquad \text{Combine like terms on the left side.}$$
$$7x - 10 + 10 = -10 + 10 \qquad \text{Add 10 to both sides.}$$
$$7x = 0 \qquad \text{Simplify.}$$
$$\frac{7x}{7} = \frac{0}{7} \qquad \text{Divide both sides by 7.}$$
$$x = 0 \qquad \text{Simplify.}$$

53. First use the distributive property to expand the expressions on both sides, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$
\begin{aligned}
4(-7x+5)+8 &= 3(-9x-1)-2 &&\text{Original equation.}\\
-28x+20+8 &= -27x-3-2 &&\text{Apply the distributive property on both sides.}\\
-28x+28 &= -27x-5 &&\text{Simplify both sides.}\\
-28x+28+27x &= -27x-5+27x &&\text{Add 27x to both sides.}\\
-x+28 &= -5 &&\text{Combine like terms.}\\
-x+28-28 &= -5-28 &&\text{Subtract 28 from both sides.}\\
-x &= -33 &&\text{Simplify.}\\
\frac{-x}{-1} &= \frac{-33}{-1} &&\text{Divide both sides by } -1.\\
x &= 33 &&\text{Simplify.}
\end{aligned}
$$

55. First use the distributive property to expand the expressions on both sides, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$
\begin{aligned}
-8(-2x-6) &= 7(5x-1)-2 &&\text{Original equation.}\\
16x+48 &= 35x-7-2 &&\text{Apply the distributive property on both sides.}\\
16x+48 &= 35x-9 &&\text{Simplify the right side.}\\
16x+48-16x &= 35x-9-16x &&\text{Subtract 16x from both sides.}\\
48 &= 19x-9 &&\text{Combine like terms.}\\
48+9 &= 19x-9+9 &&\text{Add 9 to both sides.}\\
57 &= 19x &&\text{Simplify.}\\
\frac{57}{19} &= \frac{19x}{19} &&\text{Divide both sides by 19.}\\
3 &= x &&\text{Simplify.}
\end{aligned}
$$

57. First use the distributive property to expand the expressions on both sides, then combine like terms. Next, isolate the terms containing x on one side of

the equation.

$$2(2x - 9) + 5 = -7(-x - 8)$$ Original equation.

$$4x - 18 + 5 = 7x + 56$$ Apply the distributive property on both sides.

$$4x - 13 = 7x + 56$$ Simplify the left side.

$$4x - 13 - 7x = 7x + 56 - 7x$$ Subtract 7x from both sides.

$$-3x - 13 = 56$$ Combine like terms.

$$-3x - 13 + 13 = 56 + 13$$ Add 13 to both sides.

$$-3x = 69$$ Simplify.

$$\frac{-3x}{-3} = \frac{69}{-3}$$ Divide both sides by -3.

$$x = -23$$ Simplify.

59. First use the distributive property to expand the expressions on both sides, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$6(-3x + 4) - 6 = -8(2x + 2) - 8$$ Original equation.

$$-18x + 24 - 6 = -16x - 16 - 8$$ Apply the distributive property on both sides.

$$-18x + 18 = -16x - 24$$ Simplify both sides.

$$-18x + 18 + 16x = -16x - 24 + 16x$$ Add 16x to both sides.

$$-2x + 18 = -24$$ Combine like terms.

$$-2x + 18 - 18 = -24 - 18$$ Subtract 18 from both sides.

$$-2x = -42$$ Simplify.

$$\frac{-2x}{-2} = \frac{-42}{-2}$$ Divide both sides by -2.

$$x = 21$$ Simplify.

61. First use the distributive property to expand the expressions on both sides, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$2(-2x - 3) = 3(-x + 2)$$ Original equation.

$$-4x - 6 = -3x + 6$$ Apply the distributive property on both sides.

$$-4x - 6 + 3x = -3x + 6 + 3x$$ Add 3x to both sides.

$$-x - 6 = 6$$ Combine like terms.

$$-x - 6 + 6 = 6 + 6$$ Add 6 to both sides.

$$-x = 12$$ Simplify.

$$\frac{-x}{-1} = \frac{12}{-1}$$ Divide both sides by -1.

$$x = -12$$ Simplify.

63. First use the distributive property to expand the expressions on both sides, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$
\begin{array}{ll}
-5(-9x+7)+7 = -(-9x-8) & \text{Original equation.} \\
45x - 35 + 7 = 9x + 8 & \text{Apply the distributive property on both sides.} \\
45x - 28 = 9x + 8 & \text{Simplify the left side.} \\
45x - 28 - 9x = 9x + 8 - 9x & \text{Subtract 9x from both sides.} \\
36x - 28 = 8 & \text{Combine like terms.} \\
36x - 28 + 28 = 8 + 28 & \text{Add 28 to both sides.} \\
36x = 36 & \text{Simplify.} \\
\dfrac{36x}{36} = \dfrac{36}{36} & \text{Divide both sides by 36.} \\
x = 1 & \text{Simplify.}
\end{array}
$$

65. First use the distributive property to expand the expressions on both sides, then combine like terms. Next, isolate the terms containing x on one side of the equation.

$$
\begin{array}{ll}
5(5x-2) = 4(8x+1) & \text{Original equation.} \\
25x - 10 = 32x + 4 & \text{Apply the distributive property on both sides.} \\
25x - 10 - 32x = 32x + 4 - 32x & \text{Subtract 32x from both sides.} \\
-7x - 10 = 4 & \text{Combine like terms.} \\
-7x - 10 + 10 = 4 + 10 & \text{Add 10 to both sides.} \\
-7x = 14 & \text{Simplify.} \\
\dfrac{-7x}{-7} = \dfrac{14}{-7} & \text{Divide both sides by } -7. \\
x = -2 & \text{Simplify.}
\end{array}
$$

67. First use the distributive property to expand the expressions on both sides, then combine like terms. Next, isolate the terms containing x on one side of

the equation.

$$-7(9x - 6) = 7(5x + 7) - 7 \qquad \text{Original equation.}$$
$$-63x + 42 = 35x + 49 - 7 \qquad \text{Apply the distributive property on both sides.}$$
$$-63x + 42 = 35x + 42 \qquad \text{Simplify the right side.}$$
$$-63x + 42 + 63x = 35x + 42 + 63x \qquad \text{Add 63x to both sides.}$$
$$42 = 98x + 42 \qquad \text{Combine like terms.}$$
$$42 - 42 = 98x + 42 - 42 \qquad \text{Subtract 42 from both sides.}$$
$$0 = 98x \qquad \text{Simplify.}$$
$$\frac{0}{98} = \frac{98x}{98} \qquad \text{Divide both sides by 98.}$$
$$0 = x \qquad \text{Simplify.}$$

3.6 Applications

1. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents. We let k represent an odd integer, the length of the smallest side of the triangle. Then the next two sides of the triangle are the next two consecutive odd intergers, namely $k + 2$ and $k + 4$.

In our schematic diagram, we've labeled the three sides of the triangle with expressions representing three consecutive odd integers k, $k+2$, and $k + 4$.

2. *Set up an Equation.* To find the perimeter P of the triangle, sum the three sides.
$$P = k + (k + 2) + (k + 4)$$

However, we're given the fact that the perimeter is 39 inches. Thus,

$$39 = k + (k + 2) + (k + 4)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.

$$39 = 3k + 6$$

Now, solve.

$$39 - 6 = 3k + 6 - 6 \qquad \text{Subtract 6 from both sides.}$$
$$33 = 3k \qquad \text{Simplify both sides.}$$
$$\frac{33}{3} = \frac{3k}{3} \qquad \text{Divide both sides by 3.}$$
$$11 = k \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* We've only found one side, but the question asks for the measure of all three sides. However, the remaining two sides can be found by substituting 11 for k into the expressions $k + 2$ and $k + 4$.

$$k + 2 = 11 + 2 \qquad \text{and} \qquad k + 4 = 11 + 4$$
$$= 13 \qquad \qquad \qquad = 15$$

Hence, the three sides measure 11 inches, 13 inches, and 15 inches.

5. *Look Back.* Does our solution make sense? Well, the three sides are certainly consecutive odd integers, and their sum is 11 inches + 13 inches + 15 inches = 39 inches, which was the given perimeter. Therefore, our solution is correct.

3. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents.

In our schematic diagram, if the width k is an integer, then the length $k + 1$ is the next consecutive integer.

2. *Set up an Equation.* To find the perimeter of the rectangle, sum the four sides.
$$P = k + (k + 1) + k + (k + 1)$$
However, we're given the fact that the perimeter is 142 centimeters. Thus,

$$142 = k + (k + 1) + k + (k + 1)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.

$$142 = 4k + 2$$

Now, solve.

$$142 - 2 = 4k + 2 - 2 \qquad \text{Subtract 2 from both sides.}$$
$$140 = 4k \qquad \text{Simplify both sides.}$$
$$\frac{140}{4} = \frac{4k}{4} \qquad \text{Divide both sides by 4.}$$
$$35 = k \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* We've only found the width, but the question asks for the measure of both the width and the length. However, the length can be found by substituting 35 for k into the expression $k + 1$.

$$k + 1 = 35 + 1$$
$$= 36$$

Hence, the width is 35 centimeters and the length is 36 centimeters.

5. *Look Back.* Does our solution make sense? Well, the width is 35 cm and the length is 36 cm, certainly consecutive integers. Further, the perimeter would be $35\,\text{cm} + 36\,\text{cm} + 35\,\text{cm} + 36\,\text{cm} = 142\,\text{cm}$, so our solution is correct.

5. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents. We let k represent an even integer, the length of the smallest side of the triangle. Then the next two sides of the triangle are the next two consecutive even intergers, namely $k + 2$ and $k + 4$.

In our schematic diagram, we've labeled the three sides of the triangle with expressions representing three consecutive even integers k, $k + 2$, and $k + 4$.

2. *Set up an Equation.* To find the perimeter P of the triangle, sum the three sides.
$$P = k + (k + 2) + (k + 4)$$
However, we're given the fact that the perimeter is 240 inches. Thus,
$$240 = k + (k + 2) + (k + 4)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.
$$240 = 3k + 6$$

Now, solve.

$$240 - 6 = 3k + 6 - 6 \qquad \text{Subtract 6 from both sides.}$$
$$234 = 3k \qquad \text{Simplify both sides.}$$
$$\frac{234}{3} = \frac{3k}{3} \qquad \text{Divide both sides by 3.}$$
$$78 = k \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* We've only found one side, but the question asks for the measure of all three sides. However, the remaining two sides can be found by substituting 78 for k into the expressions $k + 2$ and $k + 4$.

$$k + 2 = 78 + 2 \qquad \text{and} \qquad k + 4 = 78 + 4$$
$$= 80 \qquad\qquad\qquad = 82$$

Hence, the three sides measure 78 inches, 80 inches, and 82 inches.

5. *Look Back.* Does our solution make sense? Well, the three sides are certainly consecutive even integers, and their sum is 78 inches+80 inches+ 82 inches = 240 inches, which was the given perimeter. Therefore, our solution is correct.

7. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents.

$$k + 1$$

$$k \qquad\qquad\qquad\qquad k$$

$$k + 1$$

In our schematic diagram, if the width k is an integer, then the length $k + 1$ is the next consecutive integer.

2. *Set up an Equation.* To find the perimeter of the rectangle, sum the four sides.
$$P = k + (k + 1) + k + (k + 1)$$
However, we're given the fact that the perimeter is 374 centimeters. Thus,
$$374 = k + (k + 1) + k + (k + 1)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.
$$374 = 4k + 2$$

Now, solve.

$$374 - 2 = 4k + 2 - 2 \qquad \text{Subtract 2 from both sides.}$$
$$372 = 4k \qquad \text{Simplify both sides.}$$
$$\frac{372}{4} = \frac{4k}{4} \qquad \text{Divide both sides by 4.}$$
$$93 = k \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* We've only found the width, but the question asks for the measure of both the width and the length. However, the length can be found by substituting 93 for k into the expression $k + 1$.
$$k + 1 = 93 + 1$$
$$= 94$$

Hence, the width is 93 centimeters and the length is 94 centimeters.

5. *Look Back.* Does our solution make sense? Well, the width is 93 cm and the length is 94 cm, certainly consecutive integers. Further, the perimeter would be $93\,\text{cm} + 94\,\text{cm} + 93\,\text{cm} + 94\,\text{cm} = 374\,\text{cm}$, so our solution is correct.

9. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents.

In our schematic diagram, if the width k is an odd integer, then the length $k + 2$ is the next consecutive odd integer.

2. *Set up an Equation.* To find the perimeter of the rectangle, sum the four sides.
$$P = k + (k + 2) + k + (k + 2)$$
However, we're given the fact that the perimeter is 208 centimeters. Thus,
$$208 = k + (k + 2) + k + (k + 2)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.
$$208 = 4k + 4$$

Now, solve.

$$
\begin{aligned}
208 - 4 &= 4k + 4 - 4 & &\text{Subtract 4 from both sides.} \\
204 &= 4k & &\text{Simplify both sides.} \\
\frac{204}{4} &= \frac{4k}{4} & &\text{Divide both sides by 4.} \\
51 &= k & &\text{Simplify both sides.}
\end{aligned}
$$

4. *Answer the Question.* We've only found the width, but the question asks for the measure of both the width and the length. However, the length can be found by substituting 51 for k into the expression $k + 2$.
$$
\begin{aligned}
k + 2 &= 51 + 2 \\
&= 53
\end{aligned}
$$
Hence, the width is 51 centimeters and the length is 53 centimeters.

5. *Look Back.* Does our solution make sense? Well, the width is 51 cm and the length is 53 cm, certainly consecutive odd integers. Further, the perimeter would be $51\,\text{cm} + 53\,\text{cm} + 51\,\text{cm} + 53\,\text{cm} = 208\,\text{cm}$, so our solution is correct.

11. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents.

In our schematic diagram, if the width k is an even integer, then the length $k + 2$ is the next consecutive even integer.

2. *Set up an Equation.* To find the perimeter of the rectangle, sum the four sides.
$$P = k + (k + 2) + k + (k + 2)$$

However, we're given the fact that the perimeter is 76 centimeters. Thus,
$$76 = k + (k + 2) + k + (k + 2)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.
$$76 = 4k + 4$$

Now, solve.

$$
\begin{aligned}
76 - 4 &= 4k + 4 - 4 & &\text{Subtract 4 from both sides.}\\
72 &= 4k & &\text{Simplify both sides.}\\
\frac{72}{4} &= \frac{4k}{4} & &\text{Divide both sides by 4.}\\
18 &= k & &\text{Simplify both sides.}
\end{aligned}
$$

4. *Answer the Question.* We've only found the width, but the question asks for the measure of both the width and the length. However, the length can be found by substituting 18 for k into the expression $k + 2$.
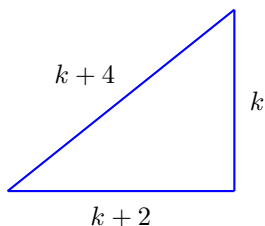
$$
\begin{aligned}
k + 2 &= 18 + 2\\
&= 20
\end{aligned}
$$

Hence, the width is 18 centimeters and the length is 20 centimeters.

5. *Look Back.* Does our solution make sense? Well, the width is 18 cm and the length is 20 cm, certainly consecutive even integers. Further, the perimeter would be $18\,\text{cm} + 20\,\text{cm} + 18\,\text{cm} + 20\,\text{cm} = 76\,\text{cm}$, so our solution is correct.

13. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents. We let k represent an even integer, the length of the smallest side of the triangle. Then the next two sides of the triangle are the next two consecutive even intergers, namely $k + 2$ and $k + 4$.

In our schematic diagram, we've labeled the three sides of the triangle with expressions representing three consecutive even integers k, $k + 2$, and $k + 4$.

2. *Set up an Equation.* To find the perimeter P of the triangle, sum the three sides.
$$P = k + (k + 2) + (k + 4)$$

However, we're given the fact that the perimeter is 144 inches. Thus,

$$144 = k + (k + 2) + (k + 4)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.

$$144 = 3k + 6$$

Now, solve.

$$144 - 6 = 3k + 6 - 6 \qquad \text{Subtract 6 from both sides.}$$
$$138 = 3k \qquad \text{Simplify both sides.}$$
$$\frac{138}{3} = \frac{3k}{3} \qquad \text{Divide both sides by 3.}$$
$$46 = k \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* We've only found one side, but the question asks for the measure of all three sides. However, the remaining two sides can be found by substituting 46 for k into the expressions $k + 2$ and $k + 4$.
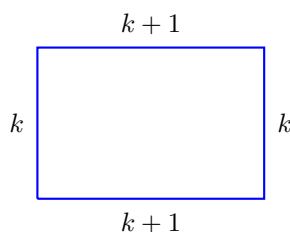
$$k + 2 = 46 + 2 \qquad \text{and} \qquad k + 4 = 46 + 4$$
$$= 48 \qquad\qquad\qquad\qquad = 50$$

Hence, the three sides measure 46 inches, 48 inches, and 50 inches.

5. *Look Back.* Does our solution make sense? Well, the three sides are certainly consecutive even integers, and their sum is 46 inches+48 inches+ 50 inches = 144 inches, which was the given perimeter. Therefore, our solution is correct.

15. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents. We let k represent one side, then the next two sides are the next two consecutive intergers, namely $k + 1$ and $k + 2$.

In our schematic diagram, we've labeled the three sides of the triangle with expressions representing the consecutive integers k, $k+1$, and $k+2$.

2. *Set up an Equation.* To find the perimeter P of the triangle, sum the three sides.

$$P = k + (k + 1) + (k + 2)$$

However, we're given the fact that the perimeter is 228 inches. Thus,

$$228 = k + (k + 1) + (k + 2)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.

$$228 = 3k + 3$$

Now, solve.

$$
\begin{aligned}
228 - 3 &= 3k + 3 - 3 && \color{red}{\text{Subtract 3 from both sides.}} \\
225 &= 3k && \color{red}{\text{Simplify both sides.}} \\
\frac{225}{3} &= \frac{3k}{3} && \color{red}{\text{Divide both sides by 3.}} \\
75 &= k && \color{red}{\text{Simplify both sides.}}
\end{aligned}
$$

4. *Answer the Question.* We've only found one side, but the question asks for the measure of all three sides. However, the remaining two sides can be found by substituting 75 for k into the expressions $k + 1$ and $k + 2$.

$$
\begin{array}{ccc}
k + 1 = 75 + 1 & \text{and} & k + 2 = 75 + 2 \\
= 76 & & = 77
\end{array}
$$

Hence, the three sides measure 75 inches, 76 inches, and 77 inches.

5. *Look Back.* Does our solution make sense? Well, the three sides are certainly consecutive integers, and their sum is 75 inches + 76 inches + 77 inches = 228 inches, which was the given perimeter. Therefore, our solution is correct.

17. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents.

$$k + 2$$

$$k + 2$$

In our schematic diagram, if the width k is an even integer, then the length $k + 2$ is the next consecutive even integer.

2. *Set up an Equation.* To find the perimeter of the rectangle, sum the four sides.

$$P = k + (k + 2) + k + (k + 2)$$

However, we're given the fact that the perimeter is 92 centimeters. Thus,

$$92 = k + (k + 2) + k + (k + 2)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.

$$92 = 4k + 4$$

Now, solve.

$$
\begin{aligned}
92 - 4 &= 4k + 4 - 4 && \text{Subtract 4 from both sides.} \\
88 &= 4k && \text{Simplify both sides.} \\
\frac{88}{4} &= \frac{4k}{4} && \text{Divide both sides by 4.} \\
22 &= k && \text{Simplify both sides.}
\end{aligned}
$$

4. *Answer the Question.* We've only found the width, but the question asks for the measure of both the width and the length. However, the length can be found by substituting 22 for k into the expression $k + 2$.
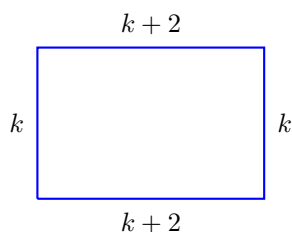
$$
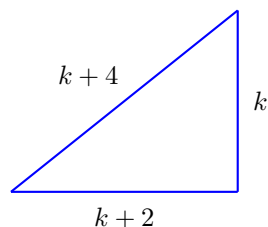\begin{aligned}
k + 2 &= 22 + 2 \\
&= 24
\end{aligned}
$$

Hence, the width is 22 centimeters and the length is 24 centimeters.

5. *Look Back.* Does our solution make sense? Well, the width is 22 cm and the length is 24 cm, certainly consecutive even integers. Further, the perimeter would be $22\,\text{cm} + 24\,\text{cm} + 22\,\text{cm} + 24\,\text{cm} = 92\,\text{cm}$, so our solution is correct.

19. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents. We let k represent one side, then the next two sides are the next two consecutive intergers, namely $k + 1$ and $k + 2$.

In our schematic diagram, we've labeled the three sides of the triangle with expressions representing the consecutive integers k, $k+1$, and $k+2$.

2. *Set up an Equation.* To find the perimeter P of the triangle, sum the three sides.
$$P = k + (k + 1) + (k + 2)$$
However, we're given the fact that the perimeter is 105 inches. Thus,
$$105 = k + (k + 1) + (k + 2)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.
$$105 = 3k + 3$$

Now, solve.

$$
\begin{aligned}
105 - 3 &= 3k + 3 - 3 && \textcolor{red}{\text{Subtract 3 from both sides.}} \\
102 &= 3k && \textcolor{red}{\text{Simplify both sides.}} \\
\frac{102}{3} &= \frac{3k}{3} && \textcolor{red}{\text{Divide both sides by 3.}} \\
34 &= k && \textcolor{red}{\text{Simplify both sides.}}
\end{aligned}
$$

4. *Answer the Question.* We've only found one side, but the question asks for the measure of all three sides. However, the remaining two sides can be found by substituting 34 for k into the expressions $k + 1$ and $k + 2$.

$$
\begin{array}{ccc}
k + 1 = 34 + 1 & \text{and} & k + 2 = 34 + 2 \\
= 35 & & = 36
\end{array}
$$

Hence, the three sides measure 34 inches, 35 inches, and 36 inches.

5. *Look Back.* Does our solution make sense? Well, the three sides are certainly consecutive integers, and their sum is $34\,\text{inches} + 35\,\text{inches} + 36\,\text{inches} = 105\,\text{inches}$, which was the given perimeter. Therefore, our solution is correct.

21. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents.

$$k + 2$$

$$k \qquad\qquad\qquad\qquad k$$

$$k + 2$$

   In our schematic diagram, if the width k is an odd integer, then the length $k + 2$ is the next consecutive odd integer.

2. *Set up an Equation.* To find the perimeter of the rectangle, sum the four sides.
$$P = k + (k + 2) + k + (k + 2)$$

   However, we're given the fact that the perimeter is 288 centimeters. Thus,

$$288 = k + (k + 2) + k + (k + 2)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.

$$288 = 4k + 4$$

   Now, solve.

$$
\begin{aligned}
288 - 4 &= 4k + 4 - 4 && \text{Subtract 4 from both sides.}\\
284 &= 4k && \text{Simplify both sides.}\\
\frac{284}{4} &= \frac{4k}{4} && \text{Divide both sides by 4.}\\
71 &= k && \text{Simplify both sides.}
\end{aligned}
$$

4. *Answer the Question.* We've only found the width, but the question asks for the measure of both the width and the length. However, the length can be found by substituting 71 for k into the expression $k + 2$.

$$
\begin{aligned}
k + 2 &= 71 + 2\\
&= 73
\end{aligned}
$$

   Hence, the width is 71 centimeters and the length is 73 centimeters.

5. *Look Back.* Does our solution make sense? Well, the width is 71 cm and the length is 73 cm, certainly consecutive odd integers. Further, the perimeter would be $71\,\text{cm} + 73\,\text{cm} + 71\,\text{cm} + 73\,\text{cm} = 288\,\text{cm}$, so our solution is correct.

23. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents. We let k represent an odd integer, the length of the smallest side of the triangle. Then the next two sides of the triangle are the next two consecutive odd intergers, namely $k + 2$ and $k + 4$.

In our schematic diagram, we've labeled the three sides of the triangle with expressions representing three consecutive odd integers k, $k + 2$, and $k + 4$.

2. *Set up an Equation.* To find the perimeter P of the triangle, sum the three sides.
$$P = k + (k + 2) + (k + 4)$$
However, we're given the fact that the perimeter is 165 inches. Thus,
$$165 = k + (k + 2) + (k + 4)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.
$$165 = 3k + 6$$

Now, solve.

$$
\begin{aligned}
165 - 6 &= 3k + 6 - 6 && \textcolor{red}{\text{Subtract 6 from both sides.}} \\
159 &= 3k && \textcolor{red}{\text{Simplify both sides.}} \\
\frac{159}{3} &= \frac{3k}{3} && \textcolor{red}{\text{Divide both sides by 3.}} \\
53 &= k && \textcolor{red}{\text{Simplify both sides.}}
\end{aligned}
$$

4. *Answer the Question.* We've only found one side, but the question asks for the measure of all three sides. However, the remaining two sides can be found by substituting 53 for k into the expressions $k + 2$ and $k + 4$.
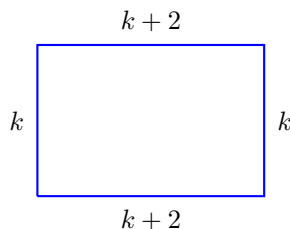
$$
\begin{array}{ccc}
k + 2 = 53 + 2 & \quad \text{and} \quad & k + 4 = 53 + 4 \\
= 55 & & = 57
\end{array}
$$

Hence, the three sides measure 53 inches, 55 inches, and 57 inches.

5. *Look Back.* Does our solution make sense? Well, the three sides are certainly consecutive odd integers, and their sum is $53\,\text{inches} + 55\,\text{inches} + 57\,\text{inches} = 165\,\text{inches}$, which was the given perimeter. Therefore, our solution is correct.

25. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let A represent the number of adult tickets purchased. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting A represent the number of adult tickets is better than letting x represent the number of adult tickets.*

	Number of Tickets	Cost (dollars)
Adults ($19 apiece)	A	$19A$
Children ($7 apiece)	$8A$	$7(8A)$
Totals	—	975

Because there are 8 times as many children's tickets purchased than adult tickets, the number of children's tickets purchased is $8A$, recorded in the second column. In the third column, $8A$ children's tickets at $7 apiece will cost $7(8A)$ dollars, and A adult tickets at $19 apiece will cost $19A$ dollars. The final entry in the column gives the total cost of all tickets as $975.

2. *Set up an Equation.* The third column of the table reveals that the sum of the costs for both children and adult tickets is $975. Hence, the equation that models this application is

$$19A + 7(8A) = 975$$

which sums the cost of children and adult tickets at $975.

3. *Solve the Equation.* On the left, use the associative property to remove parentheses.
$$19A + 56A = 975$$

Combine like terms.
$$75A = 975$$

Now, solve.

$$\frac{75A}{75} = \frac{975}{75} \qquad \textcolor{red}{\text{Divide both sides by 75.}}$$
$$A = 13 \qquad \textcolor{red}{\text{Simplify.}}$$

4. *Answer the Question.* The number of adult tickets is 13.

5. *Look Back.* Does our solution make sense? The number of children's tickets purchased is 8 times more than the 13 adult tickets purchased, or 104 children's tickets. Also, the monetary value of 104 children's tickets at $7 apiece is $728, and the monetary value of 13 adult tickets at $19 apiece is $247, a total cost of $975. Our solution is correct.

27. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let N represent the number of nickels from the piggy bank. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting N represent the number of nickels is better than letting x represent the number of nickels.*

Coins	Number of Coins	Value (cents)
Nickels (5 cents apiece)	N	$5N$
Dimes (10 cents apiece)	$N + 15$	$10(N + 15)$
Totals	—	330

Because there are 15 more dimes than nickels, the number of dimes is $N + 15$, recorded in the second column. In the third column, N nickels, worth 5 cents apiece, have a value of $5N$ cents. Next, $N + 15$ dimes, worth 10 cents apiece, have a value of $10(N + 15)$ cents. The final entry in the column gives the total value of the coins as 330 cents.

2. *Set up an Equation.* The third column of the table reveals that the sum of the coin values is 330 cents. Hence, the equation that models this application is

$$5N + 10(N + 15) = 330,$$

which sums the value of the nickels and the value of the dimes to a total of 330 cents.

3. *Solve the Equation.* On the left, use the distributive property to remove parentheses.

$$5N + 10N + 150 = 330$$

Combine like terms.

$$15N + 150 = 330$$

Now, solve.

$$15N + 150 - 150 = 330 - 150 \qquad \text{Subtract 150 from both sides.}$$
$$15N = 180 \qquad \text{Simplify.}$$
$$\frac{15N}{15} = \frac{180}{15} \qquad \text{Divide both sides by 15.}$$
$$N = 12 \qquad \text{Simplify.}$$

4. *Answer the Question.* There are 12 nickels.

5. *Look Back.* Does our solution make sense? Well, the number of dimes is 15 more than 12 nickels, which is 27 dimes. Also, the monetary value of 12 nickels is 60 cents and the monetary value of 27 dimes is 270 cents, a total of 330 cents, or $3.30, so our solution is correct.

29. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let N represent the number of nickels from the piggy bank. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting N represent the number of nickels is better than letting x represent the number of nickels.*

Coins	Number of Coins	Value (cents)
Nickels (5 cents apiece)	N	$5N$
Dimes (10 cents apiece)	$N + 7$	$10(N + 7)$
Totals	—	400

Because there are 7 more dimes than nickels, the number of dimes is $N + 7$, recorded in the second column. In the third column, N nickels, worth 5 cents apiece, have a value of $5N$ cents. Next, $N + 7$ dimes, worth 10 cents apiece, have a value of $10(N + 7)$ cents. The final entry in the column gives the total value of the coins as 400 cents.

2. *Set up an Equation.* The third column of the table reveals that the sum of the coin values is 400 cents. Hence, the equation that models this application is
$$5N + 10(N + 7) = 400,$$
which sums the value of the nickels and the value of the dimes to a total of 400 cents.

3. *Solve the Equation.* On the left, use the distributive property to remove parentheses.
$$5N + 10N + 70 = 400$$

Combine like terms.

$$15N + 70 = 400$$

Now, solve.

$$15N + 70 - 70 = 400 - 70 \qquad \text{Subtract 70 from both sides.}$$
$$15N = 330 \qquad \text{Simplify.}$$
$$\frac{15N}{15} = \frac{330}{15} \qquad \text{Divide both sides by 15.}$$
$$N = 22 \qquad \text{Simplify.}$$

4. *Answer the Question.* There are 22 nickels.

5. *Look Back.* Does our solution make sense? Well, the number of dimes is 7 more than 22 nickels, which is 29 dimes. Also, the monetary value of 22 nickels is 110 cents and the monetary value of 29 dimes is 290 cents, a total of 400 cents, or $4.00, so our solution is correct.

31. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let S represent the amount Jason invests in the savings account. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting S represent the amount invested in savings is better than letting x represent the amount invested in savings.*

Account Type	Amount Deposited
Savings Account (2.5%)	S
Certificate of Deposit (5%)	$S + 7300$
Totals	20300

Because S represents the investment in savings, and we're told that the investment in the certificate of deposit (CD) is $7300 more than the investment in savings, the investment in the CD is therefore $S + 7300$, as indicated in the table.

2. *Set up an Equation.* The second column of the table reveals that the sum of the individual investments in the CD and savings totals $TotalInvestmentF. Hence, the equation that models this application is

$$(S + 7300) + S = 20300.$$

3. *Solve the Equation.* On the left, regroup and combine like terms.

$$2S + 7300 = 20300$$

Now, solve.

$$2S + 7300 - 7300 = 20300 - 7300 \qquad \text{\color{red}Subtract 7300 from both sides.}$$
$$2S = 13000 \qquad \text{\color{red}Simplify.}$$
$$\frac{2S}{2} = \frac{13000}{2} \qquad \text{\color{red}Divide both sides by 2.}$$
$$S = 6500 \qquad \text{\color{red}Simplify.}$$

4. *Answer the Question.* The amount invested in the savings account is $\$6,500$.

5. *Look Back.* Does our solution make sense? Well, the amount invested in the CD is $\$7,300$ more than the $\$6,500$ invested in the savings account, or $\$13,800$. Secondly, the two investments total $\$6,500 + \$13,800 = \$20,300$, so our solution is correct.

33. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let N represent the number of nickels from the piggy bank. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting N represent the number of nickels is better than letting x represent the number of nickels.*

Coins	Number of Coins	Value (cents)
Nickels (5 cents apiece)	N	$5N$
Dimes (10 cents apiece)	$N + 15$	$10(N + 15)$
Totals	—	450

Because there are 15 more dimes than nickels, the number of dimes is $N + 15$, recorded in the second column. In the third column, N nickels, worth 5 cents apiece, have a value of $5N$ cents. Next, $N + 15$ dimes, worth 10 cents apiece, have a value of $10(N + 15)$ cents. The final entry in the column gives the total value of the coins as 450 cents.

2. *Set up an Equation.* The third column of the table reveals that the sum of the coin values is 450 cents. Hence, the equation that models this application is
$$5N + 10(N + 15) = 450,$$
which sums the value of the nickels and the value of the dimes to a total of 450 cents.

3. *Solve the Equation.* On the left, use the distributive property to remove parentheses.

$$5N + 10N + 150 = 450$$

Combine like terms.

$$15N + 150 = 450$$

Now, solve.

$$
\begin{aligned}
15N + 150 - 150 &= 450 - 150 & &\text{Subtract 150 from both sides.}\\
15N &= 300 & &\text{Simplify.}\\
\frac{15N}{15} &= \frac{300}{15} & &\text{Divide both sides by 15.}\\
N &= 20 & &\text{Simplify.}
\end{aligned}
$$

4. *Answer the Question.* There are 20 nickels.

5. *Look Back.* Does our solution make sense? Well, the number of dimes is 15 more than 20 nickels, which is 35 dimes. Also, the monetary value of 20 nickels is 100 cents and the monetary value of 35 dimes is 350 cents, a total of 450 cents, or $4.50, so our solution is correct.

35. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let A represent the number of adult tickets purchased. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting A represent the number of adult tickets is better than letting x represent the number of adult tickets.*

	Number of Tickets	Cost (dollars)
Adults ($10 apiece)	A	$10A$
Children ($4 apiece)	$2A$	$4(2A)$
Totals	—	216

Because there are 2 times as many children's tickets purchased than adult tickets, the number of children's tickets purchased is $2A$, recorded in the second column. In the third column, $2A$ children's tickets at $4 apiece will cost $4(2A)$ dollars, and A adult tickets at $10 apiece will cost $10A$ dollars. The final entry in the column gives the total cost of all tickets as $216.

2. *Set up an Equation.* The third column of the table reveals that the sum of the costs for both children and adult tickets is $216. Hence, the equation that models this application is

$$10A + 4(2A) = 216$$

which sums the cost of children and adult tickets at $216.

3. *Solve the Equation.* On the left, use the associative property to remove parentheses.

$$10A + 8A = 216$$

Combine like terms.

$$18A = 216$$

Now, solve.

$$\frac{18A}{18} = \frac{216}{18} \qquad \color{red}{\text{Divide both sides by 18.}}$$
$$A = 12 \qquad \color{red}{\text{Simplify.}}$$

4. *Answer the Question.* The number of adult tickets is 12.

5. *Look Back.* Does our solution make sense? The number of children's tickets purchased is 2 times more than the 12 adult tickets purchased, or 24 children's tickets. Also, the monetary value of 24 children's tickets at $4 apiece is $96, and the monetary value of 12 adult tickets at $10 apiece is $120, a total cost of $216. Our solution is correct.

37. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let N represent the number of nickels from the piggy bank. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting N represent the number of nickels is better than letting x represent the number of nickels.*

Coins	Number of Coins	Value (cents)
Nickels (5 cents apiece)	N	$5N$
Dimes (10 cents apiece)	$N + 7$	$10(N + 7)$
Totals	—	370

Because there are 7 more dimes than nickels, the number of dimes is $N + 7$, recorded in the second column. In the third column, N nickels, worth 5 cents apiece, have a value of $5N$ cents. Next, $N + 7$ dimes, worth 10 cents apiece, have a value of $10(N + 7)$ cents. The final entry in the column gives the total value of the coins as 370 cents.

2. *Set up an Equation.* The third column of the table reveals that the sum of the coin values is 370 cents. Hence, the equation that models this application is

$$5N + 10(N + 7) = 370,$$

which sums the value of the nickels and the value of the dimes to a total of 370 cents.

3. *Solve the Equation.* On the left, use the distributive property to remove parentheses.

$$5N + 10N + 70 = 370$$

Combine like terms.

$$15N + 70 = 370$$

Now, solve.

$$15N + 70 - 70 = 370 - 70 \qquad \text{Subtract 70 from both sides.}$$
$$15N = 300 \qquad \text{Simplify.}$$
$$\frac{15N}{15} = \frac{300}{15} \qquad \text{Divide both sides by 15.}$$
$$N = 20 \qquad \text{Simplify.}$$

4. *Answer the Question.* There are 20 nickels.

5. *Look Back.* Does our solution make sense? Well, the number of dimes is 7 more than 20 nickels, which is 27 dimes. Also, the monetary value of 20 nickels is 100 cents and the monetary value of 27 dimes is 270 cents, a total of 370 cents, or $3.70, so our solution is correct.

39. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let S represent the amount Mary invests in the savings account. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting S represent the amount invested in savings is better than letting x represent the amount invested in savings.*

Account Type	Amount Deposited
Savings Account (2%)	S
Certificate of Deposit (4%)	$S + 7300$
Totals	22300

Because S represents the investment in savings, and we're told that the investment in the certificate of deposit (CD) is $7300 more than the investment in savings, the investment in the CD is therefore $S + 7300$, as indicated in the table.

2. *Set up an Equation.* The second column of the table reveals that the sum of the individual investments in the CD and savings totals $TotalInvestmentF. Hence, the equation that models this application is

$$(S + 7300) + S = 22300.$$

3. *Solve the Equation.* On the left, regroup and combine like terms.

$$2S + 7300 = 22300$$

Now, solve.

$$2S + 7300 - 7300 = 22300 - 7300 \quad \text{Subtract 7300 from both sides.}$$
$$2S = 15000 \quad \text{Simplify.}$$
$$\frac{2S}{2} = \frac{15000}{2} \quad \text{Divide both sides by 2.}$$
$$S = 7500 \quad \text{Simplify.}$$

4. *Answer the Question.* The amount invested in the savings account is $7,500.

5. *Look Back.* Does our solution make sense? Well, the amount invested in the CD is $7,300 more than the $7,500 invested in the savings account, or $14,800. Secondly, the two investments total $7,500 + $14,800 = $22,300, so our solution is correct.

41. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let A represent the number of adult tickets purchased. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting A represent the number of adult tickets is better than letting x represent the number of adult tickets.*

	Number of Tickets	Cost (dollars)
Adults ($16 apiece)	A	$16A$
Children ($6 apiece)	$8A$	$6(8A)$
Totals	—	1024

Because there are 8 times as many children's tickets purchased than adult tickets, the number of children's tickets purchased is $8A$, recorded in the second column. In the third column, $8A$ children's tickets at $6 apiece will cost $6(8A)$ dollars, and A adult tickets at $16 apiece will cost $16A$ dollars. The final entry in the column gives the total cost of all tickets as $1024.

2. *Set up an Equation.* The third column of the table reveals that the sum of the costs for both children and adult tickets is \$1024. Hence, the equation that models this application is

$$16A + 6(8A) = 1024$$

which sums the cost of children and adult tickets at \$1024.

3. *Solve the Equation.* On the left, use the associative property to remove parentheses.

$$16A + 48A = 1024$$

Combine like terms.

$$64A = 1024$$

Now, solve.

$$\frac{64A}{64} = \frac{1024}{64} \qquad \text{Divide both sides by 64.}$$
$$A = 16 \qquad \text{Simplify.}$$

4. *Answer the Question.* The number of adult tickets is 16.

5. *Look Back.* Does our solution make sense? The number of children's tickets purchased is 8 times more than the 16 adult tickets purchased, or 128 children's tickets. Also, the monetary value of 128 children's tickets at \$6 apiece is \$768, and the monetary value of 16 adult tickets at \$16 apiece is \$256, a total cost of \$1,024. Our solution is correct.

43. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let S represent the amount Alan invests in the savings account. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting S represent the amount invested in savings is better than letting x represent the amount invested in savings.*

Account Type	Amount Deposited
Savings Account (3.5%)	S
Certificate of Deposit (6%)	$S + 6400$
Totals	25600

Because S represents the investment in savings, and we're told that the investment in the certificate of deposit (CD) is \$6400 more than the investment in savings, the investment in the CD is therefore $S + 6400$, as indicated in the table.

2. *Set up an Equation.* The second column of the table reveals that the sum of the individual investments in the CD and savings totals $TotalInvestmentF. Hence, the equation that models this application is

$$(S + 6400) + S = 25600.$$

3. *Solve the Equation.* On the left, regroup and combine like terms.

$$2S + 6400 = 25600$$

Now, solve.

$$
\begin{aligned}
2S + 6400 - 6400 &= 25600 - 6400 && \text{Subtract 6400 from both sides.}\\
2S &= 19200 && \text{Simplify.}\\
\frac{2S}{2} &= \frac{19200}{2} && \text{Divide both sides by 2.}\\
S &= 9600 && \text{Simplify.}
\end{aligned}
$$

4. *Answer the Question.* The amount invested in the savings account is $9,600$.

5. *Look Back.* Does our solution make sense? Well, the amount invested in the CD is $6,400 more than the $9,600 invested in the savings account, or $16,000. Secondly, the two investments total $9,600 + $16,000 = $25,600$, so our solution is correct.

45. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let S represent the amount Tony invests in the savings account. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting S represent the amount invested in savings is better than letting x represent the amount invested in savings.*

Account Type	Amount Deposited
Savings Account (2%)	S
Certificate of Deposit (4%)	$S + 9200$
Totals	20600

Because S represents the investment in savings, and we're told that the investment in the certificate of deposit (CD) is $9200 more than the investment in savings, the investment in the CD is therefore $S + 9200$, as indicated in the table.

2. *Set up an Equation.* The second column of the table reveals that the sum of the individual investments in the CD and savings totals \$TotalInvestmentF. Hence, the equation that models this application is

$$(S + 9200) + S = 20600.$$

3. *Solve the Equation.* On the left, regroup and combine like terms.

$$2S + 9200 = 20600$$

Now, solve.

$$
\begin{aligned}
2S + 9200 - 9200 &= 20600 - 9200 && \text{Subtract 9200 from both sides.} \\
2S &= 11400 && \text{Simplify.} \\
\frac{2S}{2} &= \frac{11400}{2} && \text{Divide both sides by 2.} \\
S &= 5700 && \text{Simplify.}
\end{aligned}
$$

4. *Answer the Question.* The amount invested in the savings account is \$5, 700.

5. *Look Back.* Does our solution make sense? Well, the amount invested in the CD is \$9,200 more than the \$5, 700 invested in the savings account, or \$14, 900. Secondly, the two investments total \$5, 700 + \$14, 900 = \$20, 600, so our solution is correct.

47. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let A represent the number of adult tickets purchased. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting A represent the number of adult tickets is better than letting x represent the number of adult tickets.*

	Number of Tickets	Cost (dollars)
Adults (\$14 apiece)	A	$14A$
Children (\$2 apiece)	$2A$	$2(2A)$
Totals	—	234

Because there are 2 times as many children's tickets purchased than adult tickets, the number of children's tickets purchased is $2A$, recorded in the second column. In the third column, $2A$ children's tickets at \$2 apiece will cost $2(2A)$ dollars, and A adult tickets at \$14 apiece will cost $14A$ dollars. The final entry in the column gives the total cost of all tickets as \$234.

2. *Set up an Equation.* The third column of the table reveals that the sum of the costs for both children and adult tickets is \$234. Hence, the equation that models this application is

$$14A + 2(2A) = 234$$

which sums the cost of children and adult tickets at \$234.

3. *Solve the Equation.* On the left, use the associative property to remove parentheses.

$$14A + 4A = 234$$

Combine like terms.

$$18A = 234$$

Now, solve.

$$\frac{18A}{18} = \frac{234}{18} \qquad \text{\textcolor{red}{Divide both sides by 18.}}$$
$$A = 13 \qquad \text{\textcolor{red}{Simplify.}}$$

4. *Answer the Question.* The number of adult tickets is 13.

5. *Look Back.* Does our solution make sense? The number of children's tickets purchased is 2 times more than the 13 adult tickets purchased, or 26 children's tickets. Also, the monetary value of 26 children's tickets at \$2 apiece is \$52, and the monetary value of 13 adult tickets at \$14 apiece is \$182, a total cost of \$234. Our solution is correct.

Chapter 4

Fractions

4.1 Equivalent Fractions

1. List the divisors of 72.

$$1, \ 2, \ 3, \ 4, \ 6, \ 8, \ 9, \ 12, \ 18, \ 24, \ 36, \ 72$$

List the divisiors of 8.

$$1, \ 2, \ 4, \ 8$$

The common divisors of 72 and 8 are:

$$1, \ 2, \ 4, \ 8$$

The greatest of these common divisors is the GCD of 72 and 8; that is, the GCD is 8.

3. List the divisors of 52.

$$1, \ 2, \ 4, \ 13, \ 26, \ 52$$

List the divisiors of 20.

$$1, \ 2, \ 4, \ 5, \ 10, \ 20$$

The common divisors of 52 and 20 are:

$$1, \ 2, \ 4$$

The greatest of these common divisors is the GCD of 52 and 20; that is, the GCD is 4.

5. List the divisors of 36.

$$1, \ 2, \ 3, \ 4, \ 6, \ 9, \ 12, \ 18, \ 36$$

List the divisors of 63.

$$1, \ 3, \ 7, \ 9, \ 21, \ 63$$

The common divisors of 36 and 63 are:

$$1, \ 3, \ 9$$

The greatest of these common divisors is the GCD of 36 and 63; that is, the GCD is 9.

7. List the divisors of 72.

$$1, \ 2, \ 3, \ 4, \ 6, \ 8, \ 9, \ 12, \ 18, \ 24, \ 36, \ 72$$

List the divisors of 44.

$$1, \ 2, \ 4, \ 11, \ 22, \ 44$$

The common divisors of 72 and 44 are:

$$1, \ 2, \ 4$$

The greatest of these common divisors is the GCD of 72 and 44; that is, the GCD is 4.

9. List the divisors of 16.

$$1, \ 2, \ 4, \ 8, \ 16$$

List the divisors of 56.

$$1, \ 2, \ 4, \ 7, \ 8, \ 14, \ 28, \ 56$$

The common divisors of 16 and 56 are:

$$1, \ 2, \ 4, \ 8$$

The greatest of these common divisors is the GCD of 16 and 56; that is, the GCD is 8.

11. List the divisors of 84.

$$1, \ 2, \ 3, \ 4, \ 6, \ 7, \ 12, \ 14, \ 21, \ 28, \ 42, \ 84$$

List the divisors of 24.

$$1, \ 2, \ 3, \ 4, \ 6, \ 8, \ 12, \ 24$$

The common divisors of 84 and 24 are:

$$1, \ 2, \ 3, \ 4, \ 6, \ 12$$

The greatest of these common divisors is the GCD of 84 and 24; that is, the GCD is 12.

13. The greatest common divisor of 22 and 98 is 2. Therefore,

$$\frac{22}{98} = \frac{11 \cdot 2}{49 \cdot 2} \qquad \textcolor{red}{\text{Factor out 2 in numerator and denominator.}}$$
$$= \frac{11}{49} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{22}{98} = \frac{2 \cdot 11}{2 \cdot 7 \cdot 7} \qquad \textcolor{red}{\text{Prime factorization.}}$$
$$= \frac{11}{7 \cdot 7} \qquad \textcolor{red}{\text{Cancel common factors.}}$$
$$= \frac{11}{49} \qquad \textcolor{red}{\text{Simplify numerator and denominator.}}$$

15. The greatest common divisor of 93 and 15 is 3. Therefore,

$$\frac{93}{15} = \frac{31 \cdot 3}{5 \cdot 3} \qquad \textcolor{red}{\text{Factor out 3 in numerator and denominator.}}$$
$$= \frac{31}{5} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{93}{15} = \frac{3 \cdot 31}{3 \cdot 5} \qquad \textcolor{red}{\text{Prime factorization.}}$$
$$= \frac{31}{5} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

17. The greatest common divisor of 69 and 21 is 3. Therefore,

$$\frac{69}{21} = \frac{23 \cdot 3}{7 \cdot 3} \qquad \textcolor{red}{\text{Factor out 3 in numerator and denominator.}}$$
$$= \frac{23}{7} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{69}{21} = \frac{3 \cdot 23}{3 \cdot 7} \qquad \textcolor{red}{\text{Prime factorization.}}$$
$$= \frac{23}{7} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

19. The greatest common divisor of 74 and 12 is 2. Therefore,

$$
\frac{74}{12} = \frac{37 \cdot 2}{6 \cdot 2} \qquad \text{Factor out 2 in numerator and denominator.}
$$
$$
= \frac{37}{6} \qquad \text{Cancel common factors.}
$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$
\frac{74}{12} = \frac{2 \cdot 37}{2 \cdot 2 \cdot 3} \qquad \text{Prime factorization.}
$$
$$
= \frac{37}{2 \cdot 3} \qquad \text{Cancel common factors.}
$$
$$
= \frac{37}{6} \qquad \text{Simplify numerator and denominator.}
$$

21. The greatest common divisor of 66 and 57 is 3. Therefore,

$$
\frac{66}{57} = \frac{22 \cdot 3}{19 \cdot 3} \qquad \text{Factor out 3 in numerator and denominator.}
$$
$$
= \frac{22}{19} \qquad \text{Cancel common factors.}
$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$
\frac{66}{57} = \frac{2 \cdot 3 \cdot 11}{3 \cdot 19} \qquad \text{Prime factorization.}
$$
$$
= \frac{2 \cdot 11}{19} \qquad \text{Cancel common factors.}
$$
$$
= \frac{22}{19} \qquad \text{Simplify numerator and denominator.}
$$

23. The greatest common divisor of 33 and 99 is 33. Therefore,

$$
\frac{33}{99} = \frac{1 \cdot 33}{3 \cdot 33} \qquad \text{Factor out 33 in numerator and denominator.}
$$
$$
= \frac{1}{3} \qquad \text{Cancel common factors.}
$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$
\frac{33}{99} = \frac{3 \cdot 11}{3 \cdot 3 \cdot 11} \qquad \text{Prime factorization.}
$$
$$
= \frac{1}{3} \qquad \text{Cancel common factors.}
$$

25. The greatest common divisor of 69 and 24 is 3. Therefore,

$$\begin{aligned}
\frac{69}{24} &= \frac{23 \cdot 3}{8 \cdot 3} && \text{\textcolor{red}{Factor out 3 in numerator and denominator.}} \\
&= \frac{23}{8} && \text{\textcolor{red}{Cancel common factors.}}
\end{aligned}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\begin{aligned}
\frac{69}{24} &= \frac{3 \cdot 23}{2 \cdot 2 \cdot 2 \cdot 3} && \text{\textcolor{red}{Prime factorization.}} \\
&= \frac{23}{2 \cdot 2 \cdot 2} && \text{\textcolor{red}{Cancel common factors.}} \\
&= \frac{23}{8} && \text{\textcolor{red}{Simplify numerator and denominator.}}
\end{aligned}$$

27. The greatest common divisor of 46 and 44 is 2. Therefore,

$$\begin{aligned}
\frac{46}{44} &= \frac{23 \cdot 2}{22 \cdot 2} && \text{\textcolor{red}{Factor out 2 in numerator and denominator.}} \\
&= \frac{23}{22} && \text{\textcolor{red}{Cancel common factors.}}
\end{aligned}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\begin{aligned}
\frac{46}{44} &= \frac{2 \cdot 23}{2 \cdot 2 \cdot 11} && \text{\textcolor{red}{Prime factorization.}} \\
&= \frac{23}{2 \cdot 11} && \text{\textcolor{red}{Cancel common factors.}} \\
&= \frac{23}{22} && \text{\textcolor{red}{Simplify numerator and denominator.}}
\end{aligned}$$

29. Both numerator and denominator must be multiplied by 24:

$$\begin{aligned}
3 &= \frac{3}{1} && \text{\textcolor{red}{$3 = 3/1$.}} \\
&= \frac{3 \cdot 24}{1 \cdot 24} && \text{\textcolor{red}{Multiply numerator and denominator by 24.}} \\
&= \frac{72}{24} && \text{\textcolor{red}{Simplify numerator and denominator.}}
\end{aligned}$$

31. Since $57 = 19 \cdot 3$, both numerator and denominator must be multiplied by 3:

$$\frac{25}{19} = \frac{25 \cdot 3}{19 \cdot 3} \qquad \text{Multiply numerator and denominator by 3.}$$

$$= \frac{75}{57} \qquad \text{Simplify numerator and denominator.}$$

33. Both numerator and denominator must be multiplied by 2:

$$2 = \frac{2}{1} \qquad 2 = 2/1.$$

$$= \frac{2 \cdot 2}{1 \cdot 2} \qquad \text{Multiply numerator and denominator by 2.}$$

$$= \frac{4}{2} \qquad \text{Simplify numerator and denominator.}$$

35. Since $95 = 19 \cdot 5$, both numerator and denominator must be multiplied by 5:

$$\frac{18}{19} = \frac{18 \cdot 5}{19 \cdot 5} \qquad \text{Multiply numerator and denominator by 5.}$$

$$= \frac{90}{95} \qquad \text{Simplify numerator and denominator.}$$

37. Since $24 = 3 \cdot 8$, both numerator and denominator must be multiplied by 8:

$$\frac{1}{3} = \frac{1 \cdot 8}{3 \cdot 8} \qquad \text{Multiply numerator and denominator by 8.}$$

$$= \frac{8}{24} \qquad \text{Simplify numerator and denominator.}$$

39. Both numerator and denominator must be multiplied by 4:

$$16 = \frac{16}{1} \qquad 16 = 16/1.$$

$$= \frac{16 \cdot 4}{1 \cdot 4} \qquad \text{Multiply numerator and denominator by 4.}$$

$$= \frac{64}{4} \qquad \text{Simplify numerator and denominator.}$$

41. The greatest common divisor of 34 and 86 is 2. Therefore,

$$\frac{34}{-86} = -\frac{34}{86} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{17 \cdot 2}{43 \cdot 2} \qquad \text{Factor out 2 in numerator and denominator.}$$

$$= -\frac{17}{43} \qquad \text{Cancel common factors.}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{34}{-86} = -\frac{34}{86} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 17}{2 \cdot 43} \qquad \text{Prime factorization.}$$

$$= -\frac{17}{43} \qquad \text{Cancel common factors.}$$

43. The greatest common divisor of 72 and 92 is 4. Therefore,

$$\frac{-72}{-92} = \frac{72}{92} \qquad \text{Like signs give a positive result.}$$

$$= \frac{18 \cdot 4}{23 \cdot 4} \qquad \text{Factor out 4 in numerator and denominator.}$$

$$= \frac{18}{23} \qquad \text{Cancel common factors.}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{-72}{-92} = \frac{72}{92} \qquad \text{Like signs give a positive result.}$$

$$= \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 23} \qquad \text{Prime factorization.}$$

$$= \frac{2 \cdot 3 \cdot 3}{23} \qquad \text{Cancel common factors.}$$

$$= \frac{18}{23} \qquad \text{Simplify numerator and denominator.}$$

45. The greatest common divisor of 92 and 82 is 2. Therefore,

$$\frac{-92}{82} = -\frac{92}{82} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{46 \cdot 2}{41 \cdot 2} \qquad \text{Factor out 2 in numerator and denominator.}$$

$$= -\frac{46}{41} \qquad \text{Cancel common factors.}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{-92}{82} = -\frac{92}{82}$$ Unlike signs give a negative result.

$$= -\frac{2 \cdot 2 \cdot 23}{2 \cdot 41}$$ Prime factorization.

$$= -\frac{2 \cdot 23}{41}$$ Cancel common factors.

$$= -\frac{46}{41}$$ Simplify numerator and denominator.

47. The greatest common divisor of 21 and 33 is 3. Therefore,

$$\frac{-21}{33} = -\frac{21}{33}$$ Unlike signs give a negative result.

$$= -\frac{7 \cdot 3}{11 \cdot 3}$$ Factor out 3 in numerator and denominator.

$$= -\frac{7}{11}$$ Cancel common factors.

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{-21}{33} = -\frac{21}{33}$$ Unlike signs give a negative result.

$$= -\frac{3 \cdot 7}{3 \cdot 11}$$ Prime factorization.

$$= -\frac{7}{11}$$ Cancel common factors.

49. The greatest common divisor of 22 and 98 is 2. Therefore,

$$\frac{22}{-98} = -\frac{22}{98}$$ Unlike signs give a negative result.

$$= -\frac{11 \cdot 2}{49 \cdot 2}$$ Factor out 2 in numerator and denominator.

$$= -\frac{11}{49}$$ Cancel common factors.

Alternatively, factor the numerator and denominator and cancel common

factors:

$$\frac{22}{-98} = -\frac{22}{98} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 11}{2 \cdot 7 \cdot 7} \qquad \text{Prime factorization.}$$

$$= -\frac{11}{7 \cdot 7} \qquad \text{Cancel common factors.}$$

$$= -\frac{11}{49} \qquad \text{Simplify numerator and denominator.}$$

51. The greatest common divisor of 42 and 88 is 2. Therefore,

$$\frac{42}{-88} = -\frac{42}{88} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{21 \cdot 2}{44 \cdot 2} \qquad \text{Factor out 2 in numerator and denominator.}$$

$$= -\frac{21}{44} \qquad \text{Cancel common factors.}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{42}{-88} = -\frac{42}{88} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 11} \qquad \text{Prime factorization.}$$

$$= -\frac{3 \cdot 7}{2 \cdot 2 \cdot 11} \qquad \text{Cancel common factors.}$$

$$= -\frac{21}{44} \qquad \text{Simplify numerator and denominator.}$$

53. The greatest common divisor of 94 and 6 is 2. Therefore,

$$\frac{94}{-6} = -\frac{94}{6} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{47 \cdot 2}{3 \cdot 2} \qquad \text{Factor out 2 in numerator and denominator.}$$

$$= -\frac{47}{3} \qquad \text{Cancel common factors.}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{94}{-6} = -\frac{94}{6} \qquad\qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 47}{2 \cdot 3} \qquad\qquad \text{Prime factorization.}$$

$$= -\frac{47}{3} \qquad\qquad \text{Cancel common factors.}$$

55. The greatest common divisor of 10 and 86 is 2. Therefore,

$$\frac{10}{-86} = -\frac{10}{86} \qquad\qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{5 \cdot 2}{43 \cdot 2} \qquad\qquad \text{Factor out 2 in numerator and denominator.}$$

$$= -\frac{5}{43} \qquad\qquad \text{Cancel common factors.}$$

Alternatively, factor the numerator and denominator and cancel common factors:

$$\frac{10}{-86} = -\frac{10}{86} \qquad\qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 5}{2 \cdot 43} \qquad\qquad \text{Prime factorization.}$$

$$= -\frac{5}{43} \qquad\qquad \text{Cancel common factors.}$$

57. Since $62 = 2 \cdot 31$, both numerator and denominator must be multiplied by $31n$:

$$\frac{3}{2} = \frac{3 \cdot 31n}{2 \cdot 31n} \qquad\qquad \text{Multiply numerator and denominator by } 31n.$$

$$= \frac{93n}{62n} \qquad\qquad \text{Simplify numerator and denominator.}$$

59. Since $60 = 10 \cdot 6$, both numerator and denominator must be multiplied by $6m$:

$$\frac{13}{10} = \frac{13 \cdot 6m}{10 \cdot 6m} \qquad\qquad \text{Multiply numerator and denominator by } 6m.$$

$$= \frac{78m}{60m} \qquad\qquad \text{Simplify numerator and denominator.}$$

61. Since $50 = 2 \cdot 25$, both numerator and denominator must be multiplied by $25n$:

$$\frac{3}{2} = \frac{3 \cdot 25n}{2 \cdot 25n} \qquad \text{\color{red}{Multiply numerator and denominator by } 25n.}$$
$$= \frac{75n}{50n} \qquad \text{\color{red}{Simplify numerator and denominator.}}$$

63. Both numerator and denominator must be multiplied by $4m$:

$$11 = \frac{11}{1} \qquad \text{\color{red}{$11 = 11/1$.}}$$
$$= \frac{11 \cdot 4m}{1 \cdot 4m} \qquad \text{\color{red}{Multiply numerator and denominator by } 4m.}$$
$$= \frac{44m}{4m} \qquad \text{\color{red}{Simplify numerator and denominator.}}$$

65. Both numerator and denominator must be multiplied by $10m$:

$$3 = \frac{3}{1} \qquad \text{\color{red}{$3 = 3/1$.}}$$
$$= \frac{3 \cdot 10m}{1 \cdot 10m} \qquad \text{\color{red}{Multiply numerator and denominator by } 10m.}$$
$$= \frac{30m}{10m} \qquad \text{\color{red}{Simplify numerator and denominator.}}$$

67. Both numerator and denominator must be multiplied by $5n$:

$$6 = \frac{6}{1} \qquad \text{\color{red}{$6 = 6/1$.}}$$
$$= \frac{6 \cdot 5n}{1 \cdot 5n} \qquad \text{\color{red}{Multiply numerator and denominator by } 5n.}$$
$$= \frac{30n}{5n} \qquad \text{\color{red}{Simplify numerator and denominator.}}$$

69. Factor the numerator and denominator and cancel common factors:

$$\frac{82y^5}{-48y} = -\frac{82y^5}{48y} \qquad \text{\color{red}{Unlike signs give a negative result.}}$$
$$= -\frac{2 \cdot 41 \cdot y \cdot y \cdot y \cdot y \cdot y}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot y} \qquad \text{\color{red}{Prime factorization.}}$$
$$= -\frac{41 \cdot y \cdot y \cdot y \cdot y}{2 \cdot 2 \cdot 2 \cdot 3} \qquad \text{\color{red}{Cancel common factors.}}$$
$$= -\frac{41y^4}{24} \qquad \text{\color{red}{Simplify numerator and denominator.}}$$

71. Factor the numerator and denominator and cancel common factors:

$$\frac{-77x^5}{44x^4} = -\frac{77x^5}{44x^4} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{7 \cdot 11 \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot 2 \cdot 11 \cdot x \cdot x \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{7 \cdot x}{2 \cdot 2} \qquad \text{Cancel common factors.}$$

$$= -\frac{7x}{4} \qquad \text{Simplify numerator and denominator.}$$

73. Factor the numerator and denominator and cancel common factors:

$$\frac{-14y^5}{54y^2} = -\frac{14y^5}{54y^2} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 7 \cdot y \cdot y \cdot y \cdot y \cdot y}{2 \cdot 3 \cdot 3 \cdot 3 \cdot y \cdot y} \qquad \text{Prime factorization.}$$

$$= -\frac{7 \cdot y \cdot y \cdot y}{3 \cdot 3 \cdot 3} \qquad \text{Cancel common factors.}$$

$$= -\frac{7y^3}{27} \qquad \text{Simplify numerator and denominator.}$$

75. Factor the numerator and denominator and cancel common factors:

$$\frac{42x}{81x^3} = \frac{2 \cdot 3 \cdot 7 \cdot x}{3 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= \frac{2 \cdot 7}{3 \cdot 3 \cdot 3 \cdot x \cdot x} \qquad \text{Cancel common factors.}$$

$$= \frac{14}{27x^2} \qquad \text{Simplify numerator and denominator.}$$

77. Factor the numerator and denominator and cancel common factors:

$$\frac{-12x^5}{14x^6} = -\frac{12x^5}{14x^6} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot 7 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{2 \cdot 3}{7 \cdot x} \qquad \text{Cancel common factors.}$$

$$= -\frac{6}{7x} \qquad \text{Simplify numerator and denominator.}$$

79. Factor the numerator and denominator and cancel common factors:

$$\frac{-74x}{22x^2} = -\frac{74x}{22x^2} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 37 \cdot x}{2 \cdot 11 \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{37}{11 \cdot x} \qquad \text{Cancel common factors.}$$

$$= -\frac{37}{11x} \qquad \text{Simplify numerator and denominator.}$$

81. Factor the numerator and denominator and cancel common factors:

$$\frac{-12y^5}{98y^6} = -\frac{12y^5}{98y^6} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 2 \cdot 3 \cdot y \cdot y \cdot y \cdot y \cdot y}{2 \cdot 7 \cdot 7 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y} \qquad \text{Prime factorization.}$$

$$= -\frac{2 \cdot 3}{7 \cdot 7 \cdot y} \qquad \text{Cancel common factors.}$$

$$= -\frac{6}{49y} \qquad \text{Simplify numerator and denominator.}$$

83. Factor the numerator and denominator and cancel common factors:

$$\frac{18x^6}{-54x^2} = -\frac{18x^6}{54x^2} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{x \cdot x \cdot x \cdot x}{3} \qquad \text{Cancel common factors.}$$

$$= -\frac{x^4}{3} \qquad \text{Simplify numerator and denominator.}$$

85. Factor the numerator and denominator and cancel common factors:

$$\frac{26y^2x^4}{-62y^6x^2} = -\frac{26y^2x^4}{62y^6x^2} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 13 \cdot y \cdot y \cdot x \cdot x \cdot x \cdot x}{2 \cdot 31 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{13 \cdot x \cdot x}{31 \cdot y \cdot y \cdot y \cdot y} \qquad \text{Cancel common factors.}$$

$$= -\frac{13x^2}{31y^4} \qquad \text{Simplify numerator and denominator.}$$

87. Factor the numerator and denominator and cancel common factors:

$$\frac{-2y^6x^4}{-94y^2x^5} = \frac{2y^6x^4}{94y^2x^5} \qquad \text{Like signs give a positive result.}$$

$$= \frac{2 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot x \cdot x \cdot x \cdot x}{2 \cdot 47 \cdot y \cdot y \cdot x \cdot x \cdot x \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= \frac{y \cdot y \cdot y \cdot y}{47 \cdot x} \qquad \text{Cancel common factors.}$$

$$= \frac{y^4}{47x} \qquad \text{Simplify numerator and denominator.}$$

89. Factor the numerator and denominator and cancel common factors:

$$\frac{30y^5x^5}{-26yx^4} = -\frac{30y^5x^5}{26yx^4} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 3 \cdot 5 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot 13 \cdot y \cdot x \cdot x \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{3 \cdot 5 \cdot y \cdot y \cdot y \cdot y \cdot x}{13} \qquad \text{Cancel common factors.}$$

$$= -\frac{15y^4x}{13} \qquad \text{Simplify numerator and denominator.}$$

91. Factor the numerator and denominator and cancel common factors:

$$\frac{36x^3y^2}{-98x^4y^5} = -\frac{36x^3y^2}{98x^4y^5} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y}{2 \cdot 7 \cdot 7 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y} \qquad \text{Prime factorization.}$$

$$= -\frac{2 \cdot 3 \cdot 3}{7 \cdot 7 \cdot x \cdot y \cdot y \cdot y} \qquad \text{Cancel common factors.}$$

$$= -\frac{18}{49xy^3} \qquad \text{Simplify numerator and denominator.}$$

93. Factor the numerator and denominator and cancel common factors:

$$\frac{-8x^6y^3}{54x^3y^5} = -\frac{8x^6y^3}{54x^3y^5} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y}{2 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y} \qquad \text{Prime factorization.}$$

$$= -\frac{2 \cdot 2 \cdot x \cdot x \cdot x}{3 \cdot 3 \cdot 3 \cdot y \cdot y} \qquad \text{Cancel common factors.}$$

$$= -\frac{4x^3}{27y^2} \qquad \text{Simplify numerator and denominator.}$$

95. Factor the numerator and denominator and cancel common factors:

$$\frac{34yx^6}{-58y^5x^4} = -\frac{34yx^6}{58y^5x^4} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 17 \cdot y \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{2 \cdot 29 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot x \cdot x \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{17 \cdot x \cdot x}{29 \cdot y \cdot y \cdot y \cdot y} \qquad \text{Cancel common factors.}$$

$$= -\frac{17x^2}{29y^4} \qquad \text{Simplify numerator and denominator.}$$

97. Factor the numerator and denominator and cancel common factors:

$$\frac{-36y^3x^5}{51y^2x} = -\frac{36y^3x^5}{51y^2x} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{2 \cdot 2 \cdot 3 \cdot 3 \cdot y \cdot y \cdot y \cdot x \cdot x \cdot x \cdot x \cdot x}{3 \cdot 17 \cdot y \cdot y \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{2 \cdot 2 \cdot 3 \cdot y \cdot x \cdot x \cdot x \cdot x}{17} \qquad \text{Cancel common factors.}$$

$$= -\frac{12yx^4}{17} \qquad \text{Simplify numerator and denominator.}$$

99. Factor the numerator and denominator and cancel common factors:

$$\frac{91y^3x^2}{-28y^5x^5} = -\frac{91y^3x^2}{28y^5x^5} \qquad \text{Unlike signs give a negative result.}$$

$$= -\frac{7 \cdot 13 \cdot y \cdot y \cdot y \cdot x \cdot x}{2 \cdot 2 \cdot 7 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot x \cdot x \cdot x \cdot x \cdot x} \qquad \text{Prime factorization.}$$

$$= -\frac{13}{2 \cdot 2 \cdot y \cdot y \cdot x \cdot x \cdot x} \qquad \text{Cancel common factors.}$$

$$= -\frac{13}{4y^2x^3} \qquad \text{Simplify numerator and denominator.}$$

101. i) To find the fraction of named storms that grew into hurricanes, find the number of storms that grew into hurricanes for the numerator, with the total number of named storms in the denominator. Then, reduce the fraction if possible.

$$\frac{8}{16} = \frac{1}{2}$$

Therefore, 1/2 of the named storms from 2008 grew into hurricanes.

ii) To find the fraction of named storms that became major hurricanes, create a fraction with the number of major hurricanes in the numerator and the total

number of named storms in the denominator. Therefore, 5/16 of the named storms from 2008 grew into major hurricanes. Note that this fraction is already in lowest terms.

iii) To find the fraction of hurricanes that were major, create a fraction with the number of major hurricanes in the numerator and the total number of hurricanes in the denominator. Therefore, 5/8 of the hurricanes were major hurricanes. Note that this fraction is already in lowest terms.

4.2   Multiplying Fractions

1. First, shade 1/3 of the whole. This shaded area represents 1/3.

Now shade 1/3 of the shaded area in the previous figure. The result shows that one of nine equally sized rectangles is shaded. This shows that 1/3 of 1/3 is 1/9.

3. First, shade 1/4 of the whole. This shaded area represents 1/4.

Now shade 1/3 of the shaded area in the previous figure. The result shows that one of twelve equally sized rectangles is shaded. This shows that 1/3 of 1/4 is 1/12.

5. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-21}{4} \cdot \frac{22}{19} = -\frac{(3 \cdot 7) \cdot (2 \cdot 11)}{(2 \cdot 2) \cdot (19)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{3 \cdot 7 \cdot 11}{2 \cdot 19}$$

Cancel common factors.

$$= -\frac{231}{38}$$

Multiply numerators and denominators.

7. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{20}{11} \cdot \frac{-17}{22} = -\frac{(2 \cdot 2 \cdot 5) \cdot (17)}{(11) \cdot (2 \cdot 11)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{2 \cdot 5 \cdot 17}{11 \cdot 11}$$

Cancel common factors.

$$= -\frac{170}{121}$$

Multiply numerators and denominators.

9. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{21}{8} \cdot \frac{-14}{15} = -\frac{(3 \cdot 7) \cdot (2 \cdot 7)}{(2 \cdot 2 \cdot 2) \cdot (3 \cdot 5)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{7 \cdot 7}{2 \cdot 2 \cdot 5}$$

Cancel common factors.

$$= -\frac{49}{20}$$

Multiply numerators and denominators.

11. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-5}{11} \cdot \frac{7}{20} = -\frac{(5) \cdot (7)}{(11) \cdot (2 \cdot 2 \cdot 5)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{7}{11 \cdot 2 \cdot 2}$$

Cancel common factors.

$$= -\frac{7}{44}$$

Multiply numerators and denominators.

13. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{8}{13} \cdot \frac{-1}{6} = -\frac{(2 \cdot 2 \cdot 2) \cdot (1)}{(13) \cdot (2 \cdot 3)}$$ Prime factorization.
Unlike signs give a negative product.

$$= -\frac{2 \cdot 2}{13 \cdot 3}$$ Cancel common factors.

$$= -\frac{4}{39}$$ Multiply numerators and denominators.

15. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{2}{15} \cdot \frac{-9}{8} = -\frac{(2) \cdot (3 \cdot 3)}{(3 \cdot 5) \cdot (2 \cdot 2 \cdot 2)}$$ Prime factorization.
Unlike signs give a negative product.

$$= -\frac{3}{5 \cdot 2 \cdot 2}$$ Cancel common factors.

$$= -\frac{3}{20}$$ Multiply numerators and denominators.

17. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{17}{12} \cdot \frac{3}{4} = \frac{(17) \cdot (3)}{(2 \cdot 2 \cdot 3) \cdot (2 \cdot 2)}$$ Prime factorization.
Like signs give a positive product.

$$= \frac{17}{2 \cdot 2 \cdot 2 \cdot 2}$$ Cancel common factors.

$$= \frac{17}{16}$$ Multiply numerators and denominators.

19. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-6}{23} \cdot \frac{9}{10} = -\frac{(2 \cdot 3) \cdot (3 \cdot 3)}{(23) \cdot (2 \cdot 5)}$$ Prime factorization.
Unlike signs give a negative product.

$$= -\frac{3 \cdot 3 \cdot 3}{23 \cdot 5}$$ Cancel common factors.

$$= -\frac{27}{115}$$ Multiply numerators and denominators.

21. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-23}{24} \cdot \frac{-6}{17} = \frac{(23) \cdot (2 \cdot 3)}{(2 \cdot 2 \cdot 2 \cdot 3) \cdot (17)} \qquad \text{Prime factorization.}$$

Like signs give a positive product.

$$= \frac{23}{2 \cdot 2 \cdot 17} \qquad \text{Cancel common factors.}$$

$$= \frac{23}{68} \qquad \text{Multiply numerators and denominators.}$$

23. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{24}{7} \cdot \frac{5}{2} = \frac{(2 \cdot 2 \cdot 2 \cdot 3) \cdot (5)}{(7) \cdot (2)} \qquad \text{Prime factorization.}$$

Like signs give a positive product.

$$= \frac{2 \cdot 2 \cdot 3 \cdot 5}{7} \qquad \text{Cancel common factors.}$$

$$= \frac{60}{7} \qquad \text{Multiply numerators and denominators.}$$

25. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{1}{2} \cdot \frac{-8}{11} = -\frac{(1) \cdot (2 \cdot 2 \cdot 2)}{(2) \cdot (11)} \qquad \text{Prime factorization.}$$

Unlike signs give a negative product.

$$= -\frac{2 \cdot 2}{11} \qquad \text{Cancel common factors.}$$

$$= -\frac{4}{11} \qquad \text{Multiply numerators and denominators.}$$

27. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-24}{13} \cdot \frac{-7}{18} = \frac{(2 \cdot 2 \cdot 2 \cdot 3) \cdot (7)}{(13) \cdot (2 \cdot 3 \cdot 3)} \qquad \text{Prime factorization.}$$

Like signs give a positive product.

$$= \frac{2 \cdot 2 \cdot 7}{13 \cdot 3} \qquad \text{Cancel common factors.}$$

$$= \frac{28}{39} \qquad \text{Multiply numerators and denominators.}$$

29. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-12y^3}{13} \cdot \frac{2}{9y^6} = -\frac{(2 \cdot 2 \cdot 3 \cdot y \cdot y \cdot y) \cdot (2)}{(13) \cdot (3 \cdot 3 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y)}$$ Prime factorization.
                                                                         Unlike signs give a negative product.

$$= -\frac{2 \cdot 2 \cdot 2}{13 \cdot 3 \cdot y \cdot y \cdot y}$$ Cancel common factors.

$$= -\frac{8}{39y^3}$$ Multiply numerators and denominators.

31. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{11y^3}{24} \cdot \frac{6}{5y^5} = \frac{(11 \cdot y \cdot y \cdot y) \cdot (2 \cdot 3)}{(2 \cdot 2 \cdot 2 \cdot 3) \cdot (5 \cdot y \cdot y \cdot y \cdot y \cdot y)}$$ Prime factorization.
                                                                     Like signs give a positive product.

$$= \frac{11}{2 \cdot 2 \cdot 5 \cdot y \cdot y}$$ Cancel common factors.

$$= \frac{11}{20y^2}$$ Multiply numerators and denominators.

33. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-8x^2}{21} \cdot \frac{-18}{19x} = \frac{(2 \cdot 2 \cdot 2 \cdot x \cdot x) \cdot (2 \cdot 3 \cdot 3)}{(3 \cdot 7) \cdot (19 \cdot x)}$$ Prime factorization.
                                                                Like signs give a positive product.

$$= \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot x}{7 \cdot 19}$$ Cancel common factors.

$$= \frac{48x}{133}$$ Multiply numerators and denominators.

35. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{13x^6}{15} \cdot \frac{9}{16x^2} = \frac{(13 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (3 \cdot 3)}{(3 \cdot 5) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x)}$$ Prime factorization.
                                                                          Like signs give a positive product.

$$= \frac{13 \cdot 3 \cdot x \cdot x \cdot x \cdot x}{5 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$ Cancel common factors.

$$= \frac{39x^4}{80}$$ Multiply numerators and denominators.

37. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-6y^3}{5} \cdot \frac{-20}{7y^6} = \frac{(2 \cdot 3 \cdot y \cdot y \cdot y) \cdot (2 \cdot 2 \cdot 5)}{(5) \cdot (7 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y)}$$ Prime factorization.
Like signs give a positive product.

$$= \frac{2 \cdot 3 \cdot 2 \cdot 2}{7 \cdot y \cdot y \cdot y}$$ Cancel common factors.

$$= \frac{24}{7y^3}$$ Multiply numerators and denominators.

39. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-3y^3}{4} \cdot \frac{23}{12y} = -\frac{(3 \cdot y \cdot y \cdot y) \cdot (23)}{(2 \cdot 2) \cdot (2 \cdot 2 \cdot 3 \cdot y)}$$ Prime factorization.
Unlike signs give a negative product.

$$= -\frac{23 \cdot y \cdot y}{2 \cdot 2 \cdot 2 \cdot 2}$$ Cancel common factors.

$$= -\frac{23y^2}{16}$$ Multiply numerators and denominators.

41. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{13y^6}{20x^4} \cdot \frac{2x}{7y^2} = \frac{(13 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y) \cdot (2 \cdot x)}{(2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x \cdot x) \cdot (7 \cdot y \cdot y)}$$ Prime factorization.
Like signs give a positive product.

$$= \frac{13 \cdot y \cdot y \cdot y \cdot y}{2 \cdot 5 \cdot 7 \cdot x \cdot x \cdot x}$$ Cancel common factors.

$$= \frac{13y^4}{70x^3}$$ Multiply numerators and denominators.

43. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{23y^4}{21x} \cdot \frac{-7x^6}{4y^2} = -\frac{(23 \cdot y \cdot y \cdot y \cdot y) \cdot (7 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x)}{(3 \cdot 7 \cdot x) \cdot (2 \cdot 2 \cdot y \cdot y)}$$ Prime factorization.
Unlike signs give a negative product.

$$= -\frac{23 \cdot y \cdot y \cdot x \cdot x \cdot x \cdot x \cdot x}{3 \cdot 2 \cdot 2}$$ Cancel common factors.

$$= -\frac{23y^2x^5}{12}$$ Multiply numerators and denominators.

45. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{11y^6}{12x^6} \cdot \frac{-2x^4}{7y^2} = -\frac{(11 \cdot y \cdot y \cdot y \cdot y \cdot y) \cdot (2 \cdot x \cdot x \cdot x \cdot x)}{(2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (7 \cdot y \cdot y)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{11 \cdot y \cdot y \cdot y \cdot y}{2 \cdot 3 \cdot 7 \cdot x \cdot x}$$

Cancel common factors.

$$= -\frac{11y^4}{42x^2}$$

Multiply numerators and denominators.

47. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{x^6}{21y^3} \cdot \frac{-7y^4}{9x^5} = -\frac{(x \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (7 \cdot y \cdot y \cdot y \cdot y)}{(3 \cdot 7 \cdot y \cdot y \cdot y) \cdot (3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{x \cdot y}{3 \cdot 3 \cdot 3}$$

Cancel common factors.

$$= -\frac{xy}{27}$$

Multiply numerators and denominators.

49. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{19y^2}{18x} \cdot \frac{10x^3}{7y^3} = \frac{(19 \cdot y \cdot y) \cdot (2 \cdot 5 \cdot x \cdot x \cdot x)}{(2 \cdot 3 \cdot 3 \cdot x) \cdot (7 \cdot y \cdot y \cdot y)}$$

Prime factorization.
Like signs give a positive product.

$$= \frac{19 \cdot 5 \cdot x \cdot x}{3 \cdot 3 \cdot 7 \cdot y}$$

Cancel common factors.

$$= \frac{95x^2}{63y}$$

Multiply numerators and denominators.

51. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-4y^3}{5x^5} \cdot \frac{-10x}{21y^4} = \frac{(2 \cdot 2 \cdot y \cdot y \cdot y) \cdot (2 \cdot 5 \cdot x)}{(5 \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (3 \cdot 7 \cdot y \cdot y \cdot y \cdot y)}$$

Prime factorization.
Like signs give a positive product.

$$= \frac{2 \cdot 2 \cdot 2}{3 \cdot 7 \cdot y \cdot x \cdot x \cdot x \cdot x}$$

Cancel common factors.

$$= \frac{8}{21yx^4}$$

Multiply numerators and denominators.

53. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{-16x}{21y^2} \cdot \frac{-7y^3}{5x^2} = \frac{(2 \cdot 2 \cdot 2 \cdot 2 \cdot x) \cdot (7 \cdot y \cdot y \cdot y)}{(3 \cdot 7 \cdot y \cdot y) \cdot (5 \cdot x \cdot x)} \qquad \text{\color{red}Prime factorization.}$$
$$\text{\color{red}Like signs give a positive product.}$$
$$= \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot y}{3 \cdot 5 \cdot x} \qquad \text{\color{red}Cancel common factors.}$$
$$= \frac{16y}{15x} \qquad \text{\color{red}Multiply numerators and denominators.}$$

55. Factor the numerators and denominators completely, and then cancel common factors:

$$\frac{17x^3}{3y^6} \cdot \frac{-12y^2}{7x^4} = -\frac{(17 \cdot x \cdot x \cdot x) \cdot (2 \cdot 2 \cdot 3 \cdot y \cdot y)}{(3 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y) \cdot (7 \cdot x \cdot x \cdot x \cdot x)} \qquad \text{\color{red}Prime factorization.}$$
$$\text{\color{red}Unlike signs give a negative product.}$$
$$= -\frac{17 \cdot 2 \cdot 2}{7 \cdot x \cdot y \cdot y \cdot y \cdot y} \qquad \text{\color{red}Cancel common factors.}$$
$$= -\frac{68}{7xy^4} \qquad \text{\color{red}Multiply numerators and denominators.}$$

57. To find the area of the parallelogram, find the product of the base and height.

$$A = bh \qquad \text{\color{red}Area formula for parallelogram.}$$
$$= (8\,\text{cm})(7\,\text{cm}) \qquad \text{\color{red}Substitute: } 8\,\text{cm for } b, \, 7\,\text{cm for } h.$$
$$= 56\,\text{cm}^2 \qquad \text{\color{red}Multiply.}$$

Therefore, the area of the parallelogram is 56 square centimeters.

59. To find the area of the parallelogram, find the product of the base and height.

$$A = bh \qquad \text{\color{red}Area formula for parallelogram.}$$
$$= (6\,\text{cm})(13\,\text{cm}) \qquad \text{\color{red}Substitute: } 6\,\text{cm for } b, \, 13\,\text{cm for } h.$$
$$= 78\,\text{cm}^2 \qquad \text{\color{red}Multiply.}$$

Therefore, the area of the parallelogram is 78 square centimeters.

61. To find the area of the parallelogram, find the product of the base and height.

$$A = bh \qquad \text{\color{red}Area formula for parallelogram.}$$
$$= (18\,\text{cm})(14\,\text{cm}) \qquad \text{\color{red}Substitute: } 18\,\text{cm for } b, \, 14\,\text{cm for } h.$$
$$= 252\,\text{cm}^2 \qquad \text{\color{red}Multiply.}$$

Therefore, the area of the parallelogram is 252 square centimeters.

63. To find the area of the triangle, take one-half the product of the base and height.

$$A = \frac{1}{2}bh \qquad\qquad \text{Area formula for triangle.}$$

$$= \frac{1}{2}(14\,\text{ft})(9\,\text{ft}) \qquad \text{Substitute: } 14\,\text{ft for } b,\ 9\,\text{ft for } h.$$

$$= \frac{126\,\text{ft}^2}{2} \qquad\qquad \text{Multiply numerators; multiply denominators.}$$

$$= 63\,\text{ft}^2. \qquad\qquad \text{Divide.}$$

Therefore, the area of the triangle is 63 square feet.

65. To find the area of the triangle, take one-half the product of the base and height.

$$A = \frac{1}{2}bh \qquad\qquad \text{Area formula for triangle.}$$

$$= \frac{1}{2}(12\,\text{in})(5\,\text{in}) \qquad \text{Substitute: } 12\,\text{in for } b,\ 5\,\text{in for } h.$$

$$= \frac{60\,\text{in}^2}{2} \qquad\qquad \text{Multiply numerators; multiply denominators.}$$

$$= 30\,\text{in}^2. \qquad\qquad \text{Divide.}$$

Therefore, the area of the triangle is 30 square inches.

67. To find the area of the triangle, take one-half the product of the base and height.

$$A = \frac{1}{2}bh \qquad\qquad \text{Area formula for triangle.}$$

$$= \frac{1}{2}(5\,\text{cm})(4\,\text{cm}) \qquad \text{Substitute: } 5\,\text{cm for } b,\ 4\,\text{cm for } h.$$

$$= \frac{20\,\text{cm}^2}{2} \qquad\qquad \text{Multiply numerators; multiply denominators.}$$

$$= 10\,\text{cm}^2. \qquad\qquad \text{Divide.}$$

Therefore, the area of the triangle is 10 square centimeters.

69. To find your weight on the moon, take 1/6 of your weight on Earth. In this case, "of" means multiply, so multiply your weight on Earth by 1/6.

$$
\begin{aligned}
\frac{1}{6} \cdot 138 &= \frac{1}{6} \cdot \frac{138}{1} && \text{\textcolor{red}{Understood: } } 138 = 138/1. \\
&= \frac{138}{6} && \text{\textcolor{red}{Multiply numerators; multiply denominators.}} \\
&= \frac{2 \cdot 3 \cdot 23}{2 \cdot 3} && \text{\textcolor{red}{Factor numerator and denominator.}} \\
&= \frac{\cancel{2} \cdot \cancel{3} \cdot 23}{\cancel{2} \cdot \cancel{3}} && \text{\textcolor{red}{Cancel common factors.}} \\
&= 23 && \text{\textcolor{red}{Remaining factor.}}
\end{aligned}
$$

Therefore, you would weigh only 23 pounds on the moon.

4.3 Dividing Fractions

1. To find the reciprocal of $-16/5$, we invert (turn upside down) $-16/5$. Thus, the reciprocal of $-16/5$ is $5/(-16) = -5/16$. Since the product of two negatives is positive, the computation

$$
\begin{aligned}
-\frac{16}{5} \cdot \left(-\frac{5}{16} \right) &= \frac{16 \cdot 5}{5 \cdot 16} \\
&= \frac{\cancel{16} \cdot \cancel{5}}{\cancel{5} \cdot \cancel{16}} \\
&= 1,
\end{aligned}
$$

confirms that $-5/16$ is the reciprocal of $-16/5$.

3. To find the reciprocal of -17, write $-17 = -17/1$ and invert (turn upside down). Thus, the reciprocal of -17 is $1/(-17) = -1/17$. Since the product of two negatives is positive, the computation

$$
\begin{aligned}
-17 \cdot \left(-\frac{1}{17} \right) &= -\frac{17}{1} \cdot \left(-\frac{1}{17} \right) \\
&= \frac{17}{17} \\
&= 1,
\end{aligned}
$$

confirms that $-1/17$ is the reciprocal of -17.

5. To find the reciprocal of 15/16, we invert (turn upside down) 15/16. Thus, the reciprocal of 15/16 is 16/15. Note that

$$\frac{15}{16} \cdot \frac{16}{15} = \frac{15 \cdot 16}{16 \cdot 15}$$
$$= \frac{\cancel{15} \cdot \cancel{16}}{\cancel{16} \cdot \cancel{15}}$$
$$= 1,$$

confirming that 16/15 is the reciprocal of 15/16.

7. To find the reciprocal of 30, write $30 = 30/1$ and invert (turn upside down). Thus, the reciprocal of 30 is 1/30. Note that

$$30 \cdot \frac{1}{30} = \frac{30}{1} \cdot \frac{1}{30}$$
$$= \frac{30}{30}$$
$$= 1,$$

confirming that 1/30 is the reciprocal of 30.

9. To find the reciprocal of -46, write $-46 = -46/1$ and invert (turn upside down). Thus, the reciprocal of -46 is $1/(-46) = -1/46$. Since the product of two negatives is positive, the computation

$$-46 \cdot \left(-\frac{1}{46}\right) = -\frac{46}{1} \cdot \left(-\frac{1}{46}\right)$$
$$= \frac{46}{46}$$
$$= 1,$$

confirms that $-1/46$ is the reciprocal of -46.

11. To find the reciprocal of $-9/19$, we invert (turn upside down) $-9/19$. Thus, the reciprocal of $-9/19$ is $19/(-9) = -19/9$. Since the product of two negatives is positive, the computation

$$-\frac{9}{19} \cdot \left(-\frac{19}{9}\right) = \frac{9 \cdot 19}{19 \cdot 9}$$
$$= \frac{\cancel{9} \cdot \cancel{19}}{\cancel{19} \cdot \cancel{9}}$$
$$= 1,$$

confirms that $-19/9$ is the reciprocal of $-9/19$.

13. To find the reciprocal of 3/17, we invert (turn upside down) 3/17. Thus, the reciprocal of 3/17 is 17/3. Note that

$$
\begin{aligned}
\frac{3}{17} \cdot \frac{17}{3} &= \frac{3 \cdot 17}{17 \cdot 3} \\
&= \frac{\cancel{3} \cdot \cancel{17}}{\cancel{17} \cdot \cancel{3}} \\
&= 1,
\end{aligned}
$$

confirming that 17/3 is the reciprocal of 3/17.

15. To find the reciprocal of 11, write $11 = 11/1$ and invert (turn upside down). Thus, the reciprocal of 11 is 1/11. Note that

$$
\begin{aligned}
11 \cdot \frac{1}{11} &= \frac{11}{1} \cdot \frac{1}{11} \\
&= \frac{11}{11} \\
&= 1,
\end{aligned}
$$

confirming that 1/11 is the reciprocal of 11.

17. Because the given identity

$$
\frac{2}{9} \cdot \frac{9}{2} = 1
$$

has the form

$$
\frac{a}{b} \cdot \frac{b}{a} = 1,
$$

this is an example of the multiplicative inverse property.

19. Because the given identity

$$
\frac{-19}{12} \cdot 1 = \frac{-19}{12}
$$

has the form

$$
\frac{a}{b} \cdot 1 = \frac{a}{b},
$$

this is an example of the multiplicative identity property.

21. The given identity

$$-6 \cdot \left(-\frac{1}{6}\right) = 1$$

is equivalent to

$$\frac{-6}{1} \cdot \left(\frac{1}{-6}\right) = 1,$$

which has the form

$$\frac{a}{b} \cdot \frac{b}{a} = 1.$$

Therefore, this is an example of the multiplicative inverse property.

23. Because the given identity

$$\frac{-16}{11} \cdot 1 = \frac{-16}{11}$$

has the form

$$\frac{a}{b} \cdot 1 = \frac{a}{b},$$

this is an example of the multiplicative identity property.

25. Because the given identity

$$-\frac{4}{1} \cdot \left(-\frac{1}{4}\right) = 1$$

has the form

$$\frac{a}{b} \cdot \frac{b}{a} = 1,$$

this is an example of the multiplicative inverse property.

27. Because the given identity

$$\frac{8}{1} \cdot 1 = \frac{8}{1}$$

has the form

$$\frac{a}{b} \cdot 1 = \frac{a}{b},$$

this is an example of the multiplicative identity property.

29. The given identity

$$14 \cdot \frac{1}{14} = 1$$

is equivalent to

$$\frac{14}{1} \cdot \frac{1}{14} = 1,$$

which has the form

$$\frac{a}{b} \cdot \frac{b}{a} = 1.$$

Therefore, this is an example of the multiplicative inverse property.

31. Because the given identity

$$\frac{13}{8} \cdot 1 = \frac{13}{8}$$

has the form

$$\frac{a}{b} \cdot 1 = \frac{a}{b},$$

this is an example of the multiplicative identity property.

33. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{8}{23} \div \frac{-6}{11} = \frac{8}{23} \cdot \frac{11}{-6} \qquad \text{Invert the second fraction.}$$

$$= -\frac{(2 \cdot 2 \cdot 2) \cdot (11)}{(23) \cdot (2 \cdot 3)} \qquad \begin{array}{l} \text{Prime factorization.} \\ \text{Unlike signs give a negative product.} \end{array}$$

$$= -\frac{2 \cdot 2 \cdot 11}{23 \cdot 3} \qquad \text{Cancel common factors.}$$

$$= -\frac{44}{69} \qquad \text{Multiply numerators and denominators.}$$

35. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{18}{19} \div \frac{-16}{23} = \frac{18}{19} \cdot \frac{23}{-16} \qquad \text{Invert the second fraction.}$$

$$= -\frac{(2 \cdot 3 \cdot 3) \cdot (23)}{(19) \cdot (2 \cdot 2 \cdot 2 \cdot 2)} \qquad \begin{array}{l} \text{Prime factorization.} \\ \text{Unlike signs give a negative product.} \end{array}$$

$$= -\frac{3 \cdot 3 \cdot 23}{19 \cdot 2 \cdot 2 \cdot 2} \qquad \text{Cancel common factors.}$$

$$= -\frac{207}{152} \qquad \text{Multiply numerators and denominators.}$$

37. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{4}{21} \div \frac{-6}{5} = \frac{4}{21} \cdot \frac{5}{-6} \qquad \text{Invert the second fraction.}$$

$$= -\frac{(2 \cdot 2) \cdot (5)}{(3 \cdot 7) \cdot (2 \cdot 3)} \qquad \text{Prime factorization.}$$
$$\text{Unlike signs give a negative product.}$$

$$= -\frac{2 \cdot 5}{3 \cdot 7 \cdot 3} \qquad \text{Cancel common factors.}$$

$$= -\frac{10}{63} \qquad \text{Multiply numerators and denominators.}$$

39. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{-1}{9} \div \frac{8}{3} = \frac{-1}{9} \cdot \frac{3}{8} \qquad \text{Invert the second fraction.}$$

$$= -\frac{(1) \cdot (3)}{(3 \cdot 3) \cdot (2 \cdot 2 \cdot 2)} \qquad \text{Prime factorization.}$$
$$\text{Unlike signs give a negative product.}$$

$$= -\frac{1}{3 \cdot 2 \cdot 2 \cdot 2} \qquad \text{Cancel common factors.}$$

$$= -\frac{1}{24} \qquad \text{Multiply numerators and denominators.}$$

41. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{-21}{11} \div \frac{3}{10} = \frac{-21}{11} \cdot \frac{10}{3} \qquad \text{Invert the second fraction.}$$

$$= -\frac{(3 \cdot 7) \cdot (2 \cdot 5)}{(11) \cdot (3)} \qquad \text{Prime factorization.}$$
$$\text{Unlike signs give a negative product.}$$

$$= -\frac{7 \cdot 2 \cdot 5}{11} \qquad \text{Cancel common factors.}$$

$$= -\frac{70}{11} \qquad \text{Multiply numerators and denominators.}$$

43. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{-12}{7} \div \frac{2}{3} = \frac{-12}{7} \cdot \frac{3}{2}$$ 　Invert the second fraction.

$$= -\frac{(2 \cdot 2 \cdot 3) \cdot (3)}{(7) \cdot (2)}$$ 　Prime factorization.
　Unlike signs give a negative product.

$$= -\frac{2 \cdot 3 \cdot 3}{7}$$ 　Cancel common factors.

$$= -\frac{18}{7}$$ 　Multiply numerators and denominators.

45. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{2}{19} \div \frac{24}{23} = \frac{2}{19} \cdot \frac{23}{24}$$ 　Invert the second fraction.

$$= \frac{(2) \cdot (23)}{(19) \cdot (2 \cdot 2 \cdot 2 \cdot 3)}$$ 　Prime factorization.
　Like signs give a positive product.

$$= \frac{23}{19 \cdot 2 \cdot 2 \cdot 3}$$ 　Cancel common factors.

$$= \frac{23}{228}$$ 　Multiply numerators and denominators.

47. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{-9}{5} \div \frac{-24}{19} = \frac{-9}{5} \cdot \frac{19}{-24}$$ 　Invert the second fraction.

$$= \frac{(3 \cdot 3) \cdot (19)}{(5) \cdot (2 \cdot 2 \cdot 2 \cdot 3)}$$ 　Prime factorization.
　Like signs give a positive product.

$$= \frac{3 \cdot 19}{5 \cdot 2 \cdot 2 \cdot 2}$$ 　Cancel common factors.

$$= \frac{57}{40}$$ 　Multiply numerators and denominators.

49. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{18}{11} \div \frac{14}{9} = \frac{18}{11} \cdot \frac{9}{14} \qquad \text{Invert the second fraction.}$$

$$= \frac{(2 \cdot 3 \cdot 3) \cdot (3 \cdot 3)}{(11) \cdot (2 \cdot 7)} \qquad \begin{array}{l}\text{Prime factorization.} \\ \text{Like signs give a positive product.}\end{array}$$

$$= \frac{3 \cdot 3 \cdot 3 \cdot 3}{11 \cdot 7} \qquad \text{Cancel common factors.}$$

$$= \frac{81}{77} \qquad \text{Multiply numerators and denominators.}$$

51. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{13}{18} \div \frac{4}{9} = \frac{13}{18} \cdot \frac{9}{4} \qquad \text{Invert the second fraction.}$$

$$= \frac{(13) \cdot (3 \cdot 3)}{(2 \cdot 3 \cdot 3) \cdot (2 \cdot 2)} \qquad \begin{array}{l}\text{Prime factorization.} \\ \text{Like signs give a positive product.}\end{array}$$

$$= \frac{13}{2 \cdot 2 \cdot 2} \qquad \text{Cancel common factors.}$$

$$= \frac{13}{8} \qquad \text{Multiply numerators and denominators.}$$

53. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{11}{2} \div \frac{-21}{10} = \frac{11}{2} \cdot \frac{10}{-21} \qquad \text{Invert the second fraction.}$$

$$= -\frac{(11) \cdot (2 \cdot 5)}{(2) \cdot (3 \cdot 7)} \qquad \begin{array}{l}\text{Prime factorization.} \\ \text{Unlike signs give a negative product.}\end{array}$$

$$= -\frac{11 \cdot 5}{3 \cdot 7} \qquad \text{Cancel common factors.}$$

$$= -\frac{55}{21} \qquad \text{Multiply numerators and denominators.}$$

55. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{3}{10} \div \frac{12}{5} = \frac{3}{10} \cdot \frac{5}{12} \qquad \text{Invert the second fraction.}$$

$$= \frac{(3) \cdot (5)}{(2 \cdot 5) \cdot (2 \cdot 2 \cdot 3)} \qquad \begin{array}{l} \text{Prime factorization.} \\ \text{Like signs give a positive product.} \end{array}$$

$$= \frac{1}{2 \cdot 2 \cdot 2} \qquad \text{Cancel common factors.}$$

$$= \frac{1}{8} \qquad \text{Multiply numerators and denominators.}$$

57. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{20}{17} \div 5 = \frac{20}{17} \div \frac{5}{1} \qquad 5 = 5/1.$$

$$= \frac{20}{17} \cdot \frac{1}{5} \qquad \text{Invert the second fraction.}$$

$$= \frac{(2 \cdot 2 \cdot 5) \cdot (1)}{(17) \cdot (5)} \qquad \begin{array}{l} \text{Prime factorization.} \\ \text{Like signs give a positive product.} \end{array}$$

$$= \frac{2 \cdot 2}{17} \qquad \text{Cancel common factors.}$$

$$= \frac{4}{17} \qquad \text{Multiply numerators and denominators.}$$

59. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$-7 \div \frac{21}{20} = \frac{-7}{1} \div \frac{21}{20} \qquad -7 = -7/1.$$

$$= \frac{-7}{1} \cdot \frac{20}{21} \qquad \text{Invert the second fraction.}$$

$$= -\frac{(7) \cdot (2 \cdot 2 \cdot 5)}{(1) \cdot (3 \cdot 7)} \qquad \begin{array}{l} \text{Prime factorization.} \\ \text{Unlike signs give a negative product.} \end{array}$$

$$= -\frac{2 \cdot 2 \cdot 5}{3} \qquad \text{Cancel common factors.}$$

$$= -\frac{20}{3} \qquad \text{Multiply numerators and denominators.}$$

61. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{8}{21} \div 2 = \frac{8}{21} \div \frac{2}{1} \qquad \textcolor{red}{2 = 2/1.}$$

$$= \frac{8}{21} \cdot \frac{1}{2} \qquad \textcolor{red}{\text{Invert the second fraction.}}$$

$$= \frac{(2 \cdot 2 \cdot 2) \cdot (1)}{(3 \cdot 7) \cdot (2)} \qquad \textcolor{red}{\text{Prime factorization.}}$$
$$\textcolor{red}{\text{Like signs give a positive product.}}$$

$$= \frac{2 \cdot 2}{3 \cdot 7} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

$$= \frac{4}{21} \qquad \textcolor{red}{\text{Multiply numerators and denominators.}}$$

63. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$8 \div \frac{-10}{17} = \frac{8}{1} \div \frac{-10}{17} \qquad \textcolor{red}{8 = 8/1.}$$

$$= \frac{8}{1} \cdot \frac{17}{-10} \qquad \textcolor{red}{\text{Invert the second fraction.}}$$

$$= -\frac{(2 \cdot 2 \cdot 2) \cdot (17)}{(1) \cdot (2 \cdot 5)} \qquad \textcolor{red}{\text{Prime factorization.}}$$
$$\textcolor{red}{\text{Unlike signs give a negative product.}}$$

$$= -\frac{2 \cdot 2 \cdot 17}{5} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

$$= -\frac{68}{5} \qquad \textcolor{red}{\text{Multiply numerators and denominators.}}$$

65. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$-8 \div \frac{18}{5} = \frac{-8}{1} \div \frac{18}{5} \qquad \textcolor{red}{-8 = -8/1.}$$

$$= \frac{-8}{1} \cdot \frac{5}{18} \qquad \textcolor{red}{\text{Invert the second fraction.}}$$

$$= -\frac{(2 \cdot 2 \cdot 2) \cdot (5)}{(1) \cdot (2 \cdot 3 \cdot 3)} \qquad \textcolor{red}{\text{Prime factorization.}}$$
$$\textcolor{red}{\text{Unlike signs give a negative product.}}$$

$$= -\frac{2 \cdot 2 \cdot 5}{3 \cdot 3} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

$$= -\frac{20}{9} \qquad \textcolor{red}{\text{Multiply numerators and denominators.}}$$

67. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$
\begin{aligned}
\frac{3}{4} \div (-9) &= \frac{3}{4} \div \frac{-9}{1} && \color{red}{-9 = -9/1.} \\
&= \frac{3}{4} \cdot \frac{1}{-9} && \color{red}{\text{Invert the second fraction.}} \\
&= -\frac{(3) \cdot (1)}{(2 \cdot 2) \cdot (3 \cdot 3)} && \color{red}{\text{Prime factorization.}} \\
& && \color{red}{\text{Unlike signs give a negative product.}} \\
&= -\frac{1}{2 \cdot 2 \cdot 3} && \color{red}{\text{Cancel common factors.}} \\
&= -\frac{1}{12} && \color{red}{\text{Multiply numerators and denominators.}}
\end{aligned}
$$

69. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$
\begin{aligned}
\frac{11x^2}{12} \div \frac{8x^4}{3} &= \frac{11x^2}{12} \cdot \frac{3}{8x^4} && \color{red}{\text{Invert the second fraction.}} \\
&= \frac{(11 \cdot x \cdot x) \cdot (3)}{(2 \cdot 2 \cdot 3) \cdot (2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x \cdot x)} && \color{red}{\text{Prime factorization.}} \\
& && \color{red}{\text{Like signs give a positive product.}} \\
&= \frac{11}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x} && \color{red}{\text{Cancel common factors.}} \\
&= \frac{11}{32x^2} && \color{red}{\text{Multiply numerators and denominators.}}
\end{aligned}
$$

71. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$
\begin{aligned}
\frac{17y}{9} \div \frac{10y^6}{3} &= \frac{17y}{9} \cdot \frac{3}{10y^6} && \color{red}{\text{Invert the second fraction.}} \\
&= \frac{(17 \cdot y) \cdot (3)}{(3 \cdot 3) \cdot (2 \cdot 5 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y)} && \color{red}{\text{Prime factorization.}} \\
& && \color{red}{\text{Like signs give a positive product.}} \\
&= \frac{17}{3 \cdot 2 \cdot 5 \cdot y \cdot y \cdot y \cdot y \cdot y} && \color{red}{\text{Cancel common factors.}} \\
&= \frac{17}{30y^5} && \color{red}{\text{Multiply numerators and denominators.}}
\end{aligned}
$$

73. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{-22x^4}{13} \div \frac{12x}{11} = \frac{-22x^4}{13} \cdot \frac{11}{12x}$$ Invert the second fraction.

$$= -\frac{(2 \cdot 11 \cdot x \cdot x \cdot x \cdot x) \cdot (11)}{(13) \cdot (2 \cdot 2 \cdot 3 \cdot x)}$$ Prime factorization.
Unlike signs give a negative product.

$$= -\frac{11 \cdot 11 \cdot x \cdot x \cdot x}{13 \cdot 2 \cdot 3}$$ Cancel common factors.

$$= -\frac{121x^3}{78}$$ Multiply numerators and denominators.

75. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{-3x^4}{10} \div \frac{-4x}{5} = \frac{-3x^4}{10} \cdot \frac{5}{-4x}$$ Invert the second fraction.

$$= \frac{(3 \cdot x \cdot x \cdot x \cdot x) \cdot (5)}{(2 \cdot 5) \cdot (2 \cdot 2 \cdot x)}$$ Prime factorization.
Like signs give a positive product.

$$= \frac{3 \cdot x \cdot x \cdot x}{2 \cdot 2 \cdot 2}$$ Cancel common factors.

$$= \frac{3x^3}{8}$$ Multiply numerators and denominators.

77. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{-15y^2}{14} \div \frac{-10y^5}{13} = \frac{-15y^2}{14} \cdot \frac{13}{-10y^5}$$ Invert the second fraction.

$$= \frac{(3 \cdot 5 \cdot y \cdot y) \cdot (13)}{(2 \cdot 7) \cdot (2 \cdot 5 \cdot y \cdot y \cdot y \cdot y \cdot y)}$$ Prime factorization.
Like signs give a positive product.

$$= \frac{3 \cdot 13}{2 \cdot 7 \cdot 2 \cdot y \cdot y \cdot y}$$ Cancel common factors.

$$= \frac{39}{28y^3}$$ Multiply numerators and denominators.

79. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{-15x^5}{13} \div \frac{20x^2}{19} = \frac{-15x^5}{13} \cdot \frac{19}{20x^2}$$ Invert the second fraction.

$$= -\frac{(3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (19)}{(13) \cdot (2 \cdot 2 \cdot 5 \cdot x \cdot x)}$$ Prime factorization.
Unlike signs give a negative product.

$$= -\frac{3 \cdot 19 \cdot x \cdot x \cdot x}{13 \cdot 2 \cdot 2}$$ Cancel common factors.

$$= -\frac{57x^3}{52}$$ Multiply numerators and denominators.

81. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{11y^4}{14x^2} \div \frac{-9y^2}{7x^3} = \frac{11y^4}{14x^2} \cdot \frac{7x^3}{-9y^2}$$ Invert the second fraction.

$$= -\frac{(11 \cdot y \cdot y \cdot y \cdot y) \cdot (7 \cdot x \cdot x \cdot x)}{(2 \cdot 7 \cdot x \cdot x) \cdot (3 \cdot 3 \cdot y \cdot y)}$$ Prime factorization.
Unlike signs give a negative product.

$$= -\frac{11 \cdot y \cdot y \cdot x}{2 \cdot 3 \cdot 3}$$ Cancel common factors.

$$= -\frac{11y^2x}{18}$$ Multiply numerators and denominators.

83. First rewrite as a multiplication problem. Then factor the numererators and denominators completely and cancel common factors:

$$\frac{10x^4}{3y^4} \div \frac{7x^5}{24y^2} = \frac{10x^4}{3y^4} \cdot \frac{24y^2}{7x^5}$$ Invert the second fraction.

$$= \frac{(2 \cdot 5 \cdot x \cdot x \cdot x \cdot x) \cdot (2 \cdot 2 \cdot 2 \cdot 3 \cdot y \cdot y)}{(3 \cdot y \cdot y \cdot y \cdot y) \cdot (7 \cdot x \cdot x \cdot x \cdot x \cdot x)}$$ Prime factorization.
Like signs give a positive product.

$$= \frac{2 \cdot 5 \cdot 2 \cdot 2 \cdot 2}{7 \cdot x \cdot y \cdot y}$$ Cancel common factors.

$$= \frac{80}{7xy^2}$$ Multiply numerators and denominators.

85. First rewrite as a multiplication problem. Then factor the numererators
and denominators completely and cancel common factors:

$$\frac{22y^4}{21x^5} \div \frac{-5y^2}{6x^4} = \frac{22y^4}{21x^5} \cdot \frac{6x^4}{-5y^2}$$

Invert the second fraction.

$$= -\frac{(2 \cdot 11 \cdot y \cdot y \cdot y \cdot y) \cdot (2 \cdot 3 \cdot x \cdot x \cdot x \cdot x)}{(3 \cdot 7 \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (5 \cdot y \cdot y)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{2 \cdot 11 \cdot 2 \cdot y \cdot y}{7 \cdot 5 \cdot x}$$

Cancel common factors.

$$= -\frac{44y^2}{35x}$$

Multiply numerators and denominators.

87. First rewrite as a multiplication problem. Then factor the numererators
and denominators completely and cancel common factors:

$$\frac{-22x^4}{21y^3} \div \frac{-17x^3}{3y^4} = \frac{-22x^4}{21y^3} \cdot \frac{3y^4}{-17x^3}$$

Invert the second fraction.

$$= \frac{(2 \cdot 11 \cdot x \cdot x \cdot x \cdot x) \cdot (3 \cdot y \cdot y \cdot y \cdot y)}{(3 \cdot 7 \cdot y \cdot y \cdot y) \cdot (17 \cdot x \cdot x \cdot x)}$$

Prime factorization.
Like signs give a positive product.

$$= \frac{2 \cdot 11 \cdot x \cdot y}{7 \cdot 17}$$

Cancel common factors.

$$= \frac{22xy}{119}$$

Multiply numerators and denominators.

89. First rewrite as a multiplication problem. Then factor the numererators
and denominators completely and cancel common factors:

$$\frac{-16y^2}{3x^3} \div \frac{2y^6}{11x^5} = \frac{-16y^2}{3x^3} \cdot \frac{11x^5}{2y^6}$$

Invert the second fraction.

$$= -\frac{(2 \cdot 2 \cdot 2 \cdot 2 \cdot y \cdot y) \cdot (11 \cdot x \cdot x \cdot x \cdot x \cdot x)}{(3 \cdot x \cdot x \cdot x) \cdot (2 \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{2 \cdot 2 \cdot 2 \cdot 11 \cdot x \cdot x}{3 \cdot y \cdot y \cdot y \cdot y}$$

Cancel common factors.

$$= -\frac{88x^2}{3y^4}$$

Multiply numerators and denominators.

91. First rewrite as a multiplication problem. Then factor the numererators
and denominators completely and cancel common factors:

$$\frac{-x}{12y^4} \div \frac{-23x^3}{16y^3} = \frac{-x}{12y^4} \cdot \frac{16y^3}{-23x^3}$$

Invert the second fraction.

$$= \frac{(x) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot y \cdot y \cdot y)}{(2 \cdot 2 \cdot 3 \cdot y \cdot y \cdot y \cdot y) \cdot (23 \cdot x \cdot x \cdot x)}$$

Prime factorization.
Like signs give a positive product.

$$= \frac{2 \cdot 2}{3 \cdot 23 \cdot x \cdot x \cdot y}$$

Cancel common factors.

$$= \frac{4}{69x^2y}$$

Multiply numerators and denominators.

93. First rewrite as a multiplication problem. Then factor the numererators
and denominators completely and cancel common factors:

$$\frac{y^2}{4x} \div \frac{-9y^5}{8x^3} = \frac{y^2}{4x} \cdot \frac{8x^3}{-9y^5}$$

Invert the second fraction.

$$= -\frac{(y \cdot y) \cdot (2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x)}{(2 \cdot 2 \cdot x) \cdot (3 \cdot 3 \cdot y \cdot y \cdot y \cdot y \cdot y)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{2 \cdot x \cdot x}{3 \cdot 3 \cdot y \cdot y \cdot y}$$

Cancel common factors.

$$= -\frac{2x^2}{9y^3}$$

Multiply numerators and denominators.

95. First rewrite as a multiplication problem. Then factor the numererators
and denominators completely and cancel common factors:

$$\frac{-18x^6}{13y^4} \div \frac{3x}{y^2} = \frac{-18x^6}{13y^4} \cdot \frac{y^2}{3x}$$

Invert the second fraction.

$$= -\frac{(2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) \cdot (y \cdot y)}{(13 \cdot y \cdot y \cdot y \cdot y) \cdot (3 \cdot x)}$$

Prime factorization.
Unlike signs give a negative product.

$$= -\frac{2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x}{13 \cdot y \cdot y}$$

Cancel common factors.

$$= -\frac{6x^5}{13y^2}$$

Multiply numerators and denominators.

4.4   Adding and Subtracting Fractions

1. List the multiples of 9:

$$9, 18, 27, 36, 45, 54, 63, 72, 81, 90,$$
$$99, 108, 117, 126, 135, 144, 153, 162, 171, 180, \ldots$$

List the multiples of 15:

$$15, 30, 45, 60, 75, 90, 105, 120, 135, 150,$$
$$165, 180, 195, 210, 225, 240, 255, 270, 285, 300, \ldots$$

List the common multiples of 9 and 15:

$$45, 90, 135, 180, \ldots$$

Therefore, the least common multiple is

$$\text{LCM} = 45.$$

3. List the multiples of 20:

$$20, 40, 60, 80, 100, 120, 140, 160, 180, 200,$$
$$220, 240, 260, 280, 300, 320, 340, 360, 380, 400, \ldots$$

List the multiples of 8:

$$8, 16, 24, 32, 40, 48, 56, 64, 72, 80,$$
$$88, 96, 104, 112, 120, 128, 136, 144, 152, 160, \ldots$$

List the common multiples of 20 and 8:

$$40, 80, 120, 160, \ldots$$

Therefore, the least common multiple is

$$\text{LCM} = 40.$$

5. List the multiples of 16:

$$16, 32, 48, 64, 80, 96, 112, 128, 144, 160,$$
$$176, 192, 208, 224, 240, 256, 272, 288, 304, 320, \ldots$$

List the multiples of 20:

$$20, 40, 60, 80, 100, 120, 140, 160, 180, 200,$$
$$220, 240, 260, 280, 300, 320, 340, 360, 380, 400, \ldots$$

List the common multiples of 16 and 20:

$$80, 160, 240, 320, \ldots$$

Therefore, the least common multiple is

$$\text{LCM} = 80.$$

7. List the multiples of 20:

$$20, 40, 60, 80, 100, 120, 140, 160, 180, 200,$$
$$220, 240, 260, 280, 300, 320, 340, 360, 380, 400, \ldots$$

List the multiples of 12:

$$12, 24, 36, 48, 60, 72, 84, 96, 108, 120,$$
$$132, 144, 156, 168, 180, 192, 204, 216, 228, 240, \ldots$$

List the common multiples of 20 and 12:

$$60, 120, 180, 240, \ldots$$

Therefore, the least common multiple is

$$\text{LCM} = 60.$$

9. List the multiples of 10:

$$10, 20, 30, 40, 50, 60, 70, 80, 90, 100,$$
$$110, 120, 130, 140, 150, 160, 170, 180, 190, 200, \ldots$$

List the multiples of 6:

$$6, 12, 18, 24, 30, 36, 42, 48, 54, 60,$$
$$66, 72, 78, 84, 90, 96, 102, 108, 114, 120, \ldots$$

List the common multiples of 10 and 6:

$$30, 60, 90, 120, \ldots$$

Therefore, the least common multiple is

$$\text{LCM} = 30.$$

11. Prime factor each number and place the result in compact form using exponents.

$$54 = 2^1 \cdot 3^3$$
$$12 = 2^2 \cdot 3^1$$

Write each prime factor that appears above to the highest power that appears above.

$$\text{LCM} = 2^2 \cdot 3^3 \qquad \textcolor{red}{\text{Each factor to highest power.}}$$

Expand and simplify.

$$= 4 \cdot 27 \qquad \textcolor{red}{\text{Expand: } 2^2 = 4, \ 3^3 = 27.}$$
$$= 108 \qquad \textcolor{red}{\text{Multiply.}}$$

Therefore, the LCM of 54 and 12 is 108.

13. Prime factor each number and place the result in compact form using exponents.

$$18 = 2^1 \cdot 3^2$$
$$24 = 2^3 \cdot 3^1$$

Write each prime factor that appears above to the highest power that appears above.

$$\text{LCM} = 2^3 \cdot 3^2 \qquad \textcolor{red}{\text{Each factor to highest power.}}$$

Expand and simplify.

$$= 8 \cdot 9 \qquad \textcolor{red}{\text{Expand: } 2^3 = 8, \, 3^2 = 9.}$$
$$= 72 \qquad \textcolor{red}{\text{Multiply.}}$$

Therefore, the LCM of 18 and 24 is 72.

15. Prime factor each number and place the result in compact form using exponents.

$$72 = 2^3 \cdot 3^2$$
$$108 = 2^2 \cdot 3^3$$

Write each prime factor that appears above to the highest power that appears above.

$$\text{LCM} = 2^3 \cdot 3^3 \qquad \textcolor{red}{\text{Each factor to highest power.}}$$

Expand and simplify.

$$= 8 \cdot 27 \qquad \textcolor{red}{\text{Expand: } 2^3 = 8, \, 3^3 = 27.}$$
$$= 216 \qquad \textcolor{red}{\text{Multiply.}}$$

Therefore, the LCM of 72 and 108 is 216.

17. Prime factor each number and place the result in compact form using exponents.

$$36 = 2^2 \cdot 3^2$$
$$24 = 2^3 \cdot 3^1$$

Write each prime factor that appears above to the highest power that appears above.

$$\text{LCM} = 2^3 \cdot 3^2 \qquad\qquad \textcolor{red}{\text{Each factor to highest power.}}$$

Expand and simplify.

$$
\begin{aligned}
&= 8 \cdot 9 &&\textcolor{red}{\text{Expand: } 2^3 = 8,\ 3^2 = 9.}\\
&= 72 &&\textcolor{red}{\text{Multiply.}}
\end{aligned}
$$

Therefore, the LCM of 36 and 24 is 72.

19. Prime factor each number and place the result in compact form using exponents.

$$
\begin{aligned}
12 &= 2^2 \cdot 3^1\\
18 &= 2^1 \cdot 3^2
\end{aligned}
$$

Write each prime factor that appears above to the highest power that appears above.

$$\text{LCM} = 2^2 \cdot 3^2 \qquad\qquad \textcolor{red}{\text{Each factor to highest power.}}$$

Expand and simplify.

$$
\begin{aligned}
&= 4 \cdot 9 &&\textcolor{red}{\text{Expand: } 2^2 = 4,\ 3^2 = 9.}\\
&= 36 &&\textcolor{red}{\text{Multiply.}}
\end{aligned}
$$

Therefore, the LCM of 12 and 18 is 36.

21. Since the denominators are the same, simply subtract the numerators over the common denominator and simplify.

$$
\begin{aligned}
\frac{7}{12} - \frac{1}{12} &= \frac{7-1}{12} &&\textcolor{red}{\text{Subtract numerators over common denominator.}}\\
&= \frac{6}{12} &&\textcolor{red}{\text{Simplify numerator.}}\\
&= \frac{1}{2} &&\textcolor{red}{\text{Reduce to lowest terms.}}
\end{aligned}
$$

23. Since the denominators are the same, simply add the numerators over the common denominator and simplify.

$$\frac{1}{9} + \frac{1}{9} = \frac{1+1}{9} \qquad \text{Add numerators over common denominator.}$$

$$= \frac{2}{9} \qquad \text{Simplify numerator.}$$

25. Since the denominators are the same, simply subtract the numerators over the common denominator and simplify.

$$\frac{1}{5} - \frac{4}{5} = \frac{1-4}{5} \qquad \text{Subtract numerators over common denominator.}$$

$$= -\frac{3}{5} \qquad \text{Simplify numerator.}$$

27. Since the denominators are the same, simply subtract the numerators over the common denominator and simplify.

$$\frac{3}{7} - \frac{4}{7} = \frac{3-4}{7} \qquad \text{Subtract numerators over common denominator.}$$

$$= -\frac{1}{7} \qquad \text{Simplify numerator.}$$

29. Since the denominators are the same, simply add the numerators over the common denominator and simplify.

$$\frac{4}{11} + \frac{9}{11} = \frac{4+9}{11} \qquad \text{Add numerators over common denominator.}$$

$$= \frac{13}{11} \qquad \text{Simplify numerator.}$$

31. Since the denominators are the same, simply add the numerators over the common denominator and simplify.

$$\frac{3}{11} + \frac{4}{11} = \frac{3+4}{11} \qquad \text{Add numerators over common denominator.}$$

$$= \frac{7}{11} \qquad \text{Simplify numerator.}$$

33. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator and simplify.

$$
\begin{aligned}
\frac{1}{6} - \frac{1}{8} &= \frac{1 \cdot 4}{6 \cdot 4} - \frac{1 \cdot 3}{8 \cdot 3} &&\text{Equivalent fractions with LCD} = 24. \\
&= \frac{4}{24} - \frac{3}{24} &&\text{Simplify numerators and denominators.} \\
&= \frac{4 - 3}{24} &&\text{Subtract numerators over common denominator.} \\
&= \frac{1}{24} &&\text{Simplify numerator.}
\end{aligned}
$$

35. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator and simplify.

$$
\begin{aligned}
\frac{1}{5} + \frac{2}{3} &= \frac{1 \cdot 3}{5 \cdot 3} + \frac{2 \cdot 5}{3 \cdot 5} &&\text{Equivalent fractions with LCD} = 15. \\
&= \frac{3}{15} + \frac{10}{15} &&\text{Simplify numerators and denominators.} \\
&= \frac{3 + 10}{15} &&\text{Add numerators over common denominator.} \\
&= \frac{13}{15} &&\text{Simplify numerator.}
\end{aligned}
$$

37. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator and simplify.

$$
\begin{aligned}
\frac{2}{3} + \frac{5}{8} &= \frac{2 \cdot 8}{3 \cdot 8} + \frac{5 \cdot 3}{8 \cdot 3} &&\text{Equivalent fractions with LCD} = 24. \\
&= \frac{16}{24} + \frac{15}{24} &&\text{Simplify numerators and denominators.} \\
&= \frac{16 + 15}{24} &&\text{Add numerators over common denominator.} \\
&= \frac{31}{24} &&\text{Simplify numerator.}
\end{aligned}
$$

39. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the

common denominator and simplify.

$$\frac{4}{7} - \frac{5}{9} = \frac{4 \cdot 9}{7 \cdot 9} - \frac{5 \cdot 7}{9 \cdot 7} \qquad \text{Equivalent fractions with LCD} = 63.$$

$$= \frac{36}{63} - \frac{35}{63} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{36 - 35}{63} \qquad \text{Subtract numerators over common denominator.}$$

$$= \frac{1}{63} \qquad \text{Simplify numerator.}$$

41. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator and simplify.

$$\frac{2}{3} - \frac{3}{8} = \frac{2 \cdot 8}{3 \cdot 8} - \frac{3 \cdot 3}{8 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 24.$$

$$= \frac{16}{24} - \frac{9}{24} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{16 - 9}{24} \qquad \text{Subtract numerators over common denominator.}$$

$$= \frac{7}{24} \qquad \text{Simplify numerator.}$$

43. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator and simplify.

$$\frac{6}{7} - \frac{1}{6} = \frac{6 \cdot 6}{7 \cdot 6} - \frac{1 \cdot 7}{6 \cdot 7} \qquad \text{Equivalent fractions with LCD} = 42.$$

$$= \frac{36}{42} - \frac{7}{42} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{36 - 7}{42} \qquad \text{Subtract numerators over common denominator.}$$

$$= \frac{29}{42} \qquad \text{Simplify numerator.}$$

45. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the com-

mon denominator and simplify.

$$\frac{1}{6} + \frac{2}{3} = \frac{1}{6} + \frac{2 \cdot 2}{3 \cdot 2} \qquad \text{Equivalent fractions with LCD} = 6.$$
$$= \frac{1}{6} + \frac{4}{6} \qquad \text{Simplify numerators and denominators.}$$
$$= \frac{1 + 4}{6} \qquad \text{Add numerators over common denominator.}$$
$$= \frac{5}{6} \qquad \text{Simplify numerator.}$$

47. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator and simplify.

$$\frac{7}{9} + \frac{1}{8} = \frac{7 \cdot 8}{9 \cdot 8} + \frac{1 \cdot 9}{8 \cdot 9} \qquad \text{Equivalent fractions with LCD} = 72.$$
$$= \frac{56}{72} + \frac{9}{72} \qquad \text{Simplify numerators and denominators.}$$
$$= \frac{56 + 9}{72} \qquad \text{Add numerators over common denominator.}$$
$$= \frac{65}{72} \qquad \text{Simplify numerator.}$$

49. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator and simplify.

$$\frac{1}{3} + \frac{1}{7} = \frac{1 \cdot 7}{3 \cdot 7} + \frac{1 \cdot 3}{7 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 21.$$
$$= \frac{7}{21} + \frac{3}{21} \qquad \text{Simplify numerators and denominators.}$$
$$= \frac{7 + 3}{21} \qquad \text{Add numerators over common denominator.}$$
$$= \frac{10}{21} \qquad \text{Simplify numerator.}$$

51. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the

common denominator and simplify.

$$\frac{1}{2} - \frac{2}{7} = \frac{1 \cdot 7}{2 \cdot 7} - \frac{2 \cdot 2}{7 \cdot 2} \qquad \text{Equivalent fractions with LCD} = 14.$$

$$= \frac{7}{14} - \frac{4}{14} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{7 - 4}{14} \qquad \text{Subtract numerators over common denominator.}$$

$$= \frac{3}{14} \qquad \text{Simplify numerator.}$$

53. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator and simplify.

$$\frac{5}{6} - \frac{4}{5} = \frac{5 \cdot 5}{6 \cdot 5} - \frac{4 \cdot 6}{5 \cdot 6} \qquad \text{Equivalent fractions with LCD} = 30.$$

$$= \frac{25}{30} - \frac{24}{30} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{25 - 24}{30} \qquad \text{Subtract numerators over common denominator.}$$

$$= \frac{1}{30} \qquad \text{Simplify numerator.}$$

55. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator and simplify.

$$\frac{1}{3} + \frac{1}{8} = \frac{1 \cdot 8}{3 \cdot 8} + \frac{1 \cdot 3}{8 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 24.$$

$$= \frac{8}{24} + \frac{3}{24} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{8 + 3}{24} \qquad \text{Add numerators over common denominator.}$$

$$= \frac{11}{24} \qquad \text{Simplify numerator.}$$

57. Prime factor each denominator and place the result in compact form using exponents.

$$36 = 2^2 \cdot 3^2$$
$$54 = 2^1 \cdot 3^3$$

Write each prime factor that appears above to the highest power that appears above, then simplify.

$$\text{LCD} = 2^2 \cdot 3^3 = 108$$

Next, create equivalent fractions with an LCD = 108.

$$\frac{7}{36} + \frac{11}{54} = \frac{7 \cdot 3}{36 \cdot 3} + \frac{11 \cdot 2}{54 \cdot 2} \qquad \color{red}{\text{Equivalent fractions, LCD} = 108.}$$

$$= \frac{21}{108} + \frac{22}{108} \qquad \color{red}{\text{Simplify numerators and denominators.}}$$

$$= \frac{21 + 22}{108} \qquad \color{red}{\text{Add numerators over common denominator.}}$$

$$= \frac{43}{108} \qquad \color{red}{\text{Simplify numerator.}}$$

59. Prime factor each denominator and place the result in compact form using exponents.

$$18 = 2^1 \cdot 3^2$$
$$12 = 2^2 \cdot 3^1$$

Write each prime factor that appears above to the highest power that appears above, then simplify.

$$\text{LCD} = 2^2 \cdot 3^2 = 36$$

Next, create equivalent fractions with an LCD = 36.

$$\frac{7}{18} - \frac{5}{12} = \frac{7 \cdot 2}{18 \cdot 2} - \frac{5 \cdot 3}{12 \cdot 3} \qquad \color{red}{\text{Equivalent fractions, LCD} = 36.}$$

$$= \frac{14}{36} - \frac{15}{36} \qquad \color{red}{\text{Simplify numerators and denominators.}}$$

$$= \frac{14 - 15}{36} \qquad \color{red}{\text{Subtract numerators over common denominator.}}$$

$$= \frac{-1}{36} \qquad \color{red}{\text{Simplify numerator.}}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{1}{36}$$

61. Prime factor each denominator and place the result in compact form using exponents.

$$36 = 2^2 \cdot 3^2$$
$$54 = 2^1 \cdot 3^3$$

Write each prime factor that appears above to the highest power that appears above, then simplify.

$$\text{LCD} = 2^2 \cdot 3^3 = 108$$

Next, create equivalent fractions with an LCD = 108.

$$\frac{7}{36} + \frac{7}{54} = \frac{7 \cdot 3}{36 \cdot 3} + \frac{7 \cdot 2}{54 \cdot 2} \qquad \textcolor{red}{\text{Equivalent fractions, LCD} = 108.}$$

$$= \frac{21}{108} + \frac{14}{108} \qquad \textcolor{red}{\text{Simplify numerators and denominators.}}$$

$$= \frac{21 + 14}{108} \qquad \textcolor{red}{\text{Add numerators over common denominator.}}$$

$$= \frac{35}{108} \qquad \textcolor{red}{\text{Simplify numerator.}}$$

63. Prime factor each denominator and place the result in compact form using exponents.

$$24 = 2^3 \cdot 3^1$$
$$36 = 2^2 \cdot 3^2$$

Write each prime factor that appears above to the highest power that appears above, then simplify.

$$\text{LCD} = 2^3 \cdot 3^2 = 72$$

Next, create equivalent fractions with an LCD = 72.

$$\frac{7}{24} - \frac{5}{36} = \frac{7 \cdot 3}{24 \cdot 3} - \frac{5 \cdot 2}{36 \cdot 2} \qquad \textcolor{red}{\text{Equivalent fractions, LCD} = 72.}$$

$$= \frac{21}{72} - \frac{10}{72} \qquad \textcolor{red}{\text{Simplify numerators and denominators.}}$$

$$= \frac{21 - 10}{72} \qquad \textcolor{red}{\text{Subtract numerators over common denominator.}}$$

$$= \frac{11}{72} \qquad \textcolor{red}{\text{Simplify numerator.}}$$

65. Prime factor each denominator and place the result in compact form using exponents.

$$12 = 2^2 \cdot 3^1$$
$$18 = 2^1 \cdot 3^2$$

Write each prime factor that appears above to the highest power that appears above, then simplify.

$$\text{LCD} = 2^2 \cdot 3^2 = 36$$

Next, create equivalent fractions with an LCD = 36.

$$\frac{11}{12} + \frac{5}{18} = \frac{11 \cdot 3}{12 \cdot 3} + \frac{5 \cdot 2}{18 \cdot 2} \qquad \text{\textcolor{red}{Equivalent fractions, LCD = 36.}}$$

$$= \frac{33}{36} + \frac{10}{36} \qquad \text{\textcolor{red}{Simplify numerators and denominators.}}$$

$$= \frac{33 + 10}{36} \qquad \text{\textcolor{red}{Add numerators over common denominator.}}$$

$$= \frac{43}{36} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

67. Prime factor each denominator and place the result in compact form using exponents.

$$54 = 2^1 \cdot 3^3$$
$$24 = 2^3 \cdot 3^1$$

Write each prime factor that appears above to the highest power that appears above, then simplify.

$$\text{LCD} = 2^3 \cdot 3^3 = 216$$

Next, create equivalent fractions with an LCD = 216.

$$\frac{11}{54} - \frac{5}{24} = \frac{11 \cdot 4}{54 \cdot 4} - \frac{5 \cdot 9}{24 \cdot 9} \qquad \text{\textcolor{red}{Equivalent fractions, LCD = 216.}}$$

$$= \frac{44}{216} - \frac{45}{216} \qquad \text{\textcolor{red}{Simplify numerators and denominators.}}$$

$$= \frac{44 - 45}{216} \qquad \text{\textcolor{red}{Subtract numerators over common denominator.}}$$

$$= \frac{-1}{216} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{1}{216}$$

69. First simplify by rewriting as a subtraction problem. Then, since the denominators are the same, simply subtract the numerators over the common denominator and simplify.

$$-\frac{3}{7} + \left(-\frac{3}{7}\right) = -\frac{3}{7} - \frac{3}{7} \qquad \text{\textcolor{red}{Rewrite as a subtraction problem.}}$$

$$= \frac{-3 - 3}{7} \qquad \text{\textcolor{red}{Subtract numerators over common denominator.}}$$

$$= \frac{-6}{7} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{6}{7}$$

71. First simplify by rewriting as a addition problem. Then, since the denominators are the same, simply add the numerators over the common denominator and simplify.

$$\frac{7}{9} - \left(-\frac{1}{9}\right) = \frac{7}{9} + \frac{1}{9} \qquad \text{Rewrite as an addition problem.}$$

$$= \frac{7+1}{9} \qquad \text{Add numerators over common denominator.}$$

$$= \frac{8}{9} \qquad \text{Simplify numerator.}$$

73. First simplify by rewriting as a subtraction problem. Then, since the denominators are the same, simply subtract the numerators over the common denominator and simplify.

$$\frac{7}{9} + \left(-\frac{2}{9}\right) = \frac{7}{9} - \frac{2}{9} \qquad \text{Rewrite as a subtraction problem.}$$

$$= \frac{7-2}{9} \qquad \text{Subtract numerators over common denominator.}$$

$$= \frac{5}{9} \qquad \text{Simplify numerator.}$$

75. Since the denominators are the same, simply subtract the numerators over the common denominator and simplify.

$$-\frac{3}{5} - \frac{4}{5} = \frac{-3-4}{5} \qquad \text{Subtract numerators over common denominator.}$$

$$= \frac{-7}{5} \qquad \text{Simplify numerator.}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{7}{5}$$

77. Since the denominators are the same, simply add the numerators over the common denominator and simplify.

$$-\frac{7}{8} + \frac{1}{8} = \frac{-7+1}{8} \qquad \text{Add numerators over common denominator.}$$
$$= \frac{-6}{8} \qquad \text{Simplify numerator.}$$
$$= \frac{-3}{4} \qquad \text{Reduce to lowest terms.}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{3}{4}$$

79. First simplify by rewriting as a addition problem. Then, since the denominators are the same, simply add the numerators over the common denominator and simplify.

$$-\frac{1}{3} - \left(-\frac{2}{3}\right) = -\frac{1}{3} + \frac{2}{3} \qquad \text{Rewrite as an addition problem.}$$
$$= \frac{-1+2}{3} \qquad \text{Add numerators over common denominator.}$$
$$= \frac{1}{3} \qquad \text{Simplify numerator.}$$

81. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator and simplify.

$$-\frac{2}{7} + \frac{4}{5} = -\frac{2 \cdot 5}{7 \cdot 5} + \frac{4 \cdot 7}{5 \cdot 7} \qquad \text{Equivalent fractions with LCD} = 35.$$
$$= -\frac{10}{35} + \frac{28}{35} \qquad \text{Simplify numerators and denominators.}$$
$$= \frac{-10+28}{35} \qquad \text{Add numerators over common denominator.}$$
$$= \frac{18}{35} \qquad \text{Simplify numerator.}$$

83. First simplify by rewriting as an addition problem. Then, since the denominators are different, write equivalent fractions using the least common

denominator. Finally, add the numerators over the common denominator and simplify.

$$-\frac{1}{4} - \left(-\frac{4}{9}\right) = -\frac{1}{4} + \frac{4}{9} \qquad \text{\color{red}Rewrite as an addition problem.}$$

$$= -\frac{1 \cdot 9}{4 \cdot 9} + \frac{4 \cdot 4}{9 \cdot 4} \qquad \text{\color{red}Equivalent fractions with LCD} = 36.$$

$$= -\frac{9}{36} + \frac{16}{36} \qquad \text{\color{red}Simplify numerators and denominators.}$$

$$= \frac{-9 + 16}{36} \qquad \text{\color{red}Add numerators over common denominator.}$$

$$= \frac{7}{36} \qquad \text{\color{red}Simplify numerator.}$$

85. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator and simplify.

$$-\frac{2}{7} + \frac{3}{4} = -\frac{2 \cdot 4}{7 \cdot 4} + \frac{3 \cdot 7}{4 \cdot 7} \qquad \text{\color{red}Equivalent fractions with LCD} = 28.$$

$$= -\frac{8}{28} + \frac{21}{28} \qquad \text{\color{red}Simplify numerators and denominators.}$$

$$= \frac{-8 + 21}{28} \qquad \text{\color{red}Add numerators over common denominator.}$$

$$= \frac{13}{28} \qquad \text{\color{red}Simplify numerator.}$$

87. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator and simplify.

$$-\frac{4}{9} - \frac{1}{3} = -\frac{4}{9} - \frac{1 \cdot 3}{3 \cdot 3} \qquad \text{\color{red}Equivalent fractions with LCD} = 9.$$

$$= -\frac{4}{9} - \frac{3}{9} \qquad \text{\color{red}Simplify numerators and denominators.}$$

$$= \frac{-4 - 3}{9} \qquad \text{\color{red}Subtract numerators over common denominator.}$$

$$= \frac{-7}{9} \qquad \text{\color{red}Simplify numerator.}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{7}{9}$$

89. First simplify by rewriting as an addition problem. Then, since the denominators are different, write equivalent fractions using the least common denominator. Finally, add the numerators over the common denominator and simplify.

$$-\frac{5}{7} - \left(-\frac{1}{5}\right) = -\frac{5}{7} + \frac{1}{5} \qquad \text{Rewrite as an addition problem.}$$

$$= -\frac{5 \cdot 5}{7 \cdot 5} + \frac{1 \cdot 7}{5 \cdot 7} \qquad \text{Equivalent fractions with LCD} = 35.$$

$$= -\frac{25}{35} + \frac{7}{35} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{-25 + 7}{35} \qquad \text{Add numerators over common denominator.}$$

$$= \frac{-18}{35} \qquad \text{Simplify numerator.}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{18}{35}$$

91. First simplify by rewriting as a subtraction problem. Then, since the denominators are different, write equivalent fractions using the least common denominator. Finally, subtract the numerators over the common denominator and simplify.

$$\frac{1}{9} + \left(-\frac{1}{3}\right) = \frac{1}{9} - \frac{1}{3} \qquad \text{Rewrite as a subtraction problem.}$$

$$= \frac{1}{9} - \frac{1 \cdot 3}{3 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 9.$$

$$= \frac{1}{9} - \frac{3}{9} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{1 - 3}{9} \qquad \text{Subtract numerators over common denominator.}$$

$$= \frac{-2}{9} \qquad \text{Simplify numerator.}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{2}{9}$$

93. First simplify by rewriting as a subtraction problem. Then, since the denominators are different, write equivalent fractions using the least common denominator. Finally, subtract the numerators over the common denominator and simplify.

$$
\begin{aligned}
\frac{2}{3} + \left(-\frac{1}{9}\right) &= \frac{2}{3} - \frac{1}{9} && \text{Rewrite as a subtraction problem.} \\
&= \frac{2 \cdot 3}{3 \cdot 3} - \frac{1}{9} && \text{Equivalent fractions with LCD} = 9. \\
&= \frac{6}{9} - \frac{1}{9} && \text{Simplify numerators and denominators.} \\
&= \frac{6 - 1}{9} && \text{Subtract numerators over common denominator.} \\
&= \frac{5}{9} && \text{Simplify numerator.}
\end{aligned}
$$

95. First simplify by rewriting as a subtraction problem. Then, since the denominators are different, write equivalent fractions using the least common denominator. Finally, subtract the numerators over the common denominator and simplify.

$$
\begin{aligned}
-\frac{1}{2} + \left(-\frac{6}{7}\right) &= -\frac{1}{2} - \frac{6}{7} && \text{Rewrite as a subtraction problem.} \\
&= -\frac{1 \cdot 7}{2 \cdot 7} - \frac{6 \cdot 2}{7 \cdot 2} && \text{Equivalent fractions with LCD} = 14. \\
&= -\frac{7}{14} - \frac{12}{14} && \text{Simplify numerators and denominators.} \\
&= \frac{-7 - 12}{14} && \text{Subtract numerators over common denominator.} \\
&= \frac{-19}{14} && \text{Simplify numerator.}
\end{aligned}
$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$
= -\frac{19}{14}
$$

97. First simplify by rewriting as a subtraction problem. Then, since the denominators are different, write equivalent fractions using the least common denominator. Finally, subtract the numerators over the common denominator

and simplify.

$$-\frac{1}{2}+\left(-\frac{3}{4}\right)=-\frac{1}{2}-\frac{3}{4} \qquad \text{\textcolor{red}{Rewrite as a subtraction problem.}}$$

$$=-\frac{1\cdot 2}{2\cdot 2}-\frac{3}{4} \qquad \text{\textcolor{red}{Equivalent fractions with LCD} = 4.}$$

$$=-\frac{2}{4}-\frac{3}{4} \qquad \text{\textcolor{red}{Simplify numerators and denominators.}}$$

$$=\frac{-2-3}{4} \qquad \text{\textcolor{red}{Subtract numerators over common denominator.}}$$

$$=\frac{-5}{4} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$=-\frac{5}{4}$$

99. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator and simplify.

$$-\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}-\frac{1\cdot 2}{2\cdot 2} \qquad \text{\textcolor{red}{Equivalent fractions with LCD} = 4.}$$

$$=-\frac{1}{4}-\frac{2}{4} \qquad \text{\textcolor{red}{Simplify numerators and denominators.}}$$

$$=\frac{-1-2}{4} \qquad \text{\textcolor{red}{Subtract numerators over common denominator.}}$$

$$=\frac{-3}{4} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$=-\frac{3}{4}$$

101. First simplify by rewriting as an addition problem. Then, since the denominators are different, write equivalent fractions using the least common denominator. Finally, add the numerators over the common denominator and

simplify.

$$\frac{5}{8} - \left(-\frac{3}{4}\right) = \frac{5}{8} + \frac{3}{4} \qquad \text{Rewrite as an addition problem.}$$

$$= \frac{5}{8} + \frac{3 \cdot 2}{4 \cdot 2} \qquad \text{Equivalent fractions with LCD} = 8.$$

$$= \frac{5}{8} + \frac{6}{8} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{5 + 6}{8} \qquad \text{Add numerators over common denominator.}$$

$$= \frac{11}{8} \qquad \text{Simplify numerator.}$$

103. First simplify by rewriting as an addition problem. Then, since the denominators are different, write equivalent fractions using the least common denominator. Finally, add the numerators over the common denominator and simplify.

$$\frac{1}{8} - \left(-\frac{1}{3}\right) = \frac{1}{8} + \frac{1}{3} \qquad \text{Rewrite as an addition problem.}$$

$$= \frac{1 \cdot 3}{8 \cdot 3} + \frac{1 \cdot 8}{3 \cdot 8} \qquad \text{Equivalent fractions with LCD} = 24.$$

$$= \frac{3}{24} + \frac{8}{24} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{3 + 8}{24} \qquad \text{Add numerators over common denominator.}$$

$$= \frac{11}{24} \qquad \text{Simplify numerator.}$$

105. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator.

$$\frac{1}{2} + \frac{3q}{5} = \frac{1 \cdot 5}{2 \cdot 5} + \frac{3q \cdot 2}{5 \cdot 2} \qquad \text{Equivalent fractions with LCD} = 10.$$

$$= \frac{5}{10} + \frac{6q}{10} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{5 + 6q}{10} \qquad \text{Add numerators over common denominator.}$$

107. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the

common denominator.

$$\frac{4}{9} - \frac{3a}{4} = \frac{4 \cdot 4}{9 \cdot 4} - \frac{3a \cdot 9}{4 \cdot 9} \qquad \text{Equivalent fractions with LCD} = 36.$$

$$= \frac{16}{36} - \frac{27a}{36} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{16 - 27a}{36} \qquad \text{Add numerators over common denominator.}$$

109. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator.

$$\frac{2}{s} + \frac{1}{3} = \frac{2 \cdot 3}{s \cdot 3} + \frac{1 \cdot s}{3 \cdot s} \qquad \text{Equivalent fractions with LCD} = 3s.$$

$$= \frac{6}{3s} + \frac{s}{3s} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{6 + s}{3s} \qquad \text{Add numerators over common denominator.}$$

111. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator.

$$\frac{1}{3} - \frac{7}{b} = \frac{1 \cdot b}{3 \cdot b} - \frac{7 \cdot 3}{b \cdot 3} \qquad \text{Equivalent fractions with LCD} = 3b.$$

$$= \frac{b}{3b} - \frac{21}{3b} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{b - 21}{3b} \qquad \text{Add numerators over common denominator.}$$

113. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator.

$$\frac{4b}{7} + \frac{2}{3} = \frac{4b \cdot 3}{7 \cdot 3} + \frac{2 \cdot 7}{3 \cdot 7} \qquad \text{Equivalent fractions with LCD} = 21.$$

$$= \frac{12b}{21} + \frac{14}{21} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{12b + 14}{21} \qquad \text{Add numerators over common denominator.}$$

115. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator.

$$
\begin{aligned}
\frac{2}{3} - \frac{9}{t} &= \frac{2 \cdot t}{3 \cdot t} - \frac{9 \cdot 3}{t \cdot 3} && \text{\color{red}Equivalent fractions with LCD} = 3t. \\
&= \frac{2t}{3t} - \frac{27}{3t} && \text{\color{red}Simplify numerators and denominators.} \\
&= \frac{2t - 27}{3t} && \text{\color{red}Add numerators over common denominator.}
\end{aligned}
$$

117. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator.

$$
\begin{aligned}
\frac{9}{s} + \frac{7}{8} &= \frac{9 \cdot 8}{s \cdot 8} + \frac{7 \cdot s}{8 \cdot s} && \text{\color{red}Equivalent fractions with LCD} = 8s. \\
&= \frac{72}{8s} + \frac{7s}{8s} && \text{\color{red}Simplify numerators and denominators.} \\
&= \frac{72 + 7s}{8s} && \text{\color{red}Add numerators over common denominator.}
\end{aligned}
$$

119. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then subtract the numerators over the common denominator.

$$
\begin{aligned}
\frac{7b}{8} - \frac{5}{9} &= \frac{7b \cdot 9}{8 \cdot 9} - \frac{5 \cdot 8}{9 \cdot 8} && \text{\color{red}Equivalent fractions with LCD} = 72. \\
&= \frac{63b}{72} - \frac{40}{72} && \text{\color{red}Simplify numerators and denominators.} \\
&= \frac{63b - 40}{72} && \text{\color{red}Add numerators over common denominator.}
\end{aligned}
$$

121. The least common denominator for 3 and 7 is 21. Make equivalent fractions with an LCD equal to 21:

$$
-\frac{2}{3} = -\frac{2 \cdot 7}{3 \cdot 7} = -\frac{14}{21} \qquad \text{and} \qquad -\frac{8}{7} = -\frac{8 \cdot 3}{7 \cdot 3} = -\frac{24}{21}.
$$

Now compare numerators (including the negative sign): since $-14 > -24$, it follows that

$$
-\frac{14}{21} > -\frac{24}{21},
$$

and therefore

$$
-\frac{2}{3} > -\frac{8}{7}
$$

123. The least common denominator for 7 and 3 is 21. Make equivalent fractions with an LCD equal to 21:

$$\frac{6}{7} = \frac{6 \cdot 3}{7 \cdot 3} = \frac{18}{21} \qquad \text{and} \qquad \frac{7}{3} = \frac{7 \cdot 7}{3 \cdot 7} = \frac{49}{21}.$$

Now compare numerators: since $18 < 49$, it follows that

$$\frac{18}{21} < \frac{49}{21},$$

and therefore

$$\frac{6}{7} < \frac{7}{3}$$

125. The least common denominator for 4 and 3 is 12. Make equivalent fractions with an LCD equal to 12:

$$-\frac{9}{4} = -\frac{9 \cdot 3}{4 \cdot 3} = -\frac{27}{12} \qquad \text{and} \qquad -\frac{2}{3} = -\frac{2 \cdot 4}{3 \cdot 4} = -\frac{8}{12}.$$

Now compare numerators (including the negative sign): since $-27 < -8$, it follows that

$$-\frac{27}{12} < -\frac{8}{12},$$

and therefore

$$-\frac{9}{4} < -\frac{2}{3}$$

$$-\frac{27}{12} \qquad\qquad\qquad -\frac{8}{12}$$

$$-\frac{9}{4} \qquad\qquad\qquad -\frac{2}{3}$$

127. The least common denominator for 7 and 9 is 63. Make equivalent fractions with an LCD equal to 63:

$$\frac{5}{7} = \frac{5 \cdot 9}{7 \cdot 9} = \frac{45}{63} \qquad \text{and} \qquad \frac{5}{9} = \frac{5 \cdot 7}{9 \cdot 7} = \frac{35}{63}.$$

Now compare numerators: since $45 > 35$, it follows that

$$\frac{45}{63} > \frac{35}{63},$$

and therefore

$$\frac{5}{7} > \frac{5}{9}$$

$$\frac{35}{63} \qquad\qquad\qquad \frac{45}{63}$$

$$\frac{5}{9} \qquad\qquad\qquad \frac{5}{7}$$

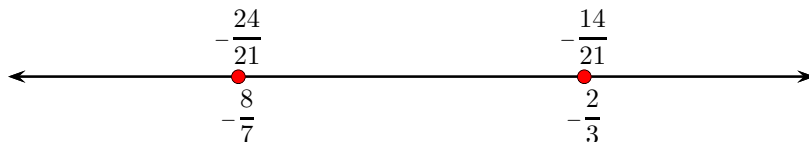**129.** The least common denominator for 2 and 5 is 10. Make equivalent fractions with an LCD equal to 10:

$$-\frac{7}{2} = -\frac{7 \cdot 5}{2 \cdot 5} = -\frac{35}{10} \qquad \text{and} \qquad -\frac{1}{5} = -\frac{1 \cdot 2}{5 \cdot 2} = -\frac{2}{10}.$$

Now compare numerators (including the negative sign): since $-35 < -2$, it follows that

$$-\frac{35}{10} < -\frac{2}{10},$$

and therefore

$$-\frac{7}{2} < -\frac{1}{5}$$

131. The least common denominator for 9 and 5 is 45. Make equivalent fractions with an LCD equal to 45:
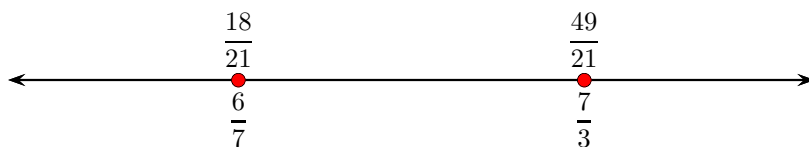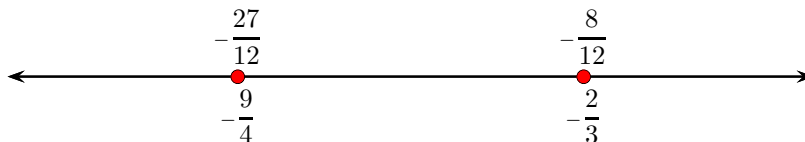
$$\frac{5}{9} = \frac{5 \cdot 5}{9 \cdot 5} = \frac{25}{45} \qquad \text{and} \qquad \frac{6}{5} = \frac{6 \cdot 9}{5 \cdot 9} = \frac{54}{45}.$$

Now compare numerators: since $25 < 54$, it follows that

$$\frac{25}{45} < \frac{54}{45},$$

and therefore

$$\frac{5}{9} < \frac{6}{5}$$

4.5 Multiplying and Dividing Mixed Fractions

1. Multiply the whole number part by the denominator, add the numerator, then place the result over the denominator.

$$2\frac{1}{3} = \frac{2 \cdot 3 + 1}{3} \qquad \text{\color{red}{Convert to an improper fraction.}}$$
$$= \frac{7}{3} \qquad \text{\color{red}{Simplify numerator.}}$$

3. Multiply the whole number part by the denominator, add the numerator, then place the result over the denominator.

$$1\frac{1}{19} = \frac{1 \cdot 19 + 1}{19} \qquad \text{\textcolor{red}{Convert to an improper fraction.}}$$
$$= \frac{20}{19} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

5. Multiply the whole number part by the denominator, add the numerator, then place the result over the denominator.

$$-1\frac{3}{7} = -\frac{1 \cdot 7 + 3}{7} \qquad \text{\textcolor{red}{Convert to an improper fraction.}}$$
$$= -\frac{10}{7} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

7. Multiply the whole number part by the denominator, add the numerator, then place the result over the denominator.

$$1\frac{1}{9} = \frac{1 \cdot 9 + 1}{9} \qquad \text{\textcolor{red}{Convert to an improper fraction.}}$$
$$= \frac{10}{9} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

9. Multiply the whole number part by the denominator, add the numerator, then place the result over the denominator.

$$-1\frac{1}{2} = -\frac{1 \cdot 2 + 1}{2} \qquad \text{\textcolor{red}{Convert to an improper fraction.}}$$
$$= -\frac{3}{2} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

11. Multiply the whole number part by the denominator, add the numerator, then place the result over the denominator.

$$1\frac{1}{3} = \frac{1 \cdot 3 + 1}{3} \qquad \text{\textcolor{red}{Convert to an improper fraction.}}$$
$$= \frac{4}{3} \qquad \text{\textcolor{red}{Simplify numerator.}}$$

13. 13 divided by 7 is 1, with a remainder of 6. Therefore,

$$\frac{13}{7} = 1\frac{6}{7}$$

15. 13 divided by 5 is 2, with a remainder of 3. Therefore,

$$-\frac{13}{5} = -2\frac{3}{5}$$

17. 16 divided by 5 is 3, with a remainder of 1. Therefore,

$$-\frac{16}{5} = -3\frac{1}{5}$$

19. 9 divided by 8 is 1, with a remainder of 1. Therefore,

$$\frac{9}{8} = 1\frac{1}{8}$$

21. 6 divided by 5 is 1, with a remainder of 1. Therefore,

$$-\frac{6}{5} = -1\frac{1}{5}$$

23. 3 divided by 2 is 1, with a remainder of 1. Therefore,

$$-\frac{3}{2} = -1\frac{1}{2}$$

25. Convert the mixed fractions to improper fractions, then multiply.

$$
\begin{aligned}
1\frac{1}{7} \cdot 2\frac{1}{2} &= \frac{8}{7} \cdot \frac{5}{2} && \text{\color{red}Convert to improper fractions.} \\
&= \frac{(2 \cdot 2 \cdot 2) \cdot (5)}{(7) \cdot (2)} && \text{\color{red}Prime factorization.} \\
&= \frac{2 \cdot 2 \cdot 5}{7} && \text{\color{red}Cancel common factors.} \\
&= \frac{20}{7} && \text{\color{red}Simplify.} \\
&= 2\frac{6}{7} && \text{\color{red}Convert to a mixed fraction.}
\end{aligned}
$$

27. Convert the numbers to improper fractions, then multiply.

$$
\begin{aligned}
4 \cdot 1\frac{1}{6} &= \frac{4}{1} \cdot \frac{7}{6} && \text{\color{red}{Convert to improper fractions.}} \\
&= \frac{(2 \cdot 2) \cdot (7)}{(1) \cdot (2 \cdot 3)} && \text{\color{red}{Prime factorization.}} \\
&= \frac{2 \cdot 7}{3} && \text{\color{red}{Cancel common factors.}} \\
&= \frac{14}{3} && \text{\color{red}{Simplify.}} \\
&= 4\frac{2}{3} && \text{\color{red}{Convert to a mixed fraction.}}
\end{aligned}
$$

29. Convert the mixed fractions to improper fractions, then multiply.

$$
\begin{aligned}
\left(-1\frac{1}{12}\right)\left(3\frac{3}{4}\right) &= \left(-\frac{13}{12}\right) \cdot \left(\frac{15}{4}\right) && \text{\color{red}{Convert to improper fractions.}} \\
&= -\frac{(13) \cdot (3 \cdot 5)}{(2 \cdot 2 \cdot 3) \cdot (2 \cdot 2)} && \text{\color{red}{Prime factorization.}} \\
& && \text{\color{red}{Unlike signs give a negative product.}} \\
&= -\frac{13 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 2} && \text{\color{red}{Cancel common factors.}} \\
&= -\frac{65}{16} && \text{\color{red}{Simplify.}} \\
&= -4\frac{1}{16} && \text{\color{red}{Convert to a mixed fraction.}}
\end{aligned}
$$

31. Convert the mixed fractions to improper fractions, then multiply.

$$
\begin{aligned}
7\frac{1}{2} \cdot 1\frac{1}{13} &= \frac{15}{2} \cdot \frac{14}{13} && \text{\color{red}{Convert to improper fractions.}} \\
&= \frac{(3 \cdot 5) \cdot (2 \cdot 7)}{(2) \cdot (13)} && \text{\color{red}{Prime factorization.}} \\
&= \frac{3 \cdot 5 \cdot 7}{13} && \text{\color{red}{Cancel common factors.}} \\
&= \frac{105}{13} && \text{\color{red}{Simplify.}} \\
&= 8\frac{1}{13} && \text{\color{red}{Convert to a mixed fraction.}}
\end{aligned}
$$

33. Convert the mixed fractions to improper fractions, then multiply.

$$\left(1\frac{2}{13}\right)\left(-4\frac{2}{3}\right) = \left(\frac{15}{13}\right)\cdot\left(-\frac{14}{3}\right) \qquad \text{Convert to improper fractions.}$$

$$= -\frac{(3\cdot 5)\cdot(2\cdot 7)}{(13)\cdot(3)} \qquad \text{Prime factorization.}$$
$$\text{Unlike signs give a negative product.}$$

$$= -\frac{5\cdot 2\cdot 7}{13} \qquad \text{Cancel common factors.}$$

$$= -\frac{70}{13} \qquad \text{Simplify.}$$

$$= -5\frac{5}{13} \qquad \text{Convert to a mixed fraction.}$$

35. Convert the mixed fractions to improper fractions, then multiply.

$$\left(1\frac{3}{7}\right)\left(-3\frac{3}{4}\right) = \left(\frac{10}{7}\right)\cdot\left(-\frac{15}{4}\right) \qquad \text{Convert to improper fractions.}$$

$$= -\frac{(2\cdot 5)\cdot(3\cdot 5)}{(7)\cdot(2\cdot 2)} \qquad \text{Prime factorization.}$$
$$\text{Unlike signs give a negative product.}$$

$$= -\frac{5\cdot 3\cdot 5}{7\cdot 2} \qquad \text{Cancel common factors.}$$

$$= -\frac{75}{14} \qquad \text{Simplify.}$$

$$= -5\frac{5}{14} \qquad \text{Convert to a mixed fraction.}$$

37. Convert the numbers to improper fractions, then multiply.

$$9\cdot\left(-1\frac{2}{15}\right) = \left(\frac{9}{1}\right)\cdot\left(-\frac{17}{15}\right) \qquad \text{Convert to improper fractions.}$$

$$= -\frac{(3\cdot 3)\cdot(17)}{(1)\cdot(3\cdot 5)} \qquad \text{Prime factorization.}$$
$$\text{Unlike signs give a negative product.}$$

$$= -\frac{3\cdot 17}{5} \qquad \text{Cancel common factors.}$$

$$= -\frac{51}{5} \qquad \text{Simplify.}$$

$$= -10\frac{1}{5} \qquad \text{Convert to a mixed fraction.}$$

39. Convert the numbers to improper fractions, then multiply.

$$\left(-2\frac{1}{8}\right)(-6) = \left(-\frac{17}{8}\right)\left(-\frac{6}{1}\right) \qquad \text{Convert to improper fractions.}$$

$$= \frac{(17)\cdot(2\cdot3)}{(2\cdot2\cdot2)\cdot(1)} \qquad \begin{array}{l}\text{Prime factorization.}\\ \text{Like signs give a positive product.}\end{array}$$

$$= \frac{17\cdot3}{2\cdot2} \qquad \text{Cancel common factors.}$$

$$= \frac{51}{4} \qquad \text{Simplify.}$$

$$= 12\frac{3}{4} \qquad \text{Convert to a mixed fraction.}$$

41. Convert the mixed fractions to improper fractions, then multiply.

$$\left(-4\frac{1}{2}\right)\left(-2\frac{2}{5}\right) = \left(-\frac{9}{2}\right)\cdot\left(-\frac{12}{5}\right) \qquad \text{Convert to improper fractions.}$$

$$= \frac{(3\cdot3)\cdot(2\cdot2\cdot3)}{(2)\cdot(5)} \qquad \begin{array}{l}\text{Prime factorization.}\\ \text{Like signs give a positive product.}\end{array}$$

$$= \frac{3\cdot3\cdot2\cdot3}{5} \qquad \text{Cancel common factors.}$$

$$= \frac{54}{5} \qquad \text{Simplify.}$$

$$= 10\frac{4}{5} \qquad \text{Convert to a mixed fraction.}$$

43. Convert the numbers to improper fractions, then multiply.

$$\left(-2\frac{1}{6}\right)\cdot4 = \left(-\frac{13}{6}\right)\cdot\left(\frac{4}{1}\right) \qquad \text{Convert to improper fractions.}$$

$$= -\frac{(13)\cdot(2\cdot2)}{(2\cdot3)\cdot(1)} \qquad \begin{array}{l}\text{Prime factorization.}\\ \text{Unlike signs give a negative product.}\end{array}$$

$$= -\frac{13\cdot2}{3} \qquad \text{Cancel common factors.}$$

$$= -\frac{26}{3} \qquad \text{Simplify.}$$

$$= -8\frac{2}{3} \qquad \text{Convert to a mixed fraction.}$$

45. Convert the mixed fractions to improper fractions, then multiply.

$$\left(-1\frac{4}{15}\right)\left(2\frac{1}{2}\right) = \left(-\frac{19}{15}\right)\cdot\left(\frac{5}{2}\right) \qquad \text{Convert to improper fractions.}$$

$$= -\frac{(19)\cdot(5)}{(3\cdot5)\cdot(2)} \qquad \text{Prime factorization.}$$
$$\text{Unlike signs give a negative product.}$$

$$= -\frac{19}{3\cdot2} \qquad \text{Cancel common factors.}$$

$$= -\frac{19}{6} \qquad \text{Simplify.}$$

$$= -3\frac{1}{6} \qquad \text{Convert to a mixed fraction.}$$

47. Convert the mixed fractions to improper fractions, then multiply.

$$\left(-2\frac{1}{2}\right)\left(-1\frac{7}{11}\right) = \left(-\frac{5}{2}\right)\cdot\left(-\frac{18}{11}\right) \qquad \text{Convert to improper fractions.}$$

$$= \frac{(5)\cdot(2\cdot3\cdot3)}{(2)\cdot(11)} \qquad \text{Prime factorization.}$$
$$\text{Like signs give a positive product.}$$

$$= \frac{5\cdot3\cdot3}{11} \qquad \text{Cancel common factors.}$$

$$= \frac{45}{11} \qquad \text{Simplify.}$$

$$= 4\frac{1}{11} \qquad \text{Convert to a mixed fraction.}$$

49. Convert the mixed fractions to improper fractions, then divide.

$$8 \div 2\frac{2}{9} = \frac{8}{1} \div \frac{20}{9} \qquad \text{Convert to improper fractions.}$$

$$= \frac{8}{1}\cdot\frac{9}{20} \qquad \text{Invert the second fraction and multiply.}$$

$$= \frac{(2\cdot2\cdot2)\cdot(3\cdot3)}{(1)\cdot(2\cdot2\cdot5)} \qquad \text{Prime factorization.}$$

$$= \frac{2\cdot3\cdot3}{5} \qquad \text{Cancel common factors.}$$

$$= \frac{18}{5} \qquad \text{Simplify.}$$

$$= 3\frac{3}{5} \qquad \text{Convert to a mixed fraction.}$$

51. Convert the mixed fractions to improper fractions, then divide.

$$
\begin{aligned}
\left(-3\frac{1}{2}\right) \div \left(1\frac{1}{16}\right) &= \left(-\frac{7}{2}\right) \div \left(\frac{17}{16}\right) && \text{Convert to improper fractions.} \\[2mm]
&= \left(-\frac{7}{2}\right) \cdot \left(\frac{16}{17}\right) && \text{Invert the second fraction and multiply.} \\[2mm]
&= -\frac{(7)\cdot(2\cdot2\cdot2\cdot2)}{(2)\cdot(17)} && \text{Prime factorization.} \\
& && \text{Unlike signs give a negative product.} \\[2mm]
&= -\frac{7\cdot2\cdot2\cdot2}{17} && \text{Cancel common factors.} \\[2mm]
&= -\frac{56}{17} && \text{Simplify.} \\[2mm]
&= -3\frac{5}{17} && \text{Convert to a mixed fraction.}
\end{aligned}
$$

53. Convert the mixed fractions to improper fractions, then divide.

$$
\begin{aligned}
6\frac{1}{2} \div 1\frac{7}{12} &= \frac{13}{2} \div \frac{19}{12} && \text{Convert to improper fractions.} \\[2mm]
&= \frac{13}{2} \cdot \frac{12}{19} && \text{Invert the second fraction and multiply.} \\[2mm]
&= \frac{(13)\cdot(2\cdot2\cdot3)}{(2)\cdot(19)} && \text{Prime factorization.} \\[2mm]
&= \frac{13\cdot2\cdot3}{19} && \text{Cancel common factors.} \\[2mm]
&= \frac{78}{19} && \text{Simplify.} \\[2mm]
&= 4\frac{2}{19} && \text{Convert to a mixed fraction.}
\end{aligned}
$$

55. Convert the mixed fractions to improper fractions, then divide.

$$(-4) \div \left(1\frac{5}{9}\right) = \left(-\frac{4}{1}\right) \div \left(\frac{14}{9}\right) \quad \text{Convert to improper fractions.}$$

$$= \left(-\frac{4}{1}\right) \cdot \left(\frac{9}{14}\right) \quad \text{Invert the second fraction and multiply.}$$

$$= -\frac{(2 \cdot 2) \cdot (3 \cdot 3)}{(1) \cdot (2 \cdot 7)} \quad \begin{array}{l} \text{Prime factorization.} \\ \text{Unlike signs give a negative product.} \end{array}$$

$$= -\frac{2 \cdot 3 \cdot 3}{7} \quad \text{Cancel common factors.}$$

$$= -\frac{18}{7} \quad \text{Simplify.}$$

$$= -2\frac{4}{7} \quad \text{Convert to a mixed fraction.}$$

57. Convert the mixed fractions to improper fractions, then divide.

$$\left(-5\frac{2}{3}\right) \div \left(-2\frac{1}{6}\right) = \left(-\frac{17}{3}\right) \div \left(-\frac{13}{6}\right) \quad \text{Convert to improper fractions.}$$

$$= \left(-\frac{17}{3}\right) \cdot \left(-\frac{6}{13}\right) \quad \text{Invert the second fraction and multiply.}$$

$$= \frac{(17) \cdot (2 \cdot 3)}{(3) \cdot (13)} \quad \begin{array}{l} \text{Prime factorization.} \\ \text{Like signs give a positive product.} \end{array}$$

$$= \frac{17 \cdot 2}{13} \quad \text{Cancel common factors.}$$

$$= \frac{34}{13} \quad \text{Simplify.}$$

$$= 2\frac{8}{13} \quad \text{Convert to a mixed fraction.}$$

59. Convert the mixed fractions to improper fractions, then divide.

$$\left(-6\frac{1}{2}\right) \div \left(4\frac{1}{4}\right) = \left(-\frac{13}{2}\right) \div \left(\frac{17}{4}\right) \quad \text{Convert to improper fractions.}$$

$$= \left(-\frac{13}{2}\right) \cdot \left(\frac{4}{17}\right) \quad \text{Invert the second fraction and multiply.}$$

$$= -\frac{(13) \cdot (2 \cdot 2)}{(2) \cdot (17)} \quad \begin{array}{l}\text{Prime factorization.} \\ \text{Unlike signs give a negative product.}\end{array}$$

$$= -\frac{13 \cdot 2}{17} \quad \text{Cancel common factors.}$$

$$= -\frac{26}{17} \quad \text{Simplify.}$$

$$= -1\frac{9}{17} \quad \text{Convert to a mixed fraction.}$$

61. Convert the mixed fractions to improper fractions, then divide.

$$(-6) \div \left(-1\frac{3}{11}\right) = \left(-\frac{6}{1}\right) \div \left(-\frac{14}{11}\right) \quad \text{Convert to improper fractions.}$$

$$= \left(-\frac{6}{1}\right) \cdot \left(-\frac{11}{14}\right) \quad \text{Invert the second fraction and multiply.}$$

$$= \frac{(2 \cdot 3) \cdot (11)}{(1) \cdot (2 \cdot 7)} \quad \begin{array}{l}\text{Prime factorization.} \\ \text{Like signs give a positive product.}\end{array}$$

$$= \frac{3 \cdot 11}{7} \quad \text{Cancel common factors.}$$

$$= \frac{33}{7} \quad \text{Simplify.}$$

$$= 4\frac{5}{7} \quad \text{Convert to a mixed fraction.}$$

63. Convert the mixed fractions to improper fractions, then divide.

$$\left(4\frac{2}{3}\right) \div (-4) = \left(\frac{14}{3}\right) \div \left(-\frac{4}{1}\right) \qquad \text{Convert to improper fractions.}$$

$$= \left(\frac{14}{3}\right) \cdot \left(-\frac{1}{4}\right) \qquad \text{Invert the second fraction and multiply.}$$

$$= -\frac{(2 \cdot 7) \cdot (1)}{(3) \cdot (2 \cdot 2)} \qquad \begin{array}{l}\text{Prime factorization.}\\ \text{Unlike signs give a negative product.}\end{array}$$

$$= -\frac{7}{3 \cdot 2} \qquad \text{Cancel common factors.}$$

$$= -\frac{7}{6} \qquad \text{Simplify.}$$

$$= -1\frac{1}{6} \qquad \text{Convert to a mixed fraction.}$$

65. Convert the mixed fractions to improper fractions, then divide.

$$\left(1\frac{3}{4}\right) \div \left(-1\frac{1}{12}\right) = \left(\frac{7}{4}\right) \div \left(-\frac{13}{12}\right) \qquad \text{Convert to improper fractions.}$$

$$= \left(\frac{7}{4}\right) \cdot \left(-\frac{12}{13}\right) \qquad \text{Invert the second fraction and multiply.}$$

$$= -\frac{(7) \cdot (2 \cdot 2 \cdot 3)}{(2 \cdot 2) \cdot (13)} \qquad \begin{array}{l}\text{Prime factorization.}\\ \text{Unlike signs give a negative product.}\end{array}$$

$$= -\frac{7 \cdot 3}{13} \qquad \text{Cancel common factors.}$$

$$= -\frac{21}{13} \qquad \text{Simplify.}$$

$$= -1\frac{8}{13} \qquad \text{Convert to a mixed fraction.}$$

67. Convert the mixed fractions to improper fractions, then divide.

$$
\begin{aligned}
5\frac{2}{3} \div 1\frac{1}{9} &= \frac{17}{3} \div \frac{10}{9} && \text{\color{red}Convert to improper fractions.}\\
&= \frac{17}{3} \cdot \frac{9}{10} && \text{\color{red}Invert the second fraction and multiply.}\\
&= \frac{(17) \cdot (3 \cdot 3)}{(3) \cdot (2 \cdot 5)} && \text{\color{red}Prime factorization.}\\
&= \frac{17 \cdot 3}{2 \cdot 5} && \text{\color{red}Cancel common factors.}\\
&= \frac{51}{10} && \text{\color{red}Simplify.}\\
&= 5\frac{1}{10} && \text{\color{red}Convert to a mixed fraction.}
\end{aligned}
$$

69. Convert the mixed fractions to improper fractions, then divide.

$$
\begin{aligned}
\left(-7\frac{1}{2}\right) \div \left(-2\frac{2}{5}\right) &= \left(-\frac{15}{2}\right) \div \left(-\frac{12}{5}\right) && \text{\color{red}Convert to improper fractions.}\\
&= \left(-\frac{15}{2}\right) \cdot \left(-\frac{5}{12}\right) && \text{\color{red}Invert the second fraction and multiply.}\\
&= \frac{(3 \cdot 5) \cdot (5)}{(2) \cdot (2 \cdot 2 \cdot 3)} && \text{\color{red}Prime factorization.}\\
& && \text{\color{red}Like signs give a positive product.}\\
&= \frac{5 \cdot 5}{2 \cdot 2 \cdot 2} && \text{\color{red}Cancel common factors.}\\
&= \frac{25}{8} && \text{\color{red}Simplify.}\\
&= 3\frac{1}{8} && \text{\color{red}Convert to a mixed fraction.}
\end{aligned}
$$

71. Convert the mixed fractions to improper fractions, then divide.

$$\left(3\frac{2}{3}\right) \div \left(-1\frac{1}{9}\right) = \left(\frac{11}{3}\right) \div \left(-\frac{10}{9}\right) \quad \text{Convert to improper fractions.}$$

$$= \left(\frac{11}{3}\right) \cdot \left(-\frac{9}{10}\right) \quad \text{Invert the second fraction and multiply.}$$

$$= -\frac{(11) \cdot (3 \cdot 3)}{(3) \cdot (2 \cdot 5)} \quad \begin{array}{l}\text{Prime factorization.}\\ \text{Unlike signs give a negative product.}\end{array}$$

$$= -\frac{11 \cdot 3}{2 \cdot 5} \quad \text{Cancel common factors.}$$

$$= -\frac{33}{10} \quad \text{Simplify.}$$

$$= -3\frac{3}{10} \quad \text{Convert to a mixed fraction.}$$

73. *Quarter-acre* means $\frac{1}{4}$ of an acre. To find the number of quarter-acres in $6\frac{1}{2}$ acres, divide the $6\frac{1}{2}$ by $\frac{1}{4}$.

$$6\frac{1}{2} \div \frac{1}{4} = \frac{13}{2} \div \frac{1}{4} \quad \text{Convert to an improper fraction.}$$

$$= \frac{13}{2} \cdot \frac{4}{1} \quad \text{Multiply by the reciprocal.}$$

$$= \frac{13 \cdot 4}{2} \quad \text{Multiply numerators; multilply denominators.}$$

$$= \frac{13 \cdot (2 \cdot 2)}{2} \quad \text{Factor numerator.}$$

$$= 26 \quad \text{Cancel common factor. Simplify.}$$

Therefore, there are 26 quarter-acre lots in $6\frac{1}{2}$ acres of land.

75. To find how many silver pieces $\frac{1}{12}$ inch long are contained in a bar $4\frac{1}{2}$ inches long, divide the $4\frac{1}{2}$ by $\frac{1}{12}$.

$$4\frac{1}{2} \div \frac{1}{12} = \frac{9}{2} \div \frac{1}{12} \quad \text{Convert to an improper fraction.}$$

$$= \frac{9}{2} \cdot \frac{12}{1} \quad \text{Multiply by the reciprocal.}$$

$$= \frac{9 \cdot 12}{2} \quad \text{Multiply numerators; multiply denominators.}$$

$$= \frac{(3 \cdot 3) \cdot (2 \cdot 2 \cdot 3)}{2} \quad \text{Factor the numerator and denominator.}$$

$$= 54 \quad \text{Cancel common factors and simplify.}$$

Therefore, 54 pieces were made from the silver bar.

4.6   Adding and Subtracting Mixed Fractions

1. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add:

$$9\frac{1}{4} + 9\frac{1}{2} = \frac{37}{4} + \frac{19}{2} \qquad \text{Convert to improper fractions.}$$

$$= \frac{37}{4} + \frac{19 \cdot 2}{2 \cdot 2} \qquad \text{Equivalent fractions with LCD} = 4.$$

$$= \frac{37}{4} + \frac{38}{4} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{75}{4} \qquad \text{Add numerators over common denominator.}$$

$$= 18\frac{3}{4} \qquad \text{Convert to a mixed fraction.}$$

3. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract:

$$6\frac{1}{2} - 1\frac{1}{3} = \frac{13}{2} - \frac{4}{3} \qquad \text{Convert to improper fractions.}$$

$$= \frac{13 \cdot 3}{2 \cdot 3} - \frac{4 \cdot 2}{3 \cdot 2} \qquad \text{Equivalent fractions with LCD} = 6.$$

$$= \frac{39}{6} - \frac{8}{6} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{31}{6} \qquad \text{Subtract numerators over common denominator.}$$

$$= 5\frac{1}{6} \qquad \text{Convert to a mixed fraction.}$$

5. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add:

$$9\frac{1}{2} + 7\frac{1}{4} = \frac{19}{2} + \frac{29}{4} \qquad \text{Convert to improper fractions.}$$

$$= \frac{19 \cdot 2}{2 \cdot 2} + \frac{29}{4} \qquad \text{Equivalent fractions with LCD} = 4.$$

$$= \frac{38}{4} + \frac{29}{4} \qquad \text{Simplify numerators and denominators.}$$

$$= \frac{67}{4} \qquad \text{Add numerators over common denominator.}$$

$$= 16\frac{3}{4} \qquad \text{Convert to a mixed fraction.}$$

7. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add:

$$
\begin{aligned}
5\frac{2}{3} + 4\frac{1}{2} &= \frac{17}{3} + \frac{9}{2} && \text{Convert to improper fractions.}\\[2mm]
&= \frac{17 \cdot 2}{3 \cdot 2} + \frac{9 \cdot 3}{2 \cdot 3} && \text{Equivalent fractions with LCD} = 6.\\[2mm]
&= \frac{34}{6} + \frac{27}{6} && \text{Simplify numerators and denominators.}\\[2mm]
&= \frac{61}{6} && \text{Add numerators over common denominator.}\\[2mm]
&= 10\frac{1}{6} && \text{Convert to a mixed fraction.}
\end{aligned}
$$

9. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract:

$$
\begin{aligned}
3\frac{1}{3} - 1\frac{1}{4} &= \frac{10}{3} - \frac{5}{4} && \text{Convert to improper fractions.}\\[2mm]
&= \frac{10 \cdot 4}{3 \cdot 4} - \frac{5 \cdot 3}{4 \cdot 3} && \text{Equivalent fractions with LCD} = 12.\\[2mm]
&= \frac{40}{12} - \frac{15}{12} && \text{Simplify numerators and denominators.}\\[2mm]
&= \frac{25}{12} && \text{Subtract numerators over common denominator.}\\[2mm]
&= 2\frac{1}{12} && \text{Convert to a mixed fraction.}
\end{aligned}
$$

11. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract:

$$
\begin{aligned}
8\frac{1}{2} - 1\frac{1}{3} &= \frac{17}{2} - \frac{4}{3} && \text{Convert to improper fractions.}\\[2mm]
&= \frac{17 \cdot 3}{2 \cdot 3} - \frac{4 \cdot 2}{3 \cdot 2} && \text{Equivalent fractions with LCD} = 6.\\[2mm]
&= \frac{51}{6} - \frac{8}{6} && \text{Simplify numerators and denominators.}\\[2mm]
&= \frac{43}{6} && \text{Subtract numerators over common denominator.}\\[2mm]
&= 7\frac{1}{6} && \text{Convert to a mixed fraction.}
\end{aligned}
$$

13. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract:

$$4\frac{1}{2} - 1\frac{1}{8} = \frac{9}{2} - \frac{9}{8} \qquad \text{\color{red}Convert to improper fractions.}$$

$$= \frac{9 \cdot 4}{2 \cdot 4} - \frac{9}{8} \qquad \text{\color{red}Equivalent fractions with LCD = 8.}$$

$$= \frac{36}{8} - \frac{9}{8} \qquad \text{\color{red}Simplify numerators and denominators.}$$

$$= \frac{27}{8} \qquad \text{\color{red}Subtract numerators over common denominator.}$$

$$= 3\frac{3}{8} \qquad \text{\color{red}Convert to a mixed fraction.}$$

15. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add:

$$4\frac{7}{8} + 1\frac{3}{4} = \frac{39}{8} + \frac{7}{4} \qquad \text{\color{red}Convert to improper fractions.}$$

$$= \frac{39}{8} + \frac{7 \cdot 2}{4 \cdot 2} \qquad \text{\color{red}Equivalent fractions with LCD = 8.}$$

$$= \frac{39}{8} + \frac{14}{8} \qquad \text{\color{red}Simplify numerators and denominators.}$$

$$= \frac{53}{8} \qquad \text{\color{red}Add numerators over common denominator.}$$

$$= 6\frac{5}{8} \qquad \text{\color{red}Convert to a mixed fraction.}$$

17. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract:

$$2\frac{1}{3} - 1\frac{1}{4} = \frac{7}{3} - \frac{5}{4} \qquad \text{\color{red}Convert to improper fractions.}$$

$$= \frac{7 \cdot 4}{3 \cdot 4} - \frac{5 \cdot 3}{4 \cdot 3} \qquad \text{\color{red}Equivalent fractions with LCD = 12.}$$

$$= \frac{28}{12} - \frac{15}{12} \qquad \text{\color{red}Simplify numerators and denominators.}$$

$$= \frac{13}{12} \qquad \text{\color{red}Subtract numerators over common denominator.}$$

$$= 1\frac{1}{12} \qquad \text{\color{red}Convert to a mixed fraction.}$$

19. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then subtract:

$$
\begin{aligned}
9\frac{1}{2} - 1\frac{3}{4} &= \frac{19}{2} - \frac{7}{4} && \text{\color{red}Convert to improper fractions.}\\
&= \frac{19 \cdot 2}{2 \cdot 2} - \frac{7}{4} && \text{\color{red}Equivalent fractions with LCD = 4.}\\
&= \frac{38}{4} - \frac{7}{4} && \text{\color{red}Simplify numerators and denominators.}\\
&= \frac{31}{4} && \text{\color{red}Subtract numerators over common denominator.}\\
&= 7\frac{3}{4} && \text{\color{red}Convert to a mixed fraction.}
\end{aligned}
$$

21. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add:

$$
\begin{aligned}
4\frac{2}{3} + 1\frac{1}{4} &= \frac{14}{3} + \frac{5}{4} && \text{\color{red}Convert to improper fractions.}\\
&= \frac{14 \cdot 4}{3 \cdot 4} + \frac{5 \cdot 3}{4 \cdot 3} && \text{\color{red}Equivalent fractions with LCD = 12.}\\
&= \frac{56}{12} + \frac{15}{12} && \text{\color{red}Simplify numerators and denominators.}\\
&= \frac{71}{12} && \text{\color{red}Add numerators over common denominator.}\\
&= 5\frac{11}{12} && \text{\color{red}Convert to a mixed fraction.}
\end{aligned}
$$

23. Change the mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add:

$$
\begin{aligned}
9\frac{1}{2} + 3\frac{1}{8} &= \frac{19}{2} + \frac{25}{8} && \text{\color{red}Convert to improper fractions.}\\
&= \frac{19 \cdot 4}{2 \cdot 4} + \frac{25}{8} && \text{\color{red}Equivalent fractions with LCD = 8.}\\
&= \frac{76}{8} + \frac{25}{8} && \text{\color{red}Simplify numerators and denominators.}\\
&= \frac{101}{8} && \text{\color{red}Add numerators over common denominator.}\\
&= 12\frac{5}{8} && \text{\color{red}Convert to a mixed fraction.}
\end{aligned}
$$

25. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$3\,\frac{1}{2} \quad = \quad 3\,\frac{1\cdot 2}{2\cdot 2} \quad = \quad 3\,\frac{2}{4}$$

$$+3\,\frac{3}{4} \quad = \quad +3\,\frac{3}{4} \quad = \quad +3\,\frac{3}{4}$$

$$6\,\frac{5}{4}$$

Since 5/4 is an improper fraction, we must convert it to a mixed fraction and then add:

$$6\,\frac{5}{4} = 6 + 1\,\frac{1}{4}$$

$$= 7\,\frac{1}{4}$$

27. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$1\,\frac{3}{8} \quad = \quad 1\,\frac{3}{8} \quad = \quad 1\,\frac{3}{8}$$

$$+1\,\frac{1}{4} \quad = \quad +1\,\frac{1\cdot 2}{4\cdot 2} \quad = \quad +1\,\frac{2}{8}$$

$$2\,\frac{5}{8}$$

29. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$1\,\frac{7}{8} \quad = \quad 1\,\frac{7}{8} \quad = \quad 1\,\frac{7}{8}$$

$$+1\,\frac{1}{2} \quad = \quad +1\,\frac{1\cdot 4}{2\cdot 4} \quad = \quad +1\,\frac{4}{8}$$

$$2\,\frac{11}{8}$$

Since 11/8 is an improper fraction, we must convert it to a mixed fraction and then add:

$$2\,\frac{11}{8} = 2 + 1\,\frac{3}{8}$$

$$= 3\,\frac{3}{8}$$

31. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{array}{rcccc}
8\dfrac{1}{2} & = & 8\dfrac{1\cdot 3}{2\cdot 3} & = & 8\dfrac{3}{6} \\[3ex]
-5\dfrac{2}{3} & = & -5\dfrac{2\cdot 2}{3\cdot 2} & = & -5\dfrac{4}{6} \\[2ex]
\hline
& & & & 2\dfrac{5}{6}
\end{array}
$$

Since $3/6 < 4/6$, we cannot subtract $4/6$ from $3/6$. Therefore, we must borrow 1 from 8 in the form of $6/6$ and add it to the $3/6$:

$$
\begin{array}{rcccc}
8\dfrac{3}{6} & = & 7 + \dfrac{6}{6} + \dfrac{3}{6} & = & 7\dfrac{9}{6} \\[3ex]
-5\dfrac{4}{6} & = & -5\dfrac{4}{6} & = & -5\dfrac{4}{6} \\[2ex]
\hline
& & & & 2\dfrac{5}{6}
\end{array}
$$

33. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{array}{rcccc}
7\dfrac{1}{2} & = & 7\dfrac{1\cdot 8}{2\cdot 8} & = & 7\dfrac{8}{16} \\[3ex]
-1\dfrac{3}{16} & = & -1\dfrac{3}{16} & = & -1\dfrac{3}{16} \\[2ex]
\hline
& & & & 6\dfrac{5}{16}
\end{array}
$$

35. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{array}{rcccc}
9\dfrac{1}{2} & = & 9\dfrac{1\cdot 3}{2\cdot 3} & = & 9\dfrac{3}{6} \\[3ex]
-1\dfrac{1}{3} & = & -1\dfrac{1\cdot 2}{3\cdot 2} & = & -1\dfrac{2}{6} \\[2ex]
\hline
& & & & 8\dfrac{1}{6}
\end{array}
$$

37. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{aligned}
5\frac{1}{3} &= 5\frac{1\cdot 2}{3\cdot 2} = 5\frac{2}{6}\\
-2\frac{1}{2} &= -2\frac{1\cdot 3}{2\cdot 3} = -2\frac{3}{6}\\
\hline
& \qquad\qquad\qquad 2\frac{5}{6}
\end{aligned}
$$

Since $2/6 < 3/6$, we cannot subtract $3/6$ from $2/6$. Therefore, we must borrow 1 from 5 in the form of $6/6$ and add it to the $2/6$:

$$
\begin{aligned}
5\frac{2}{6} &= 4+\frac{6}{6}+\frac{2}{6} = 4\frac{8}{6}\\
-2\frac{3}{6} &= \qquad -2\frac{3}{6} = -2\frac{3}{6}\\
\hline
& \qquad\qquad\qquad 2\frac{5}{6}
\end{aligned}
$$

39. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{aligned}
9\frac{1}{2} &= 9\frac{1\cdot 3}{2\cdot 3} = 9\frac{3}{6}\\
-2\frac{2}{3} &= -2\frac{2\cdot 2}{3\cdot 2} = -2\frac{4}{6}\\
\hline
& \qquad\qquad\qquad 6\frac{5}{6}
\end{aligned}
$$

Since $3/6 < 4/6$, we cannot subtract $4/6$ from $3/6$. Therefore, we must borrow 1 from 9 in the form of $6/6$ and add it to the $3/6$:

$$
\begin{aligned}
9\frac{3}{6} &= 8+\frac{6}{6}+\frac{3}{6} = 8\frac{9}{6}\\
-2\frac{4}{6} &= \qquad -2\frac{4}{6} = -2\frac{4}{6}\\
\hline
& \qquad\qquad\qquad 6\frac{5}{6}
\end{aligned}
$$

41. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{array}{rcl}
1\dfrac{1}{16} & = & 1\dfrac{1}{16} & = & 1\dfrac{1}{16} \\[2ex]
+1\dfrac{3}{4} & = & +1\dfrac{3\cdot 4}{4\cdot 4} & = & +1\dfrac{12}{16} \\[2ex]
\hline
& & & & 2\dfrac{13}{16}
\end{array}
$$

43. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{array}{rcl}
8\dfrac{1}{2} & = & 8\dfrac{1\cdot 3}{2\cdot 3} & = & 8\dfrac{3}{6} \\[2ex]
+3\dfrac{2}{3} & = & +3\dfrac{2\cdot 2}{3\cdot 2} & = & +3\dfrac{4}{6} \\[2ex]
\hline
& & & & 11\dfrac{7}{6}
\end{array}
$$

Since 7/6 is an improper fraction, we must convert it to a mixed fraction and then add:

$$
\begin{aligned}
11\dfrac{7}{6} &= 11 + 1\dfrac{1}{6} \\
&= 12\dfrac{1}{6}
\end{aligned}
$$

45. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{array}{rcl}
6\dfrac{1}{2} & = & 6\dfrac{1\cdot 8}{2\cdot 8} & = & 6\dfrac{8}{16} \\[2ex]
-1\dfrac{3}{16} & = & -1\dfrac{3}{16} & = & -1\dfrac{3}{16} \\[2ex]
\hline
& & & & 5\dfrac{5}{16}
\end{array}
$$

47. Working in vertical format, first make equivalent fractions with a common denominator, and then add the whole number and fractional parts:

$$
\begin{aligned}
2\frac{2}{3} \quad &= \quad 2\frac{2\cdot 4}{3\cdot 4} \quad = \quad 2\frac{8}{12} \\
+1\frac{1}{4} \quad &= \quad +1\frac{1\cdot 3}{4\cdot 3} \quad = \quad +1\frac{3}{12} \\
\hline
& \qquad\qquad\qquad\qquad\qquad 3\frac{11}{12}
\end{aligned}
$$

4.7   Order of Operations with Fractions

1. Apply exponents first, then multiply.

$$
\begin{aligned}
\left(-\frac{7}{3}\right)^3 &= \left(-\frac{7}{3}\right)\left(-\frac{7}{3}\right)\left(-\frac{7}{3}\right) & a^3 = a\cdot a\cdot a. \\
&= -\frac{7\cdot 7\cdot 7}{3\cdot 3\cdot 3} & \text{Multiply numerators and denominators.} \\
& & \text{An odd number of negative factors is negative.} \\
&= -\frac{343}{27} & \text{Simplify.}
\end{aligned}
$$

3. Apply exponents first, then multiply.

$$
\begin{aligned}
\left(\frac{5}{3}\right)^4 &= \left(\frac{5}{3}\right)\left(\frac{5}{3}\right)\left(\frac{5}{3}\right)\left(\frac{5}{3}\right) & a^4 = a\cdot a\cdot a\cdot a. \\
&= \frac{5\cdot 5\cdot 5\cdot 5}{3\cdot 3\cdot 3\cdot 3} & \text{Multiply numerators and denominators.} \\
&= \frac{625}{81} & \text{Simplify.}
\end{aligned}
$$

5. Apply exponents first, then multiply.

$$
\begin{aligned}
\left(\frac{1}{2}\right)^5 &= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) & a^5 = a\cdot a\cdot a\cdot a\cdot a. \\
&= \frac{1\cdot 1\cdot 1\cdot 1\cdot 1}{2\cdot 2\cdot 2\cdot 2\cdot 2} & \text{Multiply numerators and denominators.} \\
&= \frac{1}{32} & \text{Simplify.}
\end{aligned}
$$

7. Apply exponents first, then multiply.

$$\left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)\left(\frac{4}{3}\right) \qquad a^2 = a \cdot a.$$

$$= \frac{4 \cdot 4}{3 \cdot 3} \qquad \text{Multiply numerators and denominators.}$$

$$= \frac{16}{9} \qquad \text{Simplify.}$$

9. Substitute $a = 7/6$ in the expression a^3, then simplify.

$$a^3 = \left(\frac{7}{6}\right)^3 \qquad \text{Substitute } a = 7/6.$$

$$= \left(\frac{7}{6}\right)\left(\frac{7}{6}\right)\left(\frac{7}{6}\right) \qquad \left(\frac{7}{6}\right)^3 = \left(\frac{7}{6}\right)\left(\frac{7}{6}\right)\left(\frac{7}{6}\right).$$

$$= \frac{7 \cdot 7 \cdot 7}{6 \cdot 6 \cdot 6} \qquad \text{Multiply numerators and denominators.}$$

$$= \frac{343}{216} \qquad \text{Simplify.}$$

11. Substitute $e = -2/3$ in the expression $-e^2$, then simplify. Note that order of operations demands that we evaluate the exponent before negating.

$$-e^2 = -\left(-\frac{2}{3}\right)^2 \qquad \text{Substitute } e = -2/3.$$

$$= -\left[\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\right] \qquad \left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right).$$

$$= -\left[\frac{2 \cdot 2}{3 \cdot 3}\right] \qquad \begin{array}{l}\text{Multiply numerators and denominators.}\\ \text{An even number of negative factors is positive.}\end{array}$$

$$= -\frac{4}{9} \qquad \text{Simplify.}$$

13. Substitute $b = -5/9$ in the expression b^2, then simplify.

$$b^2 = \left(-\frac{5}{9}\right)^2 \qquad \text{Substitute } b = -5/9.$$

$$= \left(-\frac{5}{9}\right)\left(-\frac{5}{9}\right) \qquad \left(-\frac{5}{9}\right)^2 = \left(-\frac{5}{9}\right)\left(-\frac{5}{9}\right).$$

$$= \frac{5 \cdot 5}{9 \cdot 9} \qquad \begin{array}{l}\text{Multiply numerators and denominators.}\\ \text{An even number of negative factors is positive.}\end{array}$$

$$= \frac{25}{81} \qquad \text{Simplify.}$$

15. Substitute $b = -1/2$ in the expression $-b^3$, then simplify. Note that order of operations demands that we evaluate the exponent before negating.

$$-b^3 = -\left(-\frac{1}{2}\right)^3 \qquad\qquad \text{Substitute } b = -1/2.$$

$$= -\left[\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\right] \quad \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right).$$

$$= -\left[-\frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2}\right] \qquad \text{Multiply numerators and denominators.}$$
$$\qquad\qquad\qquad\qquad\qquad \text{An odd number of negative factors is negative.}$$

$$= \frac{1}{8} \qquad\qquad\qquad \text{Simplify.}$$

17. Multiply first, then add or subtract as needed.

$$\left(-\frac{1}{2}\right)\left(\frac{1}{6}\right) - \left(\frac{7}{8}\right)\left(-\frac{7}{9}\right) = \left(-\frac{1}{12}\right) - \left(-\frac{49}{72}\right) \quad \text{Multiply.}$$

$$= -\frac{6}{72} + \frac{49}{72} \qquad\qquad \text{Rewrite with a common denominator.}$$

$$= \frac{43}{72} \qquad\qquad\qquad \text{Add over common denominator.}$$

19. First evaluate exponents and multiply, then add or subtract as needed.

$$\left(-\frac{9}{8}\right)^2 - \left(-\frac{3}{2}\right)\left(\frac{7}{3}\right) = \frac{81}{64} - \left(-\frac{7}{2}\right) \quad \text{Evaluate exponents and multiply.}$$

$$= \frac{81}{64} + \frac{224}{64} \qquad\qquad \text{Rewrite with a common denominator.}$$

$$= \frac{305}{64} \qquad\qquad\qquad \text{Add over common denominator.}$$

21. First evaluate exponents and multiply, then subtract.

$$\left(-\frac{1}{2}\right)\left(-\frac{7}{4}\right) - \left(-\frac{1}{2}\right)^2 = \frac{7}{8} - \frac{1}{4} \quad \text{Evaluate exponents and multiply.}$$

$$= \frac{7}{8} - \frac{2}{8} \qquad \text{Rewrite with a common denominator.}$$

$$= \frac{5}{8} \qquad\qquad \text{Subtract over common denominator.}$$

23. Multiply first, then subtract.

$$-\frac{7}{6} - \frac{1}{7} \cdot \frac{7}{9} = -\frac{7}{6} - \frac{1}{9} \qquad \text{Multiply.}$$
$$= -\frac{21}{18} - \frac{2}{18} \qquad \text{Rewrite with a common denominator.}$$
$$= -\frac{23}{18} \qquad \text{Subtract over common denominator.}$$

25. Multiply first, then add or subtract as needed.

$$\frac{3}{4} + \frac{9}{7}\left(-\frac{7}{6}\right) = \frac{3}{4} + \left(-\frac{3}{2}\right) \qquad \text{Multiply.}$$
$$= \frac{3}{4} - \frac{6}{4} \qquad \text{Rewrite with a common denominator.}$$
$$= -\frac{3}{4} \qquad \text{Subtract over common denominator.}$$

27. First evaluate exponents and multiply, then add or subtract as needed.

$$\left(-\frac{1}{3}\right)^2 + \left(\frac{7}{8}\right)\left(-\frac{1}{3}\right) = \frac{1}{9} + \left(-\frac{7}{24}\right) \qquad \text{Evaluate exponents and multiply.}$$
$$= \frac{8}{72} - \frac{21}{72} \qquad \text{Rewrite with a common denominator.}$$
$$= -\frac{13}{72} \qquad \text{Subtract over common denominator.}$$

29. Multiply first, then add.

$$\frac{5}{9} + \frac{5}{9} \cdot \frac{7}{9} = \frac{5}{9} + \frac{35}{81} \qquad \text{Multiply.}$$
$$= \frac{45}{81} + \frac{35}{81} \qquad \text{Rewrite with a common denominator.}$$
$$= \frac{80}{81} \qquad \text{Add over common denominator.}$$

31. Multiply first, then add or subtract as needed.

$$\left(-\frac{5}{6}\right)\left(\frac{3}{8}\right) + \left(-\frac{7}{9}\right)\left(-\frac{3}{4}\right) = \left(-\frac{5}{16}\right) + \frac{7}{12} \qquad \text{Multiply.}$$
$$= -\frac{15}{48} + \frac{28}{48} \qquad \text{Rewrite with a common denominator.}$$
$$= \frac{13}{48} \qquad \text{Add over common denominator.}$$

33. Multiply first, then add or subtract as needed.

$$\frac{4}{3} - \frac{2}{9}\left(-\frac{3}{4}\right) = \frac{4}{3} - \left(-\frac{1}{6}\right) \qquad \text{Multiply.}$$

$$= \frac{8}{6} + \frac{1}{6} \qquad \text{Rewrite with a common denominator.}$$

$$= \frac{9}{6} \qquad \text{Add over common denominator.}$$

$$= \frac{3}{2} \qquad \text{Simplify.}$$

35. First evaluate exponents and multiply, then add.

$$\left(-\frac{5}{9}\right)\left(\frac{1}{2}\right) + \left(-\frac{1}{6}\right)^2 = \left(-\frac{5}{18}\right) + \frac{1}{36} \qquad \text{Evaluate exponents and multiply.}$$

$$= -\frac{10}{36} + \frac{1}{36} \qquad \text{Rewrite with a common denominator.}$$

$$= -\frac{9}{36} \qquad \text{Add over common denominator.}$$

$$= -\frac{1}{4} \qquad \text{Simplify.}$$

37. Substitute $a = -5/4$, $b = 1/2$, and $c = 3/8$. To evaluate the resulting expression, multiply first, then add.

$$a + bc = -\frac{5}{4} + \left(\frac{1}{2}\right)\left(\frac{3}{8}\right) \qquad \text{Substitute.}$$

$$= -\frac{5}{4} + \frac{3}{16} \qquad \text{Multiply.}$$

$$= -\frac{20}{16} + \frac{3}{16} \qquad \text{Rewrite with a common denominator.}$$

$$= -\frac{17}{16} \qquad \text{Add over common denominator.}$$

39. Substitute $x = -1/8$, $y = 5/2$, and $z = -1/2$. In the resulting expression, multiply first, then add or subtract as needed.

$$x + yz = -\frac{1}{8} + \left(\frac{5}{2}\right)\left(-\frac{1}{2}\right) \qquad \text{Substitute.}$$

$$= -\frac{1}{8} + \left(-\frac{5}{4}\right) \qquad \text{Multiply.}$$

$$= -\frac{1}{8} - \frac{10}{8} \qquad \text{Rewrite with a common denominator.}$$

$$= -\frac{11}{8} \qquad \text{Subtract over common denominator.}$$

41. Substitute $a = 3/4$, $b = 5/7$, and $c = 1/2$. In the resulting expression, multiply first, then subtract.

$$
\begin{aligned}
a - bc &= \frac{3}{4} - \frac{5}{7} \cdot \frac{1}{2} && \text{Substitute.} \\
&= \frac{3}{4} - \frac{5}{14} && \text{Multiply.} \\
&= \frac{21}{28} - \frac{10}{28} && \text{Rewrite with a common denominator.} \\
&= \frac{11}{28} && \text{Subtract over common denominator.}
\end{aligned}
$$

43. Substitute $x = -3/2$, $y = 1/4$, and $z = -5/7$. In the resulting expression, first evaluate exponents and multiply, then add or subtract as needed.

$$
\begin{aligned}
x^2 - yz &= \left(-\frac{3}{2}\right)^2 - \left(\frac{1}{4}\right)\left(-\frac{5}{7}\right) && \text{Substitute.} \\
&= \frac{9}{4} - \left(-\frac{5}{28}\right) && \text{Evaluate exponents and multiply.} \\
&= \frac{63}{28} + \frac{5}{28} && \text{Rewrite with a common denominator.} \\
&= \frac{68}{28} && \text{Add over common denominator.} \\
&= \frac{17}{7} && \text{Simplify.}
\end{aligned}
$$

45. Substitute $a = 6/7$, $b = 2/3$, $c = -8/9$, and $d = -6/7$. In the resulting expression, multiply first, then add or subtract as needed.

$$
\begin{aligned}
ab + cd &= \left(\frac{6}{7}\right)\left(\frac{2}{3}\right) + \left(-\frac{8}{9}\right)\left(-\frac{6}{7}\right) && \text{Substitute.} \\
&= \frac{4}{7} + \frac{16}{21} && \text{Multiply.} \\
&= \frac{12}{21} + \frac{16}{21} && \text{Rewrite with a common denominator.} \\
&= \frac{28}{21} && \text{Add over common denominator.} \\
&= \frac{4}{3} && \text{Simplify.}
\end{aligned}
$$

47. Substitute $w = -1/8$, $x = -2/7$, $y = -1/2$, and $z = 8/7$. In the resulting expression, multiply first, then add or subtract as needed.

$$
\begin{aligned}
wx - yz = &= \left(-\frac{1}{8}\right)\left(-\frac{2}{7}\right) - \left(-\frac{1}{2}\right)\left(\frac{8}{7}\right) && \text{\color{red}Substitute.} \\
&= \frac{1}{28} - \left(-\frac{4}{7}\right) && \text{\color{red}Multiply.} \\
&= \frac{1}{28} + \frac{16}{28} && \text{\color{red}Rewrite with a common denominator.} \\
&= \frac{17}{28} && \text{\color{red}Add over common denominator.}
\end{aligned}
$$

49. Substitute $x = 3/8$, $y = 3/5$, and $z = -3/2$. In the resulting expression, first evaluate exponents and multiply, then add.

$$
\begin{aligned}
xy + z^2 &= \left(\frac{3}{8}\right)\left(\frac{3}{5}\right) + \left(-\frac{3}{2}\right)^2 && \text{\color{red}Substitute.} \\
&= \frac{9}{40} + \frac{9}{4} && \text{\color{red}Evaluate exponents and multiply.} \\
&= \frac{9}{40} + \frac{90}{40} && \text{\color{red}Rewrite with a common denominator.} \\
&= \frac{99}{40} && \text{\color{red}Add over common denominator.}
\end{aligned}
$$

51. Substitute $u = 9/7$, $v = 2/3$, and $w = -3/7$. In the resulting expression, first evaluate exponents and multiply, then subtract.

$$
\begin{aligned}
uv - w^2 &= \left(\frac{9}{7}\right)\left(\frac{2}{3}\right) - \left(-\frac{3}{7}\right)^2 && \text{\color{red}Substitute.} \\
&= \frac{6}{7} - \frac{9}{49} && \text{\color{red}Evaluate exponents and multiply.} \\
&= \frac{42}{49} - \frac{9}{49} && \text{\color{red}Rewrite with a common denominator.} \\
&= \frac{33}{49} && \text{\color{red}Subtract over common denominator.}
\end{aligned}
$$

53. Substitute $a = 7/8$, $b = -1/4$, and $c = -3/2$. In the resulting expression,

first evaluate exponents and multiply, then add or subtract as needed.

$$a^2 + bc = \left(\frac{7}{8}\right)^2 + \left(-\frac{1}{4}\right)\left(-\frac{3}{2}\right) \qquad \text{Substitute.}$$

$$= \frac{49}{64} + \frac{3}{8} \qquad\qquad\qquad \text{Evaluate exponents and multiply.}$$

$$= \frac{49}{64} + \frac{24}{64} \qquad\qquad\quad \text{Rewrite with a common denominator.}$$

$$= \frac{73}{64} \qquad\qquad\qquad\quad \text{Add over common denominator.}$$

55. Substitute $u = 1/3$, $v = 5/2$, and $w = -2/9$. In the resulting expression, multiply first, then add or subtract as needed.

$$u - vw = \frac{1}{3} - \left(\frac{5}{2}\right)\left(-\frac{2}{9}\right) \qquad \text{Substitute.}$$

$$= \frac{1}{3} - \left(-\frac{5}{9}\right) \qquad\qquad \text{Multiply.}$$

$$= \frac{3}{9} + \frac{5}{9} \qquad\qquad\quad \text{Rewrite with a common denominator.}$$

$$= \frac{8}{9} \qquad\qquad\qquad\quad \text{Add over common denominator.}$$

57. Start by multiplying the main numerator and denominator by the least common denominator (LCD) of the four small fractions. Then simplify the numerator and denominator.

$$\frac{\dfrac{8}{3} + \dfrac{7}{6}}{-\dfrac{9}{2} - \dfrac{1}{4}} = \frac{12\left(\dfrac{8}{3} + \dfrac{7}{6}\right)}{12\left(-\dfrac{9}{2} - \dfrac{1}{4}\right)} \qquad \text{Multiply numerator and denominator by the LCD} = 12.$$

$$= \frac{12\left(\dfrac{8}{3}\right) + 12\left(\dfrac{7}{6}\right)}{12\left(-\dfrac{9}{2}\right) - 12\left(\dfrac{1}{4}\right)} \qquad \text{Distribute.}$$

$$= \frac{32 + 14}{-54 - 3} \qquad\qquad\qquad \text{Multiply.}$$

$$= \frac{46}{-57} \qquad\qquad\qquad\quad \text{Simplify numerator and denominator.}$$

$$= -\frac{46}{57} \qquad\qquad\qquad\quad \text{Simplify.}$$

59. Start by multiplying the main numerator and denominator by the least common denominator (LCD) of the four small fractions. Then simplify the numerator and denominator.

$$
\frac{\dfrac{3}{4}+\dfrac{4}{3}}{\dfrac{1}{9}+\dfrac{5}{3}}=\frac{36\left(\dfrac{3}{4}+\dfrac{4}{3}\right)}{36\left(\dfrac{1}{9}+\dfrac{5}{3}\right)}
$$
Multiply numerator and denominator by the LCD $= 36$.

$$
=\frac{36\left(\dfrac{3}{4}\right)+36\left(\dfrac{4}{3}\right)}{36\left(\dfrac{1}{9}\right)+36\left(\dfrac{5}{3}\right)}
$$
Distribute.

$$
=\frac{27+48}{4+60}
$$
Multiply.

$$
=\frac{75}{64}
$$
Simplify numerator and denominator.

61. Start by multiplying the main numerator and denominator by the least common denominator (LCD) of the four small fractions. Then simplify the numerator and denominator.

$$
\frac{\dfrac{7}{5}+\dfrac{5}{2}}{-\dfrac{1}{4}+\dfrac{1}{2}}=\frac{20\left(\dfrac{7}{5}+\dfrac{5}{2}\right)}{20\left(-\dfrac{1}{4}+\dfrac{1}{2}\right)}
$$
Multiply numerator and denominator by the LCD $= 20$.

$$
=\frac{20\left(\dfrac{7}{5}\right)+20\left(\dfrac{5}{2}\right)}{20\left(-\dfrac{1}{4}\right)+20\left(\dfrac{1}{2}\right)}
$$
Distribute.

$$
=\frac{28+50}{-5+10}
$$
Multiply.

$$
=\frac{78}{5}
$$
Simplify numerator and denominator.

63. Start by multiplying the main numerator and denominator by the least common denominator (LCD) of the four small fractions. Then simplify the

numerator and denominator.

$$\frac{-\dfrac{3}{2}-\dfrac{2}{3}}{-\dfrac{7}{4}-\dfrac{2}{3}} = \frac{12\left(-\dfrac{3}{2}-\dfrac{2}{3}\right)}{12\left(-\dfrac{7}{4}-\dfrac{2}{3}\right)} \qquad \text{Multiply numerator and denominator by the LCD} = 12.$$

$$= \frac{12\left(-\dfrac{3}{2}\right)-12\left(\dfrac{2}{3}\right)}{12\left(-\dfrac{7}{4}\right)-12\left(\dfrac{2}{3}\right)} \qquad \text{Distribute.}$$

$$= \frac{-18-8}{-21-8} \qquad \text{Multiply.}$$

$$= \frac{-26}{-29} \qquad \text{Simplify numerator and denominator.}$$

$$= \frac{26}{29} \qquad \text{Simplify.}$$

65. Start by multiplying the main numerator and denominator by the least common denominator (LCD) of the four small fractions. Then simplify the numerator and denominator.

$$\frac{-\dfrac{1}{2}-\dfrac{4}{7}}{-\dfrac{5}{7}+\dfrac{1}{6}} = \frac{42\left(-\dfrac{1}{2}-\dfrac{4}{7}\right)}{42\left(-\dfrac{5}{7}+\dfrac{1}{6}\right)} \qquad \text{Multiply numerator and denominator by the LCD} = 42.$$

$$= \frac{42\left(-\dfrac{1}{2}\right)-42\left(\dfrac{4}{7}\right)}{42\left(-\dfrac{5}{7}\right)+42\left(\dfrac{1}{6}\right)} \qquad \text{Distribute.}$$

$$= \frac{-21-24}{-30+7} \qquad \text{Multiply.}$$

$$= \frac{-45}{-23} \qquad \text{Simplify numerator and denominator.}$$

$$= \frac{45}{23} \qquad \text{Simplify.}$$

67. Start by multiplying the main numerator and denominator by the least common denominator (LCD) of the four small fractions. Then simplify the

numerator and denominator.

$$\frac{-\dfrac{3}{7}-\dfrac{1}{3}}{\dfrac{1}{3}-\dfrac{6}{7}}=\frac{21\left(-\dfrac{3}{7}-\dfrac{1}{3}\right)}{21\left(\dfrac{1}{3}-\dfrac{6}{7}\right)}$$ Multiply numerator and denominator by the LCD $= 21$.

$$=\frac{21\left(-\dfrac{3}{7}\right)-21\left(\dfrac{1}{3}\right)}{21\left(\dfrac{1}{3}\right)-21\left(\dfrac{6}{7}\right)}$$ Distribute.

$$=\frac{-9-7}{7-18}$$ Multiply.

$$=\frac{-16}{-11}$$ Simplify numerator and denominator.

$$=\frac{16}{11}$$ Simplify.

69. The formula for the area of a trapezoid is

$$A=\frac{1}{2}h\left(b_1+b_2\right)$$

Substituting the given bases and height, we get

$$A=\frac{1}{2}(7)\left(3\frac{3}{8}+5\frac{1}{2}\right).$$

Simplify the expression inside the parentheses first. Change mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add.

$$A=\frac{1}{2}(7)\left(\frac{27}{8}+\frac{11}{2}\right)$$

$$=\frac{1}{2}(7)\left(\frac{27\cdot 1}{8\cdot 1}+\frac{11\cdot 4}{2\cdot 4}\right)$$

$$=\frac{1}{2}(7)\left(\frac{27}{8}+\frac{44}{8}\right)$$

$$=\frac{1}{2}\left(\frac{7}{1}\right)\left(\frac{71}{8}\right)$$

Multiply numerators and denominators.

$$=\frac{497}{16}$$

This improper fraction is a perfectly good answer, but let's change this result to a mixed fraction (497 divided by 16 is 31 with a remainder of 1). Thus, the area of the trapezoid is

$$A = 31\frac{1}{16} \text{ square feet.}$$

71. The formula for the area of a trapezoid is

$$A = \frac{1}{2}h\left(b_1 + b_2\right)$$

Substituting the given bases and height, we get

$$A = \frac{1}{2}(7)\left(2\frac{1}{4} + 7\frac{3}{8}\right).$$

Simplify the expression inside the parentheses first. Change mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add.

$$\begin{aligned}
A &= \frac{1}{2}(7)\left(\frac{9}{4} + \frac{59}{8}\right) \\
&= \frac{1}{2}(7)\left(\frac{9 \cdot 2}{4 \cdot 2} + \frac{59 \cdot 1}{8 \cdot 1}\right) \\
&= \frac{1}{2}(7)\left(\frac{18}{8} + \frac{59}{8}\right) \\
&= \frac{1}{2}\left(\frac{7}{1}\right)\left(\frac{77}{8}\right)
\end{aligned}$$

Multiply numerators and denominators.

$$= \frac{539}{16}$$

This improper fraction is a perfectly good answer, but let's change this result to a mixed fraction (539 divided by 16 is 33 with a remainder of 11). Thus, the area of the trapezoid is

$$A = 33\frac{11}{16} \text{ square feet.}$$

73. The formula for the area of a trapezoid is

$$A = \frac{1}{2}h\left(b_1 + b_2\right)$$

Substituting the given bases and height, we get

$$A = \frac{1}{2}(3)\left(2\frac{3}{4} + 6\frac{5}{8}\right).$$

Simplify the expression inside the parentheses first. Change mixed fractions to improper fractions, make equivalent fractions with a common denominator, then add.

$$A = \frac{1}{2}(3)\left(\frac{11}{4} + \frac{53}{8}\right)$$
$$= \frac{1}{2}(3)\left(\frac{11 \cdot 2}{4 \cdot 2} + \frac{53 \cdot 1}{8 \cdot 1}\right)$$
$$= \frac{1}{2}(3)\left(\frac{22}{8} + \frac{53}{8}\right)$$
$$= \frac{1}{2}\left(\frac{3}{1}\right)\left(\frac{75}{8}\right)$$

Multiply numerators and denominators.

$$= \frac{225}{16}$$

This improper fraction is a perfectly good answer, but let's change this result to a mixed fraction (225 divided by 16 is 14 with a remainder of 1). Thus, the area of the trapezoid is

$$A = 14\frac{1}{16} \text{ square feet.}$$

4.8 Solving Equations with Fractions

1. To see if $1/4$ is a solution, we substitute $1/4$ for x in the equation and check to see if this results in a true or false statement.

$$x + \frac{5}{8} = \frac{5}{8} \qquad \text{Original equation.}$$
$$\frac{1}{4} + \frac{5}{8} = \frac{5}{8} \qquad \text{Substitute } x = 1/4.$$
$$\frac{1 \cdot 2}{4 \cdot 2} + \frac{5}{8} = \frac{5}{8} \qquad \text{Equivalent fractions on the left with LCD} = 8.$$
$$\frac{2}{8} + \frac{5}{8} = \frac{5}{8} \qquad \text{Simplify numerators and denominators.}$$
$$\frac{2 + 5}{8} = \frac{5}{8} \qquad \text{Add numerators over common denominator.}$$
$$\frac{7}{8} = \frac{5}{8} \qquad \text{Simplify numerator.}$$

This last statement is a false statement. Therefore, $1/4$ is **not** a solution of the equation.

3. To see if $-8/15$ is a solution, we substitute $-8/15$ for x in the equation and check to see if this results in a true or false statement.

$$\left(\frac{1}{4}\right)\left(-\frac{8}{15}\right) = -\frac{1}{15} \qquad \text{Original equation.}$$

$$-\frac{(1)\cdot(2\cdot2\cdot2)}{(2\cdot2)\cdot(3\cdot5)} = -\frac{1}{15} \qquad \text{Prime factorization.}$$
$$\text{Unlike signs give a negative product.}$$

$$-\frac{2}{3\cdot5} = -\frac{1}{15} \qquad \text{Cancel common factors.}$$

$$-\frac{2}{15} = -\frac{1}{15} \qquad \text{Multiply numerators and denominators.}$$

This last statement is a false statement. Therefore, $1/4$ is **not** a solution of the equation.

5. To see if $1/2$ is a solution, we substitute $1/2$ for x in the equation and check to see if this results in a true or false statement.

$$x + \frac{4}{9} = \frac{17}{18} \qquad \text{Original equation.}$$

$$\frac{1}{2} + \frac{4}{9} = \frac{17}{18} \qquad \text{Substitute } x = 1/2.$$

$$\frac{1\cdot9}{2\cdot9} + \frac{4\cdot2}{9\cdot2} = \frac{17}{18} \qquad \text{Equivalent fractions on the left with LCD} = 18.$$

$$\frac{9}{18} + \frac{8}{18} = \frac{17}{18} \qquad \text{Simplify numerators and denominators.}$$

$$\frac{9+8}{18} = \frac{17}{18} \qquad \text{Add numerators over common denominator.}$$

$$\frac{17}{18} = \frac{17}{18} \qquad \text{Simplify numerator.}$$

This last statement is a true statement. Therefore, $1/2$ is a solution of the equation.

7. To see if $3/8$ is a solution, we substitute $3/8$ for x in the equation and check to see if this results in a true or false statement.

$$x - \frac{5}{9} = -\frac{13}{72} \qquad \text{Original equation.}$$

$$\frac{3}{8} - \frac{5}{9} = -\frac{13}{72} \qquad \text{Substitute } x = 3/8.$$

$$\frac{3 \cdot 9}{8 \cdot 9} - \frac{5 \cdot 8}{9 \cdot 8} = -\frac{13}{72} \qquad \text{Equivalent fractions on the left with LCD} = 72.$$

$$\frac{27}{72} - \frac{40}{72} = -\frac{13}{72} \qquad \text{Simplify numerators and denominators.}$$

$$\frac{27 - 40}{72} = -\frac{13}{72} \qquad \text{Subtract numerators over common denominator.}$$

$$\frac{-13}{72} = -\frac{13}{72} \qquad \text{Simplify numerator.}$$

This last statement is a true statement. Therefore, $3/8$ is a solution of the equation.

9. To see if $2/7$ is a solution, we substitute $2/7$ for x in the equation and check to see if this results in a true or false statement.

$$x - \frac{4}{9} = -\frac{8}{63} \qquad \text{Original equation.}$$

$$\frac{2}{7} - \frac{4}{9} = -\frac{8}{63} \qquad \text{Substitute } x = 2/7.$$

$$\frac{2 \cdot 9}{7 \cdot 9} - \frac{4 \cdot 7}{9 \cdot 7} = -\frac{8}{63} \qquad \text{Equivalent fractions on the left with LCD} = 63.$$

$$\frac{18}{63} - \frac{28}{63} = -\frac{8}{63} \qquad \text{Simplify numerators and denominators.}$$

$$\frac{18 - 28}{63} = -\frac{8}{63} \qquad \text{Subtract numerators over common denominator.}$$

$$\frac{-10}{63} = -\frac{8}{63} \qquad \text{Simplify numerator.}$$

This last statement is a false statement. Therefore, $2/7$ is **not** a solution of the equation.

11. To see if $8/5$ is a solution, we substitute $8/5$ for x in the equation and check to see if this results in a true or false statement.

$$\left(\frac{11}{14}\right)\left(\frac{8}{5}\right) = \frac{44}{35} \qquad \text{Original equation.}$$

$$\frac{(11)\cdot(2\cdot2\cdot2)}{(2\cdot7)\cdot(5)} = \frac{44}{35} \qquad \begin{array}{l}\text{Prime factorization.}\\ \text{Like signs give a positive product.}\end{array}$$

$$\frac{11\cdot2\cdot2}{7\cdot5} = \frac{44}{35} \qquad \text{Cancel common factors.}$$

$$\frac{44}{35} = \frac{44}{35} \qquad \text{Multiply numerators and denominators.}$$

This last statement is a true statement. Therefore, $8/5$ is a solution of the equation.

13.

$$\begin{array}{ll}
2x - 3 = 6x + 7 & \text{Original equation.} \\
2x - 6x - 3 = 7 & \text{Add } -6x \text{ to both sides.} \\
-4x - 3 = 7 & \text{Combine like terms on the left side.} \\
-4x = 7 + 3 & \text{Add 3 to both sides.} \\
-4x = 10 & \text{Combine like terms on the right side.} \\
x = -\dfrac{10}{4} & \text{Divide both sides by } -4. \\
x = -\dfrac{5}{2} & \text{Simplify.}
\end{array}$$

15.

$$\begin{array}{ll}
-7x + 4 = 3x & \text{Original equation.} \\
4 = 3x + 7x & \text{Add } 7x \text{ to both sides.} \\
4 = 10x & \text{Combine like terms on the right side.} \\
\dfrac{4}{10} = x & \text{Divide both sides by 10.} \\
\dfrac{2}{5} = x & \text{Simplify.}
\end{array}$$

17.

$$\begin{array}{ll}
-2x = 9x - 4 & \text{Original equation.} \\
-2x - 9x = -4 & \text{Add } -9x \text{ to both sides.} \\
-11x = -4 & \text{Combine like terms on the left side.} \\
x = \dfrac{4}{11} & \text{Divide both sides by } -11.
\end{array}$$

19.

$$-8x = 7x - 7 \qquad \text{Original equation.}$$
$$-8x - 7x = -7 \qquad \text{Add } -7x \text{ to both sides.}$$
$$-15x = -7 \qquad \text{Combine like terms on the left side.}$$
$$x = \frac{7}{15} \qquad \text{Divide both sides by } -15.$$

21.

$$-7x + 8 = 2x \qquad \text{Original equation.}$$
$$8 = 2x + 7x \qquad \text{Add } 7x \text{ to both sides.}$$
$$8 = 9x \qquad \text{Combine like terms on the right side.}$$
$$\frac{8}{9} = x \qquad \text{Divide both sides by } 9.$$

23.

$$-9x + 4 = 4x - 6 \qquad \text{Original equation.}$$
$$-9x - 4x + 4 = -6 \qquad \text{Add } -4x \text{ to both sides.}$$
$$-13x + 4 = -6 \qquad \text{Combine like terms on the left side.}$$
$$-13x = -6 - 4 \qquad \text{Add } -4 \text{ to both sides.}$$
$$-13x = -10 \qquad \text{Combine like terms on the right side.}$$
$$x = \frac{10}{13} \qquad \text{Divide both sides by } -13.$$

25.

$$x + \frac{3}{2} = \frac{1}{2} \qquad \text{Original equation.}$$
$$2\left(x + \frac{3}{2}\right) = 2\left(\frac{1}{2}\right) \qquad \text{Multiply both sides by the LCD} = 2.$$
$$2x + 2\left(\frac{3}{2}\right) = 2\left(\frac{1}{2}\right) \qquad \text{On the left, distribute 2.}$$
$$2x + 3 = 1 \qquad \text{Cancel and simplify.}$$
$$2x = -2 \qquad \text{Add } -3 \text{ to both sides.}$$
$$x = -\frac{2}{2} \qquad \text{Divide both sides by 2.}$$
$$x = -1 \qquad \text{Simplify.}$$

27.

$$-\frac{9}{5}x = \frac{1}{2} \qquad \text{Original equation.}$$

$$10\left(-\frac{9}{5}\right) = 10\left(\frac{1}{2}\right) \qquad \text{Multiply both sides by the LCD} = 10.$$

$$-18x = 5 \qquad \text{Cancel and simplify.}$$

$$x = -\frac{5}{18} \qquad \text{Divide both sides by } -18.$$

29.

$$\frac{3}{8}x = \frac{8}{7} \qquad \text{Original equation.}$$

$$56\left(\frac{3}{8}\right) = 56\left(\frac{8}{7}\right) \qquad \text{Multiply both sides by the LCD} = 56.$$

$$21x = 64 \qquad \text{Cancel and simplify.}$$

$$x = \frac{64}{21} \qquad \text{Divide both sides by 21.}$$

31.

$$\frac{2}{5}x = -\frac{1}{6} \qquad \text{Original equation.}$$

$$30\left(\frac{2}{5}\right) = 30\left(-\frac{1}{6}\right) \qquad \text{Multiply both sides by the LCD} = 30.$$

$$12x = -5 \qquad \text{Cancel and simplify.}$$

$$x = -\frac{5}{12} \qquad \text{Divide both sides by 12.}$$

33.

$$-\frac{3}{2}x = \frac{8}{7} \qquad \text{Original equation.}$$

$$14\left(-\frac{3}{2}\right) = 14\left(\frac{8}{7}\right) \qquad \text{Multiply both sides by the LCD} = 14.$$

$$-21x = 16 \qquad \text{Cancel and simplify.}$$

$$x = -\frac{16}{21} \qquad \text{Divide both sides by } -21.$$

35.

$$x + \frac{3}{4} = \frac{5}{9} \qquad \text{Original equation.}$$

$$36\left(x + \frac{3}{4}\right) = 36\left(\frac{5}{9}\right) \qquad \text{Multiply both sides by the LCD} = 36.$$

$$36x + 36\left(\frac{3}{4}\right) = 36\left(\frac{5}{9}\right) \qquad \text{On the left, distribute 36.}$$

$$36x + 27 = 20 \qquad \text{Cancel and simplify.}$$

$$36x = -7 \qquad \text{Add } -27 \text{ to both sides.}$$

$$x = -\frac{7}{36} \qquad \text{Divide both sides by 36.}$$

37.

$$x - \frac{4}{7} = \frac{7}{8} \qquad \text{Original equation.}$$

$$56\left(x - \frac{4}{7}\right) = 56\left(\frac{7}{8}\right) \qquad \text{Multiply both sides by the LCD} = 56.$$

$$56x - 56\left(\frac{4}{7}\right) = 56\left(\frac{7}{8}\right) \qquad \text{On the left, distribute 56.}$$

$$56x - 32 = 49 \qquad \text{Cancel and simplify.}$$

$$56x = 81 \qquad \text{Add 32 to both sides.}$$

$$x = \frac{81}{56} \qquad \text{Divide both sides by 56.}$$

39.

$$x + \frac{8}{9} = \frac{2}{3} \qquad \text{Original equation.}$$

$$9\left(x + \frac{8}{9}\right) = 9\left(\frac{2}{3}\right) \qquad \text{Multiply both sides by the LCD} = 9.$$

$$9x + 9\left(\frac{8}{9}\right) = 9\left(\frac{2}{3}\right) \qquad \text{On the left, distribute 9.}$$

$$9x + 8 = 6 \qquad \text{Cancel and simplify.}$$

$$9x = -2 \qquad \text{Add } -8 \text{ to both sides.}$$

$$x = -\frac{2}{9} \qquad \text{Divide both sides by 9.}$$

41.

$$x + \frac{5}{2} = -\frac{9}{8}$$ Original equation.

$$8\left(x + \frac{5}{2}\right) = 8\left(-\frac{9}{8}\right)$$ Multiply both sides by the LCD = 8.

$$8x + 8\left(\frac{5}{2}\right) = 8\left(-\frac{9}{8}\right)$$ On the left, distribute 8.

$$8x + 20 = -9$$ Cancel and simplify.

$$8x = -29$$ Add −20 to both sides.

$$x = -\frac{29}{8}$$ Divide both sides by 8.

43.

$$-\frac{8}{5}x = \frac{7}{9}$$ Original equation.

$$45\left(-\frac{8}{5}\right) = 45\left(\frac{7}{9}\right)$$ Multiply both sides by the LCD = 45.

$$-72x = 35$$ Cancel and simplify.

$$x = -\frac{35}{72}$$ Divide both sides by −72.

45.

$$x - \frac{1}{4} = -\frac{1}{8}$$ Original equation.

$$8\left(x - \frac{1}{4}\right) = 8\left(-\frac{1}{8}\right)$$ Multiply both sides by the LCD = 8.

$$8x - 8\left(\frac{1}{4}\right) = 8\left(-\frac{1}{8}\right)$$ On the left, distribute 8.

$$8x - 2 = -1$$ Cancel and simplify.

$$8x = 1$$ Add 2 to both sides.

$$x = \frac{1}{8}$$ Divide both sides by 8.

47.

$$-\frac{1}{4}x = \frac{1}{2}$$ Original equation.

$$4\left(-\frac{1}{4}\right) = 4\left(\frac{1}{2}\right)$$ Multiply both sides by the LCD = 4.

$$-x = 2$$ Cancel and simplify.

$$x = -\frac{2}{1}$$ Divide both sides by -1.

$$x = 2$$ Simplify.

49.

$$-\frac{7}{3}x - \frac{2}{3} = \frac{3}{4}x + \frac{2}{3}$$ Original equation.

$$12\left(-\frac{7}{3}x - \frac{2}{3}\right) = 12\left(\frac{3}{4}x + \frac{2}{3}\right)$$ Multiply both sides by the LCD = 12.

$$12\left(-\frac{7}{3}x\right) - 12\left(\frac{2}{3}\right) = 12\left(\frac{3}{4}x\right) + 12\left(\frac{2}{3}\right)$$ On both sides, distribute 12.

$$-28x - 8 = 9x + 8$$ Cancel and simplify.

$$-37x - 8 = 8$$ Add $-9x$ to both sides.

$$-37x = 16$$ Add 8 to both sides.

$$x = -\frac{16}{37}$$ Divide both sides by -37.

51.

$$-\frac{7}{2}x - \frac{5}{4} = \frac{4}{5}$$ Original equation.

$$20\left(-\frac{7}{2}x - \frac{5}{4}\right) = 20\left(\frac{4}{5}\right)$$ Multiply both sides by the LCD = 20.

$$20\left(-\frac{7}{2}x\right) - 20\left(\frac{5}{4}\right) = 20\left(\frac{4}{5}\right)$$ On the left, distribute 20.

$$-70x - 25 = 16$$ Cancel and simplify.

$$-70x = 41$$ Add 25 to both sides.

$$x = -\frac{41}{70}$$ Divide both sides by -70.

53.

$$-\frac{9}{7}x + \frac{9}{2} = -\frac{5}{2}$$ Original equation.

$$14\left(-\frac{9}{7}x + \frac{9}{2}\right) = 14\left(-\frac{5}{2}\right)$$ Multiply both sides by the LCD = 14.

$$14\left(-\frac{9}{7}x\right) + 14\left(\frac{9}{2}\right) = 14\left(-\frac{5}{2}\right)$$ On the left, distribute 14.

$$-18x + 63 = -35$$ Cancel and simplify.

$$-18x = -98$$ Add -63 to both sides.

$$x = \frac{98}{18}$$ Divide both sides by -18.

$$x = \frac{49}{9}$$ Simplify.

55.

$$\frac{1}{4}x - \frac{4}{3} = -\frac{2}{3}$$ Original equation.

$$12\left(\frac{1}{4}x - \frac{4}{3}\right) = 12\left(-\frac{2}{3}\right)$$ Multiply both sides by the LCD = 12.

$$12\left(\frac{1}{4}x\right) - 12\left(\frac{4}{3}\right) = 12\left(-\frac{2}{3}\right)$$ On the left, distribute 12.

$$3x - 16 = -8$$ Cancel and simplify.

$$3x = 8$$ Add 16 to both sides.

$$x = \frac{8}{3}$$ Divide both sides by 3.

57.

$$\frac{5}{3}x + \frac{3}{2} = -\frac{1}{4}$$ Original equation.

$$12\left(\frac{5}{3}x + \frac{3}{2}\right) = 12\left(-\frac{1}{4}\right)$$ Multiply both sides by the LCD = 12.

$$12\left(\frac{5}{3}x\right) + 12\left(\frac{3}{2}\right) = 12\left(-\frac{1}{4}\right)$$ On the left, distribute 12.

$$20x + 18 = -3$$ Cancel and simplify.

$$20x = -21$$ Add -18 to both sides.

$$x = -\frac{21}{20}$$ Divide both sides by 20.

59.

$$-\frac{1}{3}x + \frac{4}{5} = -\frac{9}{5}x - \frac{5}{6}$$ Original equation.

$$30\left(-\frac{1}{3}x + \frac{4}{5}\right) = 30\left(-\frac{9}{5}x - \frac{5}{6}\right)$$ Multiply both sides by the LCD = 30.

$$30\left(-\frac{1}{3}x\right) + 30\left(\frac{4}{5}\right) = 30\left(-\frac{9}{5}x\right) - 30\left(-\frac{5}{6}\right)$$ On both sides, distribute 30.

$$-10x + 24 = -54x - 25$$ Cancel and simplify.

$$44x + 24 = -25$$ Add $54x$ to both sides.

$$44x = -49$$ Add -24 to both sides.

$$x = -\frac{49}{44}$$ Divide both sides by 44.

61.

$$-\frac{4}{9}x - \frac{8}{9} = \frac{1}{2}x - \frac{1}{2}$$ Original equation.

$$18\left(-\frac{4}{9}x - \frac{8}{9}\right) = 18\left(\frac{1}{2}x - \frac{1}{2}\right)$$ Multiply both sides by the LCD = 18.

$$18\left(-\frac{4}{9}x\right) - 18\left(\frac{8}{9}\right) = 18\left(\frac{1}{2}x\right) - 18\left(-\frac{1}{2}\right)$$ On both sides, distribute 18.

$$-8x - 16 = 9x - 9$$ Cancel and simplify.

$$-17x - 16 = -9$$ Add $-9x$ to both sides.

$$-17x = 7$$ Add 16 to both sides.

$$x = -\frac{7}{17}$$ Divide both sides by -17.

63.

$$\frac{1}{2}x - \frac{1}{8} = -\frac{1}{8}x + \frac{5}{7}$$ Original equation.

$$56\left(\frac{1}{2}x - \frac{1}{8}\right) = 56\left(-\frac{1}{8}x + \frac{5}{7}\right)$$ Multiply both sides by the LCD = 56.

$$56\left(\frac{1}{2}x\right) - 56\left(\frac{1}{8}\right) = 56\left(-\frac{1}{8}x\right) + 56\left(\frac{5}{7}\right)$$ On both sides, distribute 56.

$$28x - 7 = -7x + 40$$ Cancel and simplify.

$$35x - 7 = 40$$ Add $7x$ to both sides.

$$35x = 47$$ Add 7 to both sides.

$$x = \frac{47}{35}$$ Divide both sides by 35.

65.

$$-\frac{3}{7}x - \frac{1}{3} = -\frac{1}{9}$$ Original equation.

$$63\left(-\frac{3}{7}x - \frac{1}{3}\right) = 63\left(-\frac{1}{9}\right)$$ Multiply both sides by the LCD = 63.

$$63\left(-\frac{3}{7}x\right) - 63\left(\frac{1}{3}\right) = 63\left(-\frac{1}{9}\right)$$ On the left, distribute 63.

$$-27x - 21 = -7$$ Cancel and simplify.

$$-27x = 14$$ Add 21 to both sides.

$$x = -\frac{14}{27}$$ Divide both sides by -27.

67.

$$-\frac{3}{4}x + \frac{2}{7} = \frac{8}{7}x - \frac{1}{3}$$ Original equation.

$$84\left(-\frac{3}{4}x + \frac{2}{7}\right) = 84\left(\frac{8}{7}x - \frac{1}{3}\right)$$ Multiply both sides by the LCD = 84.

$$84\left(-\frac{3}{4}x\right) + 84\left(\frac{2}{7}\right) = 84\left(\frac{8}{7}x\right) - 84\left(-\frac{1}{3}\right)$$ On both sides, distribute 84.

$$-63x + 24 = 96x - 28$$ Cancel and simplify.

$$-159x + 24 = -28$$ Add $-96x$ to both sides.

$$-159x = -52$$ Add -24 to both sides.

$$x = \frac{52}{159}$$ Divide both sides by -159.

69.

$$-\frac{3}{4}x - \frac{2}{3} = -\frac{2}{3}x - \frac{1}{2}$$ Original equation.

$$12\left(-\frac{3}{4}x - \frac{2}{3}\right) = 12\left(-\frac{2}{3}x - \frac{1}{2}\right)$$ Multiply both sides by the LCD = 12.

$$12\left(-\frac{3}{4}x\right) - 12\left(\frac{2}{3}\right) = 12\left(-\frac{2}{3}x\right) - 12\left(-\frac{1}{2}\right)$$ On both sides, distribute 12.

$$-9x - 8 = -8x - 6$$ Cancel and simplify.

$$-x - 8 = -6$$ Add $8x$ to both sides.

$$-x = 2$$ Add 8 to both sides.

$$x = -\frac{2}{1}$$ Divide both sides by -1.

$$x = -2$$ Simplify.

71.

$$-\frac{5}{2}x + \frac{9}{5} = \frac{5}{8} \qquad \text{Original equation.}$$

$$40\left(-\frac{5}{2}x + \frac{9}{5}\right) = 40\left(\frac{5}{8}\right) \qquad \text{Multiply both sides by the LCD} = 40.$$

$$40\left(-\frac{5}{2}x\right) + 40\left(\frac{9}{5}\right) = 40\left(\frac{5}{8}\right) \qquad \text{On the left, distribute 40.}$$

$$-100x + 72 = 25 \qquad \text{Cancel and simplify.}$$

$$-100x = -47 \qquad \text{Add } -72 \text{ to both sides.}$$

$$x = \frac{47}{100} \qquad \text{Divide both sides by } -100.$$

73. We are told that 2/9 of full seating capacity is 4,302.

1. *Set up a Variable Dictionary.* Let F represent the full seating capacity.

2. *Set up an Equation.* 2/9 of the full seating capacity is 4,302.

$\frac{2}{9}$	of	Full Seating Capacity	is	4,302
$\frac{2}{9}$	$\cdot$	F	$=$	4,302

Hence, the equation is

$$\frac{2}{9}F = 4302.$$

3. *Solve the Equation.* Multiply both sides by 9 to clear fractions, then solve.

$$\frac{2}{9}F = 4302 \qquad \text{Original equation.}$$

$$9\left(\frac{2}{9}F\right) = 9(4302) \qquad \text{Multiply both sides by 9.}$$

$$2F = 38718 \qquad \text{Simplify both sides.}$$

$$\frac{2F}{2} = \frac{38718}{2} \qquad \text{Divide both sides by 2.}$$
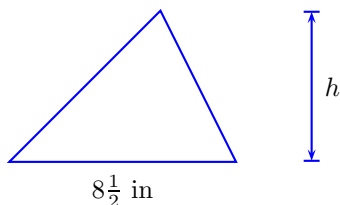
$$F = 19359 \qquad \text{Simplify both sides.}$$

4. *Answer the Question.* The full seating capacity is 19,359.

5. *Look Back.* The words of the problem state that 2/9 of the seating capacity is 4,302. Let's take 2/9 of our answer and see what we get.

$$\frac{2}{9} \cdot 19,359 = 4,302$$

This is the correct attendance, so our solution is correct.

75. We follow the requirements for word problems in our solution.

1. *Set up a Variable Dictionary.* Our variable dictionary will take the form of a well labeled diagram.

$8\frac{1}{2}$ in

2. *Set up an Equation.* The area A of a triangle with base b and height h is

$$A = \frac{1}{2}bh.$$

Substitute $A = 51$ and $b = 8\frac{1}{2}$.

$$51 = \frac{1}{2}\left(8\frac{1}{2}\right)h.$$

3. *Solve the Equation.* Change the mixed fraction to an improper fraction, then simplify.

$$51 = \frac{1}{2}\left(8\frac{1}{2}\right)h \qquad \text{Original equation.}$$

$$51 = \frac{1}{2}\left(\frac{17}{2}\right)h \qquad \text{Mixed to improper: } 8\frac{1}{2} = \frac{17}{2}.$$

$$51 = \left(\frac{1}{2}\cdot\frac{17}{2}\right)h \qquad \text{Associative property.}$$

$$51 = \frac{17}{4}h \qquad \text{Multiply: } \frac{1}{2}\cdot\frac{17}{2} = \frac{17}{4}.$$

Now, multiply both sides by 4/17 and solve.

$$\frac{4}{17}(51) = \frac{4}{17}\left(\frac{17}{4}h\right) \qquad \text{Multiply both sides by 4/17.}$$

$$12 = h \qquad \text{Simplify: } \frac{4}{17}(51) = 12$$

4. *Answer the Question.* The height of the triangle is 12 inches.

5. *Look Back.* If the height is 12 inches and the base is $8\frac{1}{2}$ inches, then the area is

$$A = \frac{1}{2}\left(8\frac{1}{2}\right)(12)$$
$$= \frac{1}{2} \cdot \frac{17}{2} \cdot \frac{12}{1}$$
$$= 51$$

This is the correct area (51 square inches), so our solution is correct.

77. We follow the requirements for word problems in our solution.

1. *Set up a Variable Dictionary.* Our variable dictionary will take the form of a well labeled diagram.

$4\frac{1}{2}$ in

2. *Set up an Equation.* The area A of a triangle with base b and height h is

$$A = \frac{1}{2}bh.$$

Substitute $A = 18$ and $b = 4\frac{1}{2}$.

$$18 = \frac{1}{2}\left(4\frac{1}{2}\right)h.$$

3. *Solve the Equation.* Change the mixed fraction to an improper fraction, then simplify.

$$18 = \frac{1}{2}\left(4\frac{1}{2}\right)h \qquad \text{Original equation.}$$
$$18 = \frac{1}{2}\left(\frac{9}{2}\right)h \qquad \text{Mixed to improper: } 4\frac{1}{2} = \frac{9}{2}.$$
$$18 = \left(\frac{1}{2} \cdot \frac{9}{2}\right)h \qquad \text{Associative property.}$$
$$18 = \frac{9}{4}h \qquad \text{Multiply: } \frac{1}{2} \cdot \frac{9}{2} = \frac{9}{4}.$$

Now, multiply both sides by 4/9 and solve.

$$\frac{4}{9}(18) = \frac{4}{9}\left(\frac{9}{4}h\right) \qquad \text{\color{red}{Multiply both sides by 4/9.}}$$

$$8 = h \qquad \text{\color{red}{Simplify: } \frac{4}{9}(18) = 8}$$

4. *Answer the Question.* The height of the triangle is 8 inches.
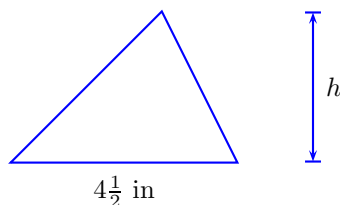
5. *Look Back.* If the height is 8 inches and the base is $4\frac{1}{2}$ inches, then the area is

$$A = \frac{1}{2}\left(4\frac{1}{2}\right)(8)$$
$$= \frac{1}{2} \cdot \frac{9}{2} \cdot \frac{8}{1}$$
$$= 18$$

This is the correct area (18 square inches), so our solution is correct.

79. We are told that 2/11 of full seating capacity is 4,536.

1. *Set up a Variable Dictionary.* Let F represent the full seating capacity.

2. *Set up an Equation.* 2/11 of the full seating capacity is 4,536.

$\frac{2}{11}$	of	Full Seating Capacity	is	4,536
$\frac{2}{11}$	$\cdot$	F	$=$	4,536

Hence, the equation is

$$\frac{2}{11}F = 4536.$$

3. *Solve the Equation.* Multiply both sides by 11 to clear fractions, then solve.

$$\frac{2}{11}F = 4536 \qquad \text{\color{red}{Original equation.}}$$

$$11\left(\frac{2}{11}F\right) = 11(4536) \qquad \text{\color{red}{Multiply both sides by 11.}}$$

$$2F = 49896 \qquad \text{\color{red}{Simplify both sides.}}$$

$$\frac{2F}{2} = \frac{49896}{2} \qquad \text{\color{red}{Divide both sides by 2.}}$$

$$F = 24948 \qquad \text{\color{red}{Simplify both sides.}}$$

4. *Answer the Question.* The full seating capacity is 24,948.

5. *Look Back.* The words of the problem state that 2/11 of the seating capacity is 4,536. Let's take 2/11 of our answer and see what we get.

$$\frac{2}{11} \cdot 24,948 = 4,536$$

This is the correct attendance, so our solution is correct.

81. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can satisfy this requirement by simply stating "Let x represent the number of pirate attacks worldwide in 2008."

2. *Set up an Equation.* "One-third of number of pirate attacks worldwide were off the Somali coast" becomes

one-third	of	number pirate attacks worldwide	were	number attacks off Somali coast
$\frac{1}{3}$	$\cdot$	x	$=$	111

3. *Solve the Equation.* To "undo" the multiplication of a fraction, multiply both sides of the equation by the reciprocal.

$$\frac{1}{3} \cdot x = 111 \qquad \text{Original equation.}$$
$$\frac{3}{1} \cdot \frac{1}{3} \cdot x = 111 \cdot \frac{3}{1} \qquad \text{Multiply both sides of the equation by } \frac{3}{1}.$$
$$x = 333 \qquad \text{On the left, multiplying by 3/1 "undoes" the effect}$$

of multiplying by 1/3 and returns x. On the right, $111 \cdot \frac{3}{1} = 333$.

4. *Answer the Question.* In 2008, the number of pirate attacks worldwide was 333.

5. *Look Back.* Do 333 pirate attacks worldwide satisfy the words in the original problem? We were told that "About one-third of the number of pirate attacks worldwide were off the Somali coast." Well, one-third of 333 is 111.

83. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can satisfy this requirement by simply stating "Let x represent the total number of the world's crop seeds."

2. *Set up an Equation.* "One-third of the world's crop seeds is half a million" becomes

one-third	of	number of world's crop seeds	is	number of seeds in seed vault
$\dfrac{1}{3}$	$\cdot$	x	$=$	500,000

3. *Solve the Equation.* To "undo" the multiplication of a fraction, multiply both sides of the equation by the reciprocal.

$$\frac{1}{3} \cdot x = 500000 \qquad \text{Original equation.}$$

$$\frac{3}{1} \cdot \frac{1}{3} \cdot x = 500000 \cdot \frac{3}{1} \qquad \text{Multiply both sides of the equation by } \frac{3}{1}.$$

$$x = 1500000 \qquad \text{On the left, multiplying by 3/1 "undoes" the effect of multiplying by 1/3 and returns } x. \text{ On the right,}$$

$$500000 \cdot \frac{3}{1} = 1500000.$$

4. *Answer the Question.* The number of crop seeds in the world is estimated at $1,500,000$.

5. *Look Back.* Does $1,500,000$ crop seeds worldwide make sense? We were told that "At least one-third of the world's crop seeds are $500,000$." Well, one-third of $1,500,000$ is $500,000$.

Chapter 5

Decimals

5.1 Introduction to Decimals

1. The tenths column is the first column after the decimal point, so the correct digit is 0.

3. The tenths column is the first column after the decimal point, so the correct digit is 2.

5. The ten-thousandths column is the fourth column after the decimal point, so the correct digit is 7.

7. The hundredths column is the second column after the decimal point, so the correct digit is 4.

9. The hundredths column is the second column after the decimal point, so the correct digit is 1.

11. The tenths column is the first column after the decimal point, so the correct digit is 3.

13. In expanded form,

$$46.139 = 40 + 6 + \frac{1}{10} + \frac{3}{100} + \frac{9}{1000}$$

15. In expanded form,

$$643.19 = 600 + 40 + 3 + \frac{1}{10} + \frac{9}{100}$$

17. In expanded form,

$$14.829 = 10 + 4 + \frac{8}{10} + \frac{2}{100} + \frac{9}{1000}$$

19. In expanded form,

$$658.71 = 600 + 50 + 8 + \frac{7}{10} + \frac{1}{100}$$

21. First we expand 32.187, then we sum whole number and fractional parts over a common denominator.

$$\begin{aligned}
32.187 &= 30 + 2 + \frac{1}{10} + \frac{8}{100} + \frac{7}{1000} \\
&= 32 + \frac{1 \cdot 100}{10 \cdot 100} + \frac{8 \cdot 10}{100 \cdot 10} + \frac{7}{1000} \\
&= 32 + \frac{100}{1000} + \frac{80}{1000} + \frac{7}{1000} \\
&= 32 + \frac{187}{1000}
\end{aligned}$$

23. First we expand 36.754, then we sum whole number and fractional parts over a common denominator.

$$\begin{aligned}
36.754 &= 30 + 6 + \frac{7}{10} + \frac{5}{100} + \frac{4}{1000} \\
&= 36 + \frac{7 \cdot 100}{10 \cdot 100} + \frac{5 \cdot 10}{100 \cdot 10} + \frac{4}{1000} \\
&= 36 + \frac{700}{1000} + \frac{50}{1000} + \frac{4}{1000} \\
&= 36 + \frac{754}{1000}
\end{aligned}$$

25. First we expand 596.71, then we sum whole number and fractional parts over a common denominator.

$$\begin{aligned}
596.71 &= 500 + 90 + 6 + \frac{7}{10} + \frac{1}{100} \\
&= 596 + \frac{7 \cdot 10}{10 \cdot 10} + \frac{7}{100} \\
&= 596 + \frac{70}{100} + \frac{1}{100} \\
&= 596 + \frac{71}{100}
\end{aligned}$$

27. First we expand 527.49, then we sum whole number and fractional parts over a common denominator.

$$527.49 = 500 + 20 + 7 + \frac{4}{10} + \frac{9}{100}$$
$$= 527 + \frac{4 \cdot 10}{10 \cdot 10} + \frac{4}{100}$$
$$= 527 + \frac{40}{100} + \frac{9}{100}$$
$$= 527 + \frac{49}{100}$$

29. In the number 0.9837, the last digit occurs in the ten-thousandths place. There is no whole number part, so we'll omit pronunciation of the whole number part and the word "and." Pronounce the fractional part as if it were a whole number and end with the word "ten-thousandths." Thus, we pronounce 0.9837 as "nine thousand eight hundred thirty-seven ten-thousandths."

31. In the number 0.2653, the last digit occurs in the ten-thousandths place. There is no whole number part, so we'll omit pronunciation of the whole number part and the word "and." Pronounce the fractional part as if it were a whole number and end with the word "ten-thousandths." Thus, we pronounce 0.2653 as "two thousand six hundred fifty-three ten-thousandths."

33. In the number 925.47, the last digit occurs in the hundredths place. Pronounce the whole number part, then say "and" for the decimal point. Pronounce the fractional part as if it were a whole number and end with the word "hundredths." Thus, we pronounce 925.47 as "nine hundred twenty-five and forty-seven hundredths."

35. In the number 83.427, the last digit occurs in the thousandths place. Pronounce the whole number part, then say "and" for the decimal point. Pronounce the fractional part as if it were a whole number and end with the word "thousandths." Thus, we pronounce 83.427 as "eighty-three and four hundred twenty-seven thousandths."

37. In the number 63.729, the last digit occurs in the thousandths place. Pronounce the whole number part, then say "and" for the decimal point. Pronounce the fractional part as if it were a whole number and end with the word "thousandths." Thus, we pronounce 63.729 as "sixty-three and seven hundred twenty-nine thousandths."

39. In the number 826.57, the last digit occurs in the hundredths place. Pronounce the whole number part, then say "and" for the decimal point. Pronounce the fractional part as if it were a whole number and end with the word "hundredths." Thus, we pronounce 826.57 as "eight hundred twenty-six and fifty-seven hundredths."

41. There is one decimal place after the decimal point, so there will be 1 zero in the denominator of the fractional part. Thus,

$$98.1 = 98\frac{1}{10}$$

43. There is one decimal place after the decimal point, so there will be 1 zero in the denominator of the fractional part. Thus,

$$781.7 = 781\frac{7}{10}$$

45. There are three decimal places after the decimal point, so there will be 3 zeros in the denominator of the fractional part. Thus,

$$915.239 = 915\frac{239}{1000}$$

47. There are three decimal places after the decimal point, so there will be 3 zeros in the denominator of the fractional part. Thus,

$$560.453 = 560\frac{453}{1000}$$

49. There are three decimal places after the decimal point, so there will be 3 zeros in the denominator of the fractional part. Thus,

$$414.939 = 414\frac{939}{1000}$$

51. There are two decimal places after the decimal point, so there will be 2 zeros in the denominator of the fractional part. Thus,

$$446.73 = 446\frac{73}{100}$$

53. There is one decimal place after the decimal point, so so there will be 1 zero in the denominator of the fraction. Thus,

$$8.7 = \frac{87}{10}$$

55. There are two decimal places after the decimal point, so so there will be 2 zeros in the denominator of the fraction. Thus,

$$5.47 = \frac{547}{100}$$

57. There are three decimal places after the decimal point, so so there will be 3 zeros in the denominator of the fraction. Thus,

$$2.133 = \frac{2133}{1000}$$

59. There is one decimal place after the decimal point, so so there will be 1 zero in the denominator of the fraction. Thus,

$$3.9 = \frac{39}{10}$$

61. Note that the last digit occurs in the hundredths place. To convert 0.35 to a fraction, we place the number (without the decimal point) over 100 and reduce.

$$
\begin{aligned}
0.35 &= \frac{35}{100} && \textcolor{red}{\text{Place over 100.}} \\
&= \frac{5 \cdot 7}{2 \cdot 2 \cdot 5 \cdot 5} && \textcolor{red}{\text{Prime factor.}} \\
&= \frac{7}{2 \cdot 2 \cdot 5} && \textcolor{red}{\text{Cancel common factors.}} \\
&= \frac{7}{20} && \textcolor{red}{\text{Simplify.}}
\end{aligned}
$$

63. Note that the last digit occurs in the hundredths place. To convert 0.06 to a fraction, we place the number (without the decimal point) over 100 and reduce.

$$
\begin{aligned}
0.06 &= \frac{6}{100} && \textcolor{red}{\text{Place over 100.}} \\
&= \frac{2 \cdot 3}{2 \cdot 2 \cdot 5 \cdot 5} && \textcolor{red}{\text{Prime factor.}} \\
&= \frac{3}{2 \cdot 5 \cdot 5} && \textcolor{red}{\text{Cancel common factors.}} \\
&= \frac{3}{50} && \textcolor{red}{\text{Simplify.}}
\end{aligned}
$$

65. Note that the last digit occurs in the hundredths place. To convert 0.98 to a fraction, we place the number (without the decimal point) over 100 and reduce.

$$
\begin{aligned}
0.98 &= \frac{98}{100} && \text{Place over 100.}\\
&= \frac{2 \cdot 7 \cdot 7}{2 \cdot 2 \cdot 5 \cdot 5} && \text{Prime factor.}\\
&= \frac{7 \cdot 7}{2 \cdot 5 \cdot 5} && \text{Cancel common factors.}\\
&= \frac{49}{50} && \text{Simplify.}
\end{aligned}
$$

67. Note that the last digit occurs in the hundredths place. To convert 0.72 to a fraction, we place the number (without the decimal point) over 100 and reduce.

$$
\begin{aligned}
0.72 &= \frac{72}{100} && \text{Place over 100.}\\
&= \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 5 \cdot 5} && \text{Prime factor.}\\
&= \frac{2 \cdot 3 \cdot 3}{5 \cdot 5} && \text{Cancel common factors.}\\
&= \frac{18}{25} && \text{Simplify.}
\end{aligned}
$$

69. Locate the rounding digit in the hundredths place and the test digit in the thousandths place.
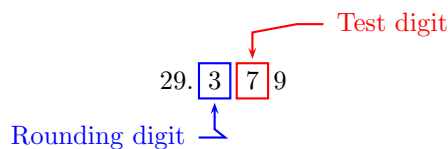

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest hundredth, 79.369 is approximately 79.37.

71. Locate the rounding digit in the thousandths place and the test digit in the ten-thousandths place.
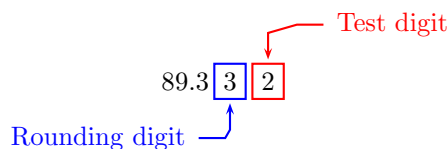
Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest thousandth, 71.2427 is approximately 71.243.

73. Locate the rounding digit in the tenths place and the test digit in the hundredths place.

29. 3 7 9 — Test digit, Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth, 29.379 is approximately 29.4.

75. Locate the rounding digit in the thousandths place and the test digit in the ten-thousandths place.

89.3 3 2 — Test digit, Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest thousandth, 89.3033 is approximately 89.303.

77. Locate the rounding digit in the tenths place and the test digit in the hundredths place.

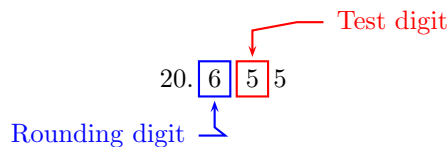20. 6 5 5 — Test digit, Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth, 20.655 is approximately 20.7.

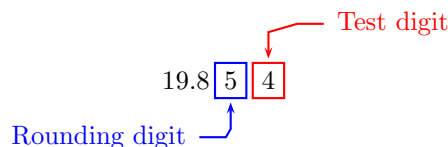**79.** Locate the rounding digit in the hundredths place and the test digit in the thousandths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest hundredth, 19.854 is approximately 19.85.

81. The leftmost digit at which these two positive numbers disagree is in the ten-thousandth place. The first number has a larger ten-thousandth digit than the second number, so

$$0.30387617 > 0.3036562$$

83. The leftmost digit at which these two negative numbers disagree is in the hundredth place. The first number has a smaller hundredth digit than the second number, so

$$-0.034 > -0.040493$$

85. The leftmost digit at which these two negative numbers disagree is in the hundredth place. The first number has a smaller hundredth digit than the second number, so

$$-8.3527 > -8.36553$$

87. The leftmost digit at which these two positive numbers disagree is in the thousandth place. The first number has a smaller thousandth digit than the second number, so

$$18.62192 < 18.6293549$$

89. The leftmost digit at which these two positive numbers disagree is in the thousandth place. The first number has a larger thousandth digit than the second number, so

$$36.8298 > 36.8266595$$

91. The leftmost digit at which these two negative numbers disagree is in the thousandth place. The first number has a larger thousandth digit than the second number, so

$$-15.188392 < -15.187157$$

93.

i) The last digit 3 is in the hundredths column. Therefore, we write the number as *seven and 3 hundredths.*

ii) The last digit 5 is in the hundredths column. Therefore, we write the number as *one and seventy-five hundredths.*

iii) The last digit 5 is in the ten-thousandths column. Therefore, we write the number as *nine thousand nine hundred eighty-five ten-thousandths.*

iv) The last non-zero digit 1 is in the ten-thousandths column. Therefore, we write the number as *one ten-thousandths.*

95.

i) First, write the decimal as a fraction with the numerator 1.26 and the denominator $1,000,000$ and divide.

$$\frac{1.26}{1,000,000} = 0.00000126$$

Therefore, written as a decimal, 1.26 millionths is 0.00000126.

ii) Writing five millionths as a fraction, and dividing, gives

$$\frac{5}{1,000,000} = 0.000005$$

Therefore, written as a decimal, five millionths is 0.000005

iii) We compare the decimals 0.00000126 and 0.000005 column by column. In the millionths column, $1 < 5$ so $0.00000126 < 0.000005$. Because the theoretical change of 0.00000126 seconds is less than the measurable changes of 0.000005 seconds, scientists are unable to observe the computed change in the length of an Earth day.

5.2 Adding and Subtracting Decimals

1. Add trailing zeros if necessary to align the decimal points. Then add.

$$\begin{array}{r} 31.9 \\ +\,84.7 \\ \hline 116.6 \end{array}$$

Thus, $31.9 + 84.7 = 116.6$.

3. Add trailing zeros if necessary to align the decimal points. Then add.

$$\begin{array}{r} 4.00 \\ +\,97.18 \\ \hline 101.18 \end{array}$$

Thus, $4 + 97.18 = 101.18$.

5. Add trailing zeros if necessary to align the decimal points. Then add.

$$\begin{array}{r} 4.000 \\ +\,87.502 \\ \hline 91.502 \end{array}$$

Thus, $4 + 87.502 = 91.502$.

7. Add trailing zeros if necessary to align the decimal points. Then add.

$$\begin{array}{r} 95.57 \\ +\,7.88 \\ \hline 103.45 \end{array}$$

Thus, $95.57 + 7.88 = 103.45$.

9. Add trailing zeros if necessary to align the decimal points. Then add.

$$\begin{array}{r} 52.671 \\ +\,5.970 \\ \hline 58.641 \end{array}$$

Thus, $52.671 + 5.97 = 58.641$.

11. Add trailing zeros if necessary to align the decimal points. Then add.

$$\begin{array}{r} 4.76 \\ +\,2.10 \\ \hline 6.86 \end{array}$$

Thus, $4.76 + 2.1 = 6.86$.

13. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude:

$$
\begin{array}{r}
9.000 \\
-\ 2.261 \\
\hline
6.739
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$9 - 2.261 = 6.739$$

15. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude:

$$
\begin{array}{r}
80.9 \\
-\ 6.0 \\
\hline
74.9
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$80.9 - 6 = 74.9$$

17. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude:

$$
\begin{array}{r}
55.672 \\
-\ 3.300 \\
\hline
52.372
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$55.672 - 3.3 = 52.372$$

19. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude:

$$
\begin{array}{r}
60.575 \\
-\ 6.000 \\
\hline
54.575
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$60.575 - 6 = 54.575$$

21. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude:

$$\begin{array}{r} 39.8 \\ -\ 4.5 \\ \hline 35.3 \end{array}$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$39.8 - 4.5 = 35.3$$

23. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude:

$$\begin{array}{r} 8.10 \\ -\ 2.12 \\ \hline 5.98 \end{array}$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$8.1 - 2.12 = 5.98$$

25. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$-19.13 - 7 = -19.13 + (-7)$$

In this addition problem, the decimals have like signs. Therefore, start by adding the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 19.13 \\ +\ 7.00 \\ \hline 26.13 \end{array}$$

Finish by prefixing the common negative sign. Thus,

$$\begin{aligned} -19.13 - 7 &= -19.13 + (-7) \\ &= -26.13 \end{aligned}$$

27. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude.

$$\begin{array}{r} 76.80 \\ -\ 6.08 \\ \hline 70.72 \end{array}$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$6.08 - 76.8 = -70.72$$

29. The two decimals are both negative. First add the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
34.700 \\
+\,56.214 \\
\hline
90.914
\end{array}
$$

Finish by prefixing the common negative sign. Hence,

$$-34.7 + (-56.214) = -90.914$$

31. The decimals have unlike signs. First subtract the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
8.400 \\
-\,6.757 \\
\hline
1.643
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$8.4 + (-6.757) = 1.643$$

33. The decimals have unlike signs. First subtract the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
50.4 \\
-\,7.6 \\
\hline
42.8
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$-50.4 + 7.6 = -42.8$$

35. The decimals have unlike signs. First subtract the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
43.3 \\
-\,2.2 \\
\hline
41.1
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$-43.3 + 2.2 = -41.1$$

37. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude.

$$
\begin{array}{r}
0.70 \\
-\,0.19 \\
\hline
0.51
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$0.19 - 0.7 = -0.51$$

39. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$-7 - 1.504 = -7 + (-1.504)$$

In this addition problem, the decimals have like signs. Therefore, start by adding the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
7.000 \\
+\,1.504 \\
\hline
8.504
\end{array}
$$

Finish by prefixing the common negative sign. Thus,

$$
\begin{aligned}
-7 - 1.504 &= -7 + (-1.504) \\
&= -8.504
\end{aligned}
$$

41. The two decimals are both negative. First add the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
4.47 \\
+\,2.00 \\
\hline
6.47
\end{array}
$$

Finish by prefixing the common negative sign. Hence,

$$-4.47 + (-2) = -6.47$$

43. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$71.72 - (-6) = 71.72 + 6$$

Then compute the sum. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
71.720 \\
+\,6.000 \\
\hline
77.720
\end{array}
$$

Thus,

$$
\begin{aligned}
71.72 - (-6) &= 71.72 + 6 \\
&= 77.72
\end{aligned}
$$

45. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$-9.829 - (-17.33) = -9.829 + 17.33$$

In this addition problem, the decimals have unlike signs. Therefore, start by subtracting the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
17.330 \\
-\,9.829 \\
\hline
7.501
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Thus,

$$
\begin{aligned}
-9.829 - (-17.33) &= -9.829 + 17.33 \\
&= 7.501
\end{aligned}
$$

47. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude.

$$
\begin{array}{r}
4.202 \\
-\,2.001 \\
\hline
2.201
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$2.001 - 4.202 = -2.201$$

49. Add trailing zeros if necessary to align the decimal points. Then subtract the smaller magnitude from the larger magnitude.

$$\begin{array}{r} 2.99 \\ -\,2.60 \\ \hline 0.39 \end{array}$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$2.6 - 2.99 = -0.39$$

51. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$-4.560 - 2.335 = -4.560 + (-2.335)$$

In this addition problem, the decimals have like signs. Therefore, start by adding the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 4.560 \\ +\,2.335 \\ \hline 6.895 \end{array}$$

Finish by prefixing the common negative sign. Thus,

$$\begin{aligned} -4.560 - 2.335 &= -4.560 + (-2.335) \\ &= -6.895 \end{aligned}$$

53. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$-54.3 - 3.97 = -54.3 + (-3.97)$$

In this addition problem, the decimals have like signs. Therefore, start by adding the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 54.30 \\ +\,3.97 \\ \hline 58.27 \end{array}$$

Finish by prefixing the common negative sign. Thus,

$$\begin{aligned} -54.3 - 3.97 &= -54.3 + (-3.97) \\ &= -58.27 \end{aligned}$$

55. The two decimals are both negative. First add the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
6.320 \\
+\,48.663 \\
\hline
54.983
\end{array}
$$

Finish by prefixing the common negative sign. Hence,

$$-6.32 + (-48.663) = -54.983$$

57. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$-8 - (-3.686) = -8 + 3.686$$

In this addition problem, the decimals have unlike signs. Therefore, start by subtracting the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
8.000 \\
-\,3.686 \\
\hline
4.314
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Thus,

$$
\begin{aligned}
-8 - (-3.686) &= -8 + 3.686 \\
&= -4.314
\end{aligned}
$$

59. The decimals have unlike signs. First subtract the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
9.365 \\
-\,5.000 \\
\hline
4.365
\end{array}
$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$9.365 + (-5) = 4.365$$

61. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$2.762 - (-7.3) = 2.762 + 7.3$$

Then compute the sum. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
2.762 \\
+\,7.300 \\
\hline
10.062
\end{array}
$$

Thus,

$$
\begin{aligned}
2.762 - (-7.3) &= 2.762 + 7.3 \\
&= 10.062
\end{aligned}
$$

63. The two decimals are both negative. First add the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$
\begin{array}{r}
96.10 \\
+\,9.65 \\
\hline
105.75
\end{array}
$$

Finish by prefixing the common negative sign. Hence,

$$-96.1 + (-9.65) = -105.75$$

65. Simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
-12.05 - |17.83 - (-17.16)| &= -12.05 - |17.83 + 17.16| && \text{Subtract: Add the opposite.} \\
&= -12.05 - |34.99| && \text{Add: } 17.83 + 17.16 = 34.99. \\
&= -12.05 - 34.99 && \text{Take absolute value: } |34.99| = 34.99. \\
&= -12.05 + (-34.99) && \text{Subtract: Add the opposite.} \\
&= -47.04 && \text{Add: } -12.05 + (-34.99) = -47.04.
\end{aligned}
$$

67. Simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
-6.4 + |9.38 - (-9.39)| &= -6.4 + |9.38 + 9.39| && \text{Subtract: Add the opposite.} \\
&= -6.4 + |18.77| && \text{Add: } 9.38 + 9.39 = 18.77. \\
&= -6.4 + 18.77 && \text{Take absolute value: } |18.77| = 18.77. \\
&= 12.37 && \text{Add: } -6.4 + 18.77 = 12.37.
\end{aligned}
$$

69. Simplify the expression inside the parentheses first.

$$
\begin{aligned}
-19.1 - (1.51 - (-17.35)) &= -19.1 - (1.51 + 17.35) && \text{Subtract: Add the opposite.} \\
&= -19.1 - 18.86 && \text{Add: } 1.51 + 17.35 = 18.86. \\
&= -19.1 + (-18.86) && \text{Subtract: Add the opposite.} \\
&= -37.96 && \text{Add: } -19.1 + (-18.86) = -37.96.
\end{aligned}
$$

71. Simplify the expression inside the parentheses first.

$$
\begin{aligned}
11.55 + (6.3 - (-1.9)) &= 11.55 + (6.3 + 1.9) && \text{Subtract: Add the opposite.} \\
&= 11.55 + 8.2 && \text{Add: } 6.3 + 1.9 = 8.2. \\
&= 19.75 && \text{Add: } 11.55 + 8.2 = 19.75.
\end{aligned}
$$

73. Simplify the expression inside the parentheses first.

$$
\begin{aligned}
-1.7 - (1.9 - (-16.25)) &= -1.7 - (1.9 + 16.25) && \text{Subtract: Add the opposite.} \\
&= -1.7 - 18.15 && \text{Add: } 1.9 + 16.25 = 18.15. \\
&= -1.7 + (-18.15) && \text{Subtract: Add the opposite.} \\
&= -19.85 && \text{Add: } -1.7 + (-18.15) = -19.85.
\end{aligned}
$$

75. Simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
1.2 + |8.74 - 16.5| &= 1.2 + |8.74 + (-16.5)| && \text{Subtract: Add the opposite.} \\
&= 1.2 + |-7.76| && \text{Add: } 8.74 + (-16.5) = -7.76. \\
&= 1.2 + 7.76 && \text{Take absolute value: } |-7.76| = 7.76. \\
&= 8.96 && \text{Add: } 1.2 + 7.76 = 8.96.
\end{aligned}
$$

77. Simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
-12.4 - |3.81 - 16.4| &= -12.4 - |3.81 + (-16.4)| && \text{Subtract: Add the opposite.} \\
&= -12.4 - |-12.59| && \text{Add: } 3.81 + (-16.4) = -12.59. \\
&= -12.4 - 12.59 && \text{Take absolute value: } |-12.59| = 12.59. \\
&= -12.4 + (-12.59) && \text{Subtract: Add the opposite.} \\
&= -24.99 && \text{Add: } -12.4 + (-12.59) = -24.99.
\end{aligned}
$$

79. Simplify the expression inside the parentheses first.

$$
\begin{aligned}
-11.15 + (11.6 - (-16.68)) &= -11.15 + (11.6 + 16.68) && \text{Subtract: Add the opposite.} \\
&= -11.15 + 28.28 && \text{Add: } 11.6 + 16.68 = 28.28. \\
&= 17.13 && \text{Add: } -11.15 + 28.28 = 17.13.
\end{aligned}
$$

81. To find the total dollar value of the nation's four largest banks, line up the decimal points and add like columns. Your answer will be in the billions.

$$124.8 + 85.3 + 61.8 + 56.4 = 328.3$$

Therefore, the dollar value of the nation's four largest banks is $328.3 billion.

83. To find the change in temperature, subtract the earlier temperature from the later temperature.

Change in Temperature	=	Latter Temperature	−	Former Temperature
	=	60.9° F	−	93.8° F
	=	−32.9° F		

Hence, the change in temperature is −32.9° F.

85. To find the net worth, subtract the debts from the assets. Your answer should be in the billions.

Net Worth	=	Assets	−	Liabilities
	=	29.6	−	27
	=	2.6		

Therefore, General Growth Properties had a net worth of $2.6 billion.

87. To find how many more people lost their jobs than were hired, find the difference between the job loss number and the hired number.

Net job loss	=	Number jobs lost	−	Number people hired
	=	4.12	−	4.08
	=	0.04		

Therefore, the net number of people who lost their job in January of 2010 was 0.04 million, or four one-hundredths of a million. Remembering that "of" means multiply, we write this as a whole number by multiplying 0.04 and one million. This requires only to move the decimal six places to the right, as in:

$$0.04 \cdot 1,000,000 = 40,000$$

Thus, 40,000 more people lost their jobs in the month of January than were hired.

5.3 Multiplying Decimals

1. Use vertical format. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
6.7 \\
\times\ 0.03 \\
\hline
0.201
\end{array}
$$

Thus, $(6.7)(0.03) = 0.201$.

3. Use vertical format. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
28.9 \\
\times\ 5.9 \\
\hline
26\ 01 \\
144\ 5\ \\
\hline
170.51
\end{array}
$$

Thus, $(28.9)(5.9) = 170.51$.

5. Use vertical format. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
4.1 \\
\times\ 4.6 \\
\hline
2\ 46 \\
16\ 4\ \\
\hline
18.86
\end{array}
$$

Thus, $(4.1)(4.6) = 18.86$.

7. Use vertical format. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
75.3 \\
\times\ 0.4 \\
\hline
30.12
\end{array}
$$

Thus, $(75.3)(0.4) = 30.12$.

9. Use vertical format. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
6.98 \\
\times\ 0.9 \\
\hline
6.282
\end{array}
$$

Thus, $(6.98)(0.9) = 6.282$.

11. Use vertical format. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
57.9 \\
\times\ 3.29 \\
\hline
5\ 211 \\
11\ 58 \\
173\ 7 \\
\hline
190.491
\end{array}
$$

Thus, $(57.9)(3.29) = 190.491$.

13. Use vertical format. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
47.3 \\
\times\ 0.9 \\
\hline
42.57
\end{array}
$$

Thus, $(47.3)(0.9) = 42.57$.

15. Use vertical format. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
9.9 \\
\times\ 6.7 \\
\hline
6\ 93 \\
59\ 4 \\
\hline
66.33
\end{array}
$$

Thus, $(9.9)(6.7) = 66.33$.

17. Use vertical format. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
19.5 \\
\times\ 7.9 \\
\hline
17\ 55 \\
136\ 5 \\
\hline
154.05
\end{array}
$$

Thus, $(19.5)(7.9) = 154.05$.

19. Use vertical format. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
6.9 \\
\times\ 0.3 \\
\hline
2.07
\end{array}
$$

Thus, $(6.9)(0.3) = 2.07$.

21. Use vertical format. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
35.3 \\
\times\ 3.81 \\
\hline
353 \\
28\ 24 \\
105\ 9 \\
\hline
134.493
\end{array}
$$

Thus, $(35.3)(3.81) = 134.493$.

23. Use vertical format. Since there are a total of 4 digits to the right of the decimal point in the original numbers, the answer also has 4 digits to the right of the decimal point.

$$
\begin{array}{r}
2.32 \\
\times\ 0.03 \\
\hline
0.0696
\end{array}
$$

Thus, $(2.32)(0.03) = 0.0696$.

25. Use vertical format. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
3.02 \\
\times\ 6.7 \\
\hline
2\ 114 \\
18\ 12 \\
\hline
20.234
\end{array}
$$

Thus, $(3.02)(6.7) = 20.234$.

27. Use vertical format. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
4.98 \\
\times\ 6.2 \\
\hline
996 \\
29\ 88 \\
\hline
30.876
\end{array}
$$

Thus, $(4.98)(6.2) = 30.876$.

29. Use vertical format with the unsigned numbers. Since there are a total of 4 digits to the right of the decimal point in the original numbers, the answer also has 4 digits to the right of the decimal point.

$$
\begin{array}{r}
9.41 \\
\times\ 0.07 \\
\hline
0.6587
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(-9.41)(0.07) = -0.6587$$

31. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
7.4 \\
\times\ 0.9 \\
\hline
6.66
\end{array}
$$

Like signs give a positive result. Therefore,

$$(-7.4)(-0.9) = 6.66$$

33. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
8.2 \\
\times\ 3.7 \\
\hline
5\ 74 \\
24\ 6 \\
\hline
30.34
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(-8.2)(3.7) = -30.34$$

35. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
9.72 \\
\times\ 9.1 \\
\hline
972 \\
87\ 48 \\
\hline
88.452
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(9.72)(-9.1) = -88.452$$

37. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
6.4 \\
\times\ 2.6 \\
\hline
3\ 84 \\
12\ 8 \\
\hline
16.64
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(-6.4)(2.6) = -16.64$$

39. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
39.3 \\
\times\ 0.8 \\
\hline
31.44
\end{array}
$$

Like signs give a positive result. Therefore,

$$(-39.3)(-0.8) = 31.44$$

41. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
63.1 \\
\times\ 0.02 \\
\hline
1.262
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(63.1)(-0.02) = -1.262$$

43. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
90.8 \\
\times\ 3.1 \\
\hline
9\ 08 \\
272\ 4\ \\
\hline
281.48
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(-90.8)(3.1) = -281.48$$

45. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer

also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
47.5 \\
\times\ 82.1 \\
\hline
4\ 75 \\
95\ 0\ \\
3800\ \ \\
\hline
3899.75
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(47.5)(-82.1) = -3899.75$$

47. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$
\begin{array}{r}
31.1 \\
\times\ 4.8 \\
\hline
24\ 88 \\
124\ 4\ \\
\hline
149.28
\end{array}
$$

Like signs give a positive result. Therefore,

$$(-31.1)(-4.8) = 149.28$$

49. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
2.5 \\
\times\ 0.07 \\
\hline
0.175
\end{array}
$$

Like signs give a positive result. Therefore,

$$(-2.5)(-0.07) = 0.175$$

51. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
1.02 \\
\times\ 0.2 \\
\hline
0.204
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(1.02)(-0.2) = -0.204$$

53. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
7.81 \\
\times\ 5.5 \\
\hline
3\ 905 \\
39\ 05 \\
\hline
42.955
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(7.81)(-5.5) = -42.955$$

55. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$
\begin{array}{r}
2.09 \\
\times\ 37.9 \\
\hline
1\ 881 \\
14\ 63 \\
62\ 7 \\
\hline
79.211
\end{array}
$$

Unlike signs give a negative result. Therefore,

$$(-2.09)(37.9) = -79.211$$

57. Move the decimal point 1 place to the right: $24.264 \cdot 10 = 242.64$

59. Move the decimal point 4 places to the right: $53.867 \cdot 10^4 = 538670$

61. Move the decimal point 3 places to the right: $5.096 \cdot 10^3 = 5096$

63. Move the decimal point 3 places to the right: $37.968 \cdot 10^3 = 37968$

65. Move the decimal point 2 places to the right: $61.303 \cdot 100 = 6130.3$

67. Move the decimal point 3 places to the right: $74.896 \cdot 1000 = 74896$

69. First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
(0.36)(7.4) - (-2.8)^2 &= (0.36)(7.4) - 7.84 & &\text{Exponents first: } (-2.8)^2 = 7.84. \\
&= 2.664 - 7.84 & &\text{Multiply: } (0.36)(7.4) = 2.664. \\
&= 2.664 + (-7.84) & &\text{Subtract: Add the opposite.} \\
&= -5.176 & &\text{Add: } 2.664 + (-7.84) = -5.176.
\end{aligned}
$$

71. First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
9.4 - (-7.7)(1.2)^2 &= 9.4 - (-7.7)(1.44) & &\text{Exponents first: } 1.2^2 = 1.44. \\
&= 9.4 - (-11.088) & &\text{Multiply: } (-7.7)(1.44) = -11.088. \\
&= 9.4 + 11.088 & &\text{Subtract: Add the opposite.} \\
&= 20.488 & &\text{Add: } 9.4 + 11.088 = 20.488.
\end{aligned}
$$

73. First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
5.94 - (-1.2)(-1.8)^2 &= 5.94 - (-1.2)(3.24) & &\text{Exponents first: } (-1.8)^2 = 3.24. \\
&= 5.94 - (-3.888) & &\text{Multiply: } (-1.2)(3.24) = -3.888. \\
&= 5.94 + 3.888 & &\text{Subtract: Add the opposite.} \\
&= 9.828 & &\text{Add: } 5.94 + 3.888 = 9.828.
\end{aligned}
$$

75. First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
6.3 - 4.2(9.3)^2 &= 6.3 - 4.2 \cdot 86.49 & &\text{Exponents first: } 9.3^2 = 86.49. \\
&= 6.3 - 363.258 & &\text{Multiply: } 4.2 \cdot 86.49 = 363.258. \\
&= 6.3 + (-363.258) & &\text{Subtract: Add the opposite.} \\
&= -356.958 & &\text{Add: } 6.3 + (-363.258) = -356.958.
\end{aligned}
$$

77. First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
(6.3)(1.88) - (-2.2)^2 &= (6.3)(1.88) - 4.84 &&\text{Exponents first: } (-2.2)^2 = 4.84.\\
&= 11.844 - 4.84 &&\text{Multiply: } (6.3)(1.88) = 11.844.\\
&= 11.844 + (-4.84) &&\text{Subtract: Add the opposite.}\\
&= 7.004 &&\text{Add: } 11.844 + (-4.84) = 7.004.
\end{aligned}
$$

79. First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
(-8.1)(9.4) - 1.8^2 &= (-8.1)(9.4) - 3.24 &&\text{Exponents first: } 1.8^2 = 3.24.\\
&= -76.14 - 3.24 &&\text{Multiply: } (-8.1)(9.4) = -76.14.\\
&= -76.14 + (-3.24) &&\text{Subtract: Add the opposite.}\\
&= -79.38 &&\text{Add: } -76.14 + (-3.24) = -79.38.
\end{aligned}
$$

81. Substitute $a = -6.24$, $b = 0.4$, and $c = 7.2$ in $a - bc^2$.

$$
a - bc^2 = (-6.24) - (0.4)(7.2)^2 \qquad \text{Substitute.}
$$

First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
&= (-6.24) - (0.4)(51.84) &&\text{Exponents first: } (7.2)^2 = 51.84.\\
&= -6.24 - 20.736 &&\text{Multiply: } 0.4 \cdot 51.84 = 20.736.\\
&= -6.24 + (-20.736) &&\text{Subtract: Add the opposite.}\\
&= -26.976 &&\text{Add: } -6.24 + (-20.736) = -26.976.
\end{aligned}
$$

83. Substitute $a = -2.4$, $b = -2.1$, and $c = -4.6$ in $a - bc^2$.

$$
ab - c^2 = (-2.4)(-2.1) - (-4.6)^2 \qquad \text{Substitute.}
$$

First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
&= (-2.4)(-2.1) - 21.16 &&\text{Exponents first: } (-4.6)^2 = 21.16.\\
&= 5.04 - 21.16 &&\text{Multiply: } (-2.4)(-2.1) = 5.04.\\
&= 5.04 + (-21.16) &&\text{Subtract: Add the opposite.}\\
&= -16.12 &&\text{Add: } 5.04 + (-21.16) = -16.12.
\end{aligned}
$$

85. Substitute $a = -3.21$, $b = 3.5$, and $c = 8.3$ in $a - bc^2$.

$$a - bc^2 = (-3.21) - (3.5)(8.3)^2 \qquad \text{Substitute.}$$

First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
&= (-3.21) - (3.5)(68.89) && \text{Exponents first: } (8.3)^2 = 68.89. \\
&= -3.21 - 241.115 && \text{Multiply: } 3.5 \cdot 68.89 = 241.115. \\
&= -3.21 + (-241.115) && \text{Subtract: Add the opposite.} \\
&= -244.325 && \text{Add: } -3.21 + (-241.115) = -244.325.
\end{aligned}
$$

87. Substitute $a = -4.5$, $b = -6.9$, and $c = 4.6$ in $a - bc^2$.

$$ab - c^2 = (-4.5)(-6.9) - (4.6)^2 \qquad \text{Substitute.}$$

First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
&= (-4.5)(-6.9) - 21.16 && \text{Exponents first: } (4.6)^2 = 21.16. \\
&= 31.05 - 21.16 && \text{Multiply: } (-4.5)(-6.9) = 31.05. \\
&= 31.05 + (-21.16) && \text{Subtract: Add the opposite.} \\
&= 9.89 && \text{Add: } 31.05 + (-21.16) = 9.89.
\end{aligned}
$$

89. To find the circumference, use the formula $C = \pi d$. Substitute 3.14 for π and 8.56 for d, then multiply.

$$
\begin{aligned}
C &= \pi d && \text{Circumference formula.} \\
C &= 3.14(8.56) && \text{Substitute: 3.14 for } \pi, \text{ 8.56 for } d. \\
C &= 26.8784 && \text{Multiply: } 3.14(8.56) = 26.8784.
\end{aligned}
$$

To round to the nearest tenth of an inch, locate the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest tenth of an inch, the circumference of the circle is approximately $C \approx 26.9$ inches.

91. To find the circumference, use the formula $C = \pi d$. Substitute 3.14 for π and 12.04 for d, then multiply.

$$C = \pi d \qquad\qquad \text{Circumference formula.}$$
$$C = 3.14(12.04) \qquad\qquad \text{Substitute: 3.14 for } \pi, \text{ 12.04 for } d.$$
$$C = 37.8056 \qquad\qquad \text{Multiply: } 3.14(12.04) = 37.8056.$$

To round to the nearest tenth of an inch, locate the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest tenth of an inch, the circumference of the circle is approximately $C \approx 37.8$ inches.

93. The first task is to find the radius r. But the diameter is twice the radius.

$$d = 2r \qquad\qquad \text{The diameter is twice the radius.}$$
$$10.75 = 2r \qquad\qquad \text{Substitute: 10.75 for } d.$$
$$\frac{10.75}{2} = \frac{2r}{2} \qquad\qquad \text{Divide both sides by 2.}$$
$$5.375 = r \qquad\qquad \text{Simplify.}$$

Thus, the radius is $r = 5.375$ inches. To find the area, use the formula $A = \pi r^2$.

$$A = \pi r^2 \qquad\qquad \text{Area formula.}$$
$$A = 3.14(5.375)^2 \qquad\qquad \text{Substitute: 3.14 for } \pi, \text{ 5.375 for } r.$$
$$A = 3.14(28.890625) \qquad\qquad \text{Square: } (5.375)^2 = 28.890625.$$
$$A = 90.7165625 \qquad\qquad \text{Multiply: } 3.14(28.890625) = 90.7165625.$$

To round to the nearest hundredth of a square inch, locate the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest hundredth of a square inch, the area of the circle is approximately $A \approx 90.72$ inches.

95. The first task is to find the radius r. But the diameter is twice the radius.

$$d = 2r \qquad \text{The diameter is twice the radius.}$$
$$13.96 = 2r \qquad \text{Substitute: } 13.96 \text{ for } d.$$
$$\frac{13.96}{2} = \frac{2r}{2} \qquad \text{Divide both sides by 2.}$$
$$6.98 = r \qquad \text{Simplify.}$$

Thus, the radius is $r = 6.98$ inches. To find the area, use the formula $A = \pi r^2$.

$$A = \pi r^2 \qquad \text{Area formula.}$$
$$A = 3.14(6.98)^2 \qquad \text{Substitute: } 3.14 \text{ for } \pi, 6.98 \text{ for } r.$$
$$A = 3.14(48.7204) \qquad \text{Square: } (6.98)^2 = 48.7204.$$
$$A = 152.982056 \qquad \text{Multiply: } 3.14(48.7204) = 152.982056.$$

To round to the nearest hundredth of a square inch, locate the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest hundredth of a square inch, the area of the circle is approximately $A \approx 152.98$ inches.

97. Because the pond has the shape of a cylinder, we can use the formula $V = \pi r^2 h$ to find the volume. Because the diameter of the base is 15 feet, the radius is 7.5 feet (half the diameter). Thus, the volume is

$$V = \pi r^2 h \qquad \text{Formula for volume of a cylinder.}$$
$$= (3.14)(7.5)^2(1.5) \qquad \text{Substitute: } 3.14 \text{ for } \pi,$$
$$\qquad \qquad \qquad \qquad 7.5 \text{ for } r, \text{ and } 1.5 \text{ for } h.$$
$$= (3.14)(56.25)(1.5) \qquad \text{Exponent first: } (7.5)^2 = 56.25.$$
$$= 264.9375$$

Thus, the volume is 264.9375 cubic feet.

99.

a) New income:

New Income	=	Overtime Income	+	Regular Income
	=	$32 \cdot 10.30 \cdot 1.5$	+	$136 \cdot 10.30$
	=	$494.40	+	$1400.80
	=	$1895.20		

b) This represents an increase over his original (see Exercise **??**) income of $1730.40.

Income Increase	=	New Income	−	Old income
	=	$1895.20	−	$1730.40
	=	$164.80		

101. To find the dollar amount of grape revenue for one acre, multiply the price per ton of grapes by the number of tons of grapes that can be grown on one acre.

Revenue	=	Price per ton	·	number of tons
	=	$3,414$	·	3.5
	=	$11,949$		

Therefore, the dollars generated on one acre of premium cabernet are about $11,949$.

103.

a) To find the total number of pounds, multiply the number of pigs by the number of pounds for each pig. Since the numbers are all powers of ten, you can move the decimal point the appropriate number of places and add zeros.

Total pounds of pig	=	number of pigs	·	pounds per pig
	=	$1,000$	·	100
	=	$100,000$		

Therefore, a typical corporate agribarn houses approximately 100,000 pounds of pig.

b) To find the cash value for the entire warehouse, multiply the total number of pounds by the price per pound.

Cash value	=	number of pounds	·	price per pound
	=	100,000	·	1.29
	=	129,000		

Therefore, the cash value for one corporate agribarn is about $129,000.

105. To find the circumference, use the formula $C = \pi d$. Substitute 3.14 for π and 230 for d, then multiply.

$C = \pi d$ Circumference formula.

$C = 3.14(230)$ Substitute: 3.14 for π, 230 for d.

$C = 722.2$ Multiply: $3.14(230) = 722.2$.

Therefore, to the nearest tenth, the circumference of the radio dish is 722.2 feet.

5.4 Dividing Decimals

1. By long division,

$$
\begin{array}{r}
0.75 \\
52\overline{)39.00} \\
\underline{36\ 4} \\
2\ 60 \\
\underline{2\ 60} \\
0
\end{array}
$$

Therefore, $\dfrac{39}{52} = 0.75$.

3. By long division,

$$
\begin{array}{r}
9.1 \\
83\overline{)755.3} \\
\underline{747} \\
8\ 3 \\
\underline{8\ 3} \\
0
\end{array}
$$

Therefore, $\dfrac{755.3}{83} = 9.1$.

5. By long division,

$$
\begin{array}{r}
4.5 \\
74\overline{)333.0} \\
296 \\
\hline
37\,0 \\
37\,0 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{333}{74} = 4.5$.

7. By long division,

$$
\begin{array}{r}
0.44 \\
73\overline{)32.12} \\
29\,2 \\
\hline
2\,92 \\
2\,92 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{32.12}{73} = 0.44$.

9. By long division,

$$
\begin{array}{r}
0.53 \\
71\overline{)37.63} \\
35\,5 \\
\hline
2\,13 \\
2\,13 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{37.63}{71} = 0.53$.

11. By long division,

$$
\begin{array}{r}
1.5 \\
92\overline{)138.0} \\
92 \\
\hline
46\,0 \\
46\,0 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{138}{92} = 1.5$.

13. By long division,

$$
\begin{array}{r}
0.68 \\
25\overline{)17.00} \\
15\ 0 \\
\hline
2\ 00 \\
2\ 00 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{17}{25} = 0.68$.

15. By long division,

$$
\begin{array}{r}
4.5 \\
51\overline{)229.5} \\
204 \\
\hline
25\ 5 \\
25\ 5 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{229.5}{51} = 4.5$.

17. Move the decimal point in the divisor and dividend two places to the right:

$$
\frac{0.3478}{0.47} = \frac{34.78}{47}
$$

Then, by long division,

$$
\begin{array}{r}
0.74 \\
47\overline{)34.78} \\
32\ 9 \\
\hline
1\ 88 \\
1\ 88 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{0.3478}{0.47} = 0.74$.

19. Move the decimal point in the divisor and dividend one place to the right:

$$
\frac{1.694}{2.2} = \frac{16.94}{22}
$$

Then, by long division,

$$
\begin{array}{r}
0.77 \\
22\overline{)16.94} \\
15\ 4 \\
\hline
1\ 54 \\
1\ 54 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{1.694}{2.2} = 0.77$.

21. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{43.61}{4.9} = \frac{436.1}{49}$$

Then, by long division,

$$
\begin{array}{r}
8.9 \\
49\overline{)436.1} \\
392 \\
\hline
44\ 1 \\
44\ 1 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{43.61}{4.9} = 8.9$.

23. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{1.107}{0.41} = \frac{110.7}{41}$$

Then, by long division,

$$
\begin{array}{r}
2.7 \\
41\overline{)110.7} \\
82 \\
\hline
28\ 7 \\
28\ 7 \\
\hline
0
\end{array}
$$

Therefore, $\dfrac{1.107}{0.41} = 2.7$.

25. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{2.958}{0.51} = \frac{295.8}{51}$$

Then, by long division,

```
         5.8
   51)295.8
      255
       40 8
       40 8
          0
```

Therefore, $\dfrac{2.958}{0.51} = 5.8$.

27. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{71.76}{7.8} = \frac{717.6}{78}$$

Then, by long division,

```
         9.2
   78)717.6
      702
       15 6
       15 6
          0
```

Therefore, $\dfrac{71.76}{7.8} = 9.2$.

29. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{0.8649}{0.93} = \frac{86.49}{93}$$

Then, by long division,

```
          0.93
   93)86.49
      83 7
       2 79
       2 79
          0
```

Therefore, $\dfrac{0.8649}{0.93} = 0.93$.

31. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{0.6958}{0.71} = \frac{69.58}{71}$$

Then, by long division,

```
        0.98
    71)69.58
       63 9
        5 68
        5 68
           0
```

Therefore, $\dfrac{0.6958}{0.71} = 0.98$.

33. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{1.248}{0.52} = \frac{124.8}{52}$$

Then, by long division,

```
         2.4
    52)124.8
       104
        20 8
        20 8
           0
```

Therefore, $\dfrac{1.248}{0.52} = 2.4$.

35. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{62.56}{9.2} = \frac{625.6}{92}$$

Then, by long division,

```
         6.8
    92)625.6
       552
        73 6
        73 6
           0
```

Therefore, $\dfrac{62.56}{9.2} = 6.8$.

37. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{6.278}{8.6} = \frac{62.78}{86}$$

Then, by long division,

$$\begin{array}{r} 0.73 \\ 86\overline{)62.78} \\ \underline{60\ 2} \\ 2\ 58 \\ \underline{2\ 58} \\ 0 \end{array}$$

Therefore, $\dfrac{6.278}{8.6} = 0.73$.

39. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{2.698}{7.1} = \frac{26.98}{71}$$

Then, by long division,

$$\begin{array}{r} 0.38 \\ 71\overline{)26.98} \\ \underline{21\ 3} \\ 5\ 68 \\ \underline{5\ 68} \\ 0 \end{array}$$

Therefore, $\dfrac{2.698}{7.1} = 0.38$.

41. First divide the magnitudes. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{-11.04}{1.6} = \frac{110.4}{16}$$

Then, by long division,

$$\begin{array}{r} 6.9 \\ 16\overline{)110.4} \\ \underline{96} \\ 14\ 4 \\ \underline{14\ 4} \\ 0 \end{array}$$

Unlike signs give a negative quotient, so $\dfrac{-11.04}{1.6} = -6.9$.

43. First divide the magnitudes. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{3.024}{5.6} = \frac{30.24}{56}$$

Then, by long division,

$$
\begin{array}{r}
0.54 \\
56\overline{)30.24} \\
28\ 0 \\
\hline
2\ 24 \\
2\ 24 \\
\hline
0
\end{array}
$$

Unlike signs give a negative quotient, so $\dfrac{-3.024}{5.6} = -0.54$.

45. First divide the magnitudes. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{0.1056}{0.22} = \frac{10.56}{22}$$

Then, by long division,

$$
\begin{array}{r}
0.48 \\
22\overline{)10.56} \\
8\ 8 \\
\hline
1\ 76 \\
1\ 76 \\
\hline
0
\end{array}
$$

Unlike signs give a negative quotient, so $\dfrac{-0.1056}{0.22} = -0.48$.

47. First divide the magnitudes. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{0.3204}{0.89} = \frac{32.04}{89}$$

Then, by long division,

$$
\begin{array}{r}
0.36 \\
89\overline{)32.04} \\
26\ 7 \\
\hline
5\ 34 \\
5\ 34 \\
\hline
0
\end{array}
$$

Unlike signs give a negative quotient, so $\dfrac{0.3204}{-0.89} = -0.36$.

49. First divide the magnitudes. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{1.419}{0.43} = \frac{141.9}{43}$$

Then, by long division,

$$
\begin{array}{r}
3.3 \\
43\overline{)141.9} \\
\underline{129} \\
12\,9 \\
\underline{12\,9} \\
0
\end{array}
$$

Unlike signs give a negative quotient, so $\dfrac{-1.419}{0.43} = -3.3$.

51. First divide the magnitudes. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{-16.72}{-2.2} = \frac{167.2}{22}$$

Then, by long division,

$$
\begin{array}{r}
7.6 \\
22\overline{)167.2} \\
\underline{154} \\
13\,2 \\
\underline{13\,2} \\
0
\end{array}
$$

Like signs give a positive quotient, so $\dfrac{-16.72}{-2.2} = 7.6$.

53. First divide the magnitudes. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{2.088}{0.87} = \frac{208.8}{87}$$

Then, by long division,

$$
\begin{array}{r}
2.4 \\
87\overline{)208.8} \\
\underline{174} \\
34\,8 \\
\underline{34\,8} \\
0
\end{array}
$$

Like signs give a positive quotient, so $\dfrac{-2.088}{-0.87} = 2.4$.

55. First divide the magnitudes. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{1.634}{8.6} = \frac{16.34}{86}$$

Then, by long division,

$$
\begin{array}{r}
0.19 \\
86\overline{)16.34} \\
\underline{8\ 6} \\
7\ 74 \\
\underline{7\ 74} \\
0
\end{array}
$$

Like signs give a positive quotient, so $\dfrac{-1.634}{-8.6} = 0.19$.

57. First divide the magnitudes. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{0.119}{0.85} = \frac{11.9}{85}$$

Then, by long division,

$$
\begin{array}{r}
0.14 \\
85\overline{)11.90} \\
\underline{8\ 5} \\
3\ 40 \\
\underline{3\ 40} \\
0
\end{array}
$$

Unlike signs give a negative quotient, so $\dfrac{-0.119}{0.85} = -0.14$.

59. First divide the magnitudes. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{3.591}{6.3} = \frac{35.91}{63}$$

Then, by long division,

$$
\begin{array}{r}
0.57 \\
63\overline{)35.91} \\
\underline{31\ 5} \\
4\ 41 \\
\underline{4\ 41} \\
0
\end{array}
$$

Like signs give a positive quotient, so $\dfrac{-3.591}{-6.3} = 0.57$.

61. First divide the magnitudes. Move the decimal point in the divisor and dividend one place to the right:

$$\frac{36.96}{-4.4} = \frac{369.6}{44}$$

Then, by long division,

$$
\begin{array}{r}
8.4 \\
44\overline{)369.6} \\
352 \\
\hline
17\,6 \\
17\,6 \\
\hline
0
\end{array}
$$

Unlike signs give a negative quotient, so $\dfrac{36.96}{-4.4} = -8.4$.

63. First divide the magnitudes. Move the decimal point in the divisor and dividend two places to the right:

$$\frac{2.156}{0.98} = \frac{215.6}{98}$$

Then, by long division,

$$
\begin{array}{r}
2.2 \\
98\overline{)215.6} \\
196 \\
\hline
19\,6 \\
19\,6 \\
\hline
0
\end{array}
$$

Like signs give a positive quotient, so $\dfrac{-2.156}{-0.98} = 2.2$.

65. Move the decimal point 2 places to the left:

$$\frac{524.35}{100} = 5.2435$$

67. Move the decimal point 3 places to the left:

$$\frac{563.94}{10^3} = 0.56394$$

69. Move the decimal point 2 places to the left:

$$\frac{116.81}{10^2} = 1.1681$$

71. Move the decimal point 1 places to the left:

$$\frac{694.55}{10} = 69.455$$

73. Move the decimal point 3 places to the left:

$$\frac{341.16}{10^3} = 0.34116$$

75. Move the decimal point 3 places to the left:

$$\frac{113.02}{1000} = 0.11302$$

77. First use long division to find the hundredth place of the quotient.

$$
\begin{array}{r}
0.62 \\
83\overline{)52.00} \\
\underline{49\ 8} \\
2\ 20 \\
\underline{1\ 66} \\
54
\end{array}
$$

Then round to the nearest tenth. Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, rounded to the nearest tenth, $52/83 \approx 0.6$.

79. First use long division to find the hundredth place of the quotient.

$$
\begin{array}{r}
0.86 \\
59\overline{)51.00} \\
47\ 2 \\
\hline
3\ 80 \\
3\ 54 \\
\hline
26
\end{array}
$$

Then round to the nearest tenth. Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Test digit

0. 8 6

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, rounded to the nearest tenth, $51/59 \approx 0.9$.

81. First use long division to find the thousandth place of the quotient.

$$
\begin{array}{r}
0.067 \\
74\overline{)5.000} \\
4\ 44 \\
\hline
560 \\
518 \\
\hline
42
\end{array}
$$

Then round to the nearest hundredth. Locate the rounding digit in the hundredths place and the test digit in the thousandths place.

Test digit

0.0 6 7

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, rounded to the nearest hundredth, $5/74 \approx 0.07$.

83. First use long division to find the thousandth place of the quotient.

$$
\begin{array}{r}
0.053 \\
94\overline{)5.000} \\
4\ 70 \\
\hline
300 \\
282 \\
\hline
18
\end{array}
$$

Then round to the nearest hundredth. Locate the rounding digit in the hundredths place and the test digit in the thousandths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, rounded to the nearest hundredth, $5/94 \approx 0.05$.

85. First use long division to find the thousandth place of the quotient.

$$
\begin{array}{r}
0.097 \\
72\overline{)7.000} \\
6\ 48 \\
\hline
520 \\
504 \\
\hline
16
\end{array}
$$

Then round to the nearest hundredth. Locate the rounding digit in the hundredths place and the test digit in the thousandths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, rounded to the nearest hundredth, $7/72 \approx 0.10$.

87. First use long division to find the hundredth place of the quotient.

$$
\begin{array}{r}
0.18 \\
86\overline{)16.00} \\
\underline{8\;6} \\
7\;40 \\
\underline{6\;88} \\
52
\end{array}
$$

Then round to the nearest tenth. Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, rounded to the nearest tenth, $16/86 \approx 0.2$.

89. The numerator must be simplified first. This requires multiplying and then subtracting.

$$
\begin{aligned}
\frac{7.5 \cdot 7.1 - 19.5}{0.54} &= \frac{53.25 - 19.5}{0.54} && \text{Multiply: } 7.5 \cdot 7.1 = 53.25. \\
&= \frac{53.25 + (-19.5)}{0.54} && \text{Subtract: Add the opposite.} \\
&= \frac{33.75}{0.54} && \text{Add: } 53.25 + (-19.5) = 33.75. \\
&= 62.5 && \text{Divide: } 33.75/0.54 = 62.5.
\end{aligned}
$$

91. With a fractional expression, simplify numerator and denominator first, then divide.

$$
\begin{aligned}
\frac{17.76 - (-11.7)}{0.5^2} &= \frac{17.76 - (-11.7)}{0.25} && \text{Exponents: } 0.5^2 = 0.25. \\
&= \frac{17.76 + 11.7}{0.25} && \text{Subtract: Add the opposite.} \\
&= \frac{29.46}{0.25} && \text{Add: } 17.76 + 11.7 = 29.46. \\
&= 117.84 && \text{Divide: } 29.46/0.25 = 117.84.
\end{aligned}
$$

93. With a fractional expression, simplify numerator and denominator first, then divide.

$$\frac{-18.22 - 6.7}{14.75 - 7.75} = \frac{-18.22 + (-6.7)}{14.75 + (-7.75)}$$

In numerator and denominator, add the opposite.

$$= \frac{-24.92}{7}$$

Numerator: $-18.22 + (-6.7) = -24.92$.
Denominator: $14.75 + (-7.75) = 7$.

$$= -3.56$$

Divide: $-24.92/7 = -3.56$.

95. With a fractional expression, simplify numerator and denominator first, then divide.

$$\frac{-12.9 - (-10.98)}{0.5^2} = \frac{-12.9 - (-10.98)}{0.25}$$

Exponents: $0.5^2 = 0.25$.

$$= \frac{-12.9 + 10.98}{0.25}$$

Subtract: Add the opposite.

$$= \frac{-1.92}{0.25}$$

Add: $-12.9 + 10.98 = -1.92$.

$$= -7.68$$

Divide: $-1.92/0.25 = -7.68$.

97. The numerator must be simplified first. This requires multiplying and then subtracting.

$$\frac{-9.5 \cdot 1.6 - 3.7}{-3.6} = \frac{-15.2 - 3.7}{-3.6}$$

Multiply: $-9.5 \cdot 1.6 = -15.2$.

$$= \frac{-15.2 + (-3.7)}{-3.6}$$

Subtract: Add the opposite.

$$= \frac{-18.9}{-3.6}$$

Add: $-15.2 + (-3.7) = -18.9$.

$$= 5.25$$

Divide: $-18.9/(-3.6) = 5.25$.

99. With a fractional expression, simplify numerator and denominator first, then divide.

$$\frac{-14.98 - 9.6}{17.99 - 19.99} = \frac{-14.98 + (-9.6)}{17.99 + (-19.99)}$$

In numerator and denominator, add the opposite.

$$= \frac{-24.58}{-2}$$

Numerator: $-14.98 + (-9.6) = -24.58$.
Denominator: $17.99 + (-19.99) = -2$.

$$= 12.29$$

Divide: $-24.58/(-2) = 12.29$.

101. First, substitute $a = -2.21$, $c = 3.3$, and $d = 0.5$ in the given expression.

$$\frac{a-c}{d^2} = \frac{-2.21 - 3.3}{(0.5)^2} \qquad \text{Substitute.}$$

With a fractional expression, simplify numerator and denominator first, then divide.

$$= \frac{-2.21 - 3.3}{0.25} \qquad \text{Exponents: } 0.5^2 = 0.25.$$

$$= \frac{-2.21 + (-3.3)}{0.25} \qquad \text{Subtract: Add the opposite.}$$

$$= \frac{-5.51}{0.25} \qquad \text{Add: } -2.21 + (-3.3) = -5.51.$$

$$= -22.04 \qquad \text{Divide: } -5.51/0.25 = -22.04.$$

103. First substitute $a = -5.8$, $b = 10.37$, $c = 4.8$, and $d = 5.64$.

$$\frac{a-b}{c-d} = \frac{-5.8 - 10.37}{4.8 - 5.64} \qquad \text{Substitute.}$$

With a fractional expression, simplify numerator and denominator first, then divide.

$$= \frac{-5.8 + (-10.37)}{4.8 + (-5.64)} \qquad \begin{array}{l} \text{In numerator and denominator,} \\ \text{add the opposite.} \end{array}$$

$$= \frac{-16.17}{-0.84} \qquad \begin{array}{l} \text{Numerator: } -5.8 + (-10.37) = -16.17. \\ \text{Denominator: } 4.8 + (-5.64) = -0.84. \end{array}$$

$$= 19.25 \qquad \text{Divide: } -16.17/(-0.84) = 19.25.$$

105. First, substitute $a = -1.5$, $b = 4.7$, $c = 18.8$, and $d = -11.75$ in the given expression.

$$\frac{ab-c}{d} = \frac{(-1.5)(4.7) - (18.8)}{-11.75} \qquad \text{Substitute.}$$

The numerator must be simplified first. This requires multiplying and then subtracting.

$$= \frac{-7.05 - 18.8}{-11.75} \qquad \text{Multiply: } (-1.5)(4.7) = -7.05.$$

$$= \frac{-7.05 + (-18.8)}{-11.75} \qquad \text{Subtract: Add the opposite.}$$

$$= \frac{-25.85}{-11.75} \qquad \text{Add: } -7.05 + (-18.8) = -25.85.$$

$$= 2.2 \qquad \text{Divide: } -25.85/(-11.75) = 2.2.$$

107. To find the average number of biodiesel plants operating in each state, divide the total number of biodiesel plants by the number of states that have biodiesel plants.

Average biodiesel plants per state	=	Total number biodiesel plants	÷	number of states with biodiesel plants
	=	180	÷	40
	=	4.5		

Of the states that have biodiesel plants, there is an average of 4.5 plants per state.

109. To find the backlog for each examiner, divide the number of applications by the number of examiners.

Number applications per examiner	=	total applications	÷	number examiners
	=	770,000	÷	6000
	=	128.33...		

To the nearest tenth, each examiner has about 128.3 applications to catch up on.

111. To find the average money spent each month, divide the total amount of money spent by the number of months.

Average spent each month	=	total spent	÷	number months
	=	100	÷	6
	=	16.666...		

To the nearest hundredth of a million, the Pentagon spent about $16.67 million each month defending against cyber-attacks.

113. First, we need to find the total cost of the mailing. Add up the costs of each piece of mail.
$$2.77 + 3.16 + 3.94 = 9.87$$
So the total cost of the mailing was $9.87. Now, we need to find the total number of pounds in the mailing. Add up the weights of each piece of mail.
$$2 + 3 + 5 = 10$$
The total weight of the mailing was 10 pounds. Now, to find the average cost per pound, divide the total cost by the total number of pounds of mail.

$$
\begin{array}{c|c|c|c|c}
\boxed{\begin{array}{c}\text{cost to mail}\\ \text{per pound}\end{array}} & = & \boxed{\begin{array}{c}\text{total cost}\\ \text{of mailing}\end{array}} & \div & \boxed{\begin{array}{c}\text{total number}\\ \text{pounds of mail}\end{array}}
\end{array}
$$

$$
\begin{aligned}
&= \quad 9.87 \quad \div \quad 10\\
&= \quad 0.987
\end{aligned}
$$

To the nearest penny, it cost about \$0.99 for each pound of mail.

5.5   Fractions and Decimals

1. The fraction 59/16 is already reduced to lowest terms. The prime factorization of the denominator is $16 = 2 \cdot 2 \cdot 2 \cdot 2$. Since the prime factorization consists of only twos and fives, dividing 59 by 16 will yield a terminating decimal:

$$\frac{59}{16} = 3.6875$$

3. The fraction 35/4 is already reduced to lowest terms. The prime factorization of the denominator is $4 = 2 \cdot 2$. Since the prime factorization consists of only twos and fives, dividing 35 by 4 will yield a terminating decimal:

$$\frac{35}{4} = 8.75$$

5. The fraction 1/16 is already reduced to lowest terms. The prime factorization of the denominator is $16 = 2 \cdot 2 \cdot 2 \cdot 2$. Since the prime factorization consists of only twos and fives, dividing 1 by 16 will yield a terminating decimal:

$$\frac{1}{16} = 0.0625$$

7. First reduce the fraction 6/8 to lowest terms:

$$
\begin{aligned}
\frac{6}{8} &= \frac{2 \cdot 3}{2 \cdot 2 \cdot 2} && \textcolor{red}{\text{Prime factorization.}}\\
&= \frac{3}{2 \cdot 2} && \textcolor{red}{\text{Cancel common factors.}}\\
&= \frac{3}{4} && \textcolor{red}{\text{Simplify numerator and denominator.}}
\end{aligned}
$$

The prime factorization of the new denominator is $4 = 2 \cdot 2$. Since the prime factorization consists of only twos and fives, dividing 3 by 4 will yield a terminating decimal:

$$\frac{3}{4} = 0.75$$

9. The fraction $3/2$ is already reduced to lowest terms. The denominator is the prime number 2. Therefore, since the prime factorization of the denominator consists of only twos and fives, dividing 3 by 2 will yield a terminating decimal:

$$\frac{3}{2} = 1.5$$

11. First reduce the fraction $119/175$ to lowest terms:

$$\frac{119}{175} = \frac{7 \cdot 17}{5 \cdot 5 \cdot 7} \qquad \text{Prime factorization.}$$
$$= \frac{17}{5 \cdot 5} \qquad \text{Cancel common factors.}$$
$$= \frac{17}{25} \qquad \text{Simplify numerator and denominator.}$$

The prime factorization of the new denominator is $25 = 5 \cdot 5$. Since the prime factorization consists of only twos and fives, dividing 17 by 25 will yield a terminating decimal:

$$\frac{17}{25} = 0.68$$

13. The fraction $9/8$ is already reduced to lowest terms. The prime factorization of the denominator is $8 = 2 \cdot 2 \cdot 2$. Since the prime factorization consists of only twos and fives, dividing 9 by 8 will yield a terminating decimal:

$$\frac{9}{8} = 1.125$$

15. First reduce the fraction $78/240$ to lowest terms:

$$\frac{78}{240} = \frac{2 \cdot 3 \cdot 13}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5} \qquad \text{Prime factorization.}$$
$$= \frac{13}{2 \cdot 2 \cdot 2 \cdot 5} \qquad \text{Cancel common factors.}$$
$$= \frac{13}{40} \qquad \text{Simplify numerator and denominator.}$$

The prime factorization of the new denominator is $40 = 2 \cdot 2 \cdot 2 \cdot 5$. Since the prime factorization consists of only twos and fives, dividing 13 by 40 will yield a terminating decimal:

$$\frac{13}{40} = 0.325$$

17. First reduce the fraction 25/10 to lowest terms:

$$\frac{25}{10} = \frac{5 \cdot 5}{2 \cdot 5} \qquad \text{\color{red}Prime factorization.}$$

$$= \frac{5}{2} \qquad \text{\color{red}Cancel common factors.}$$

$$= \frac{5}{2} \qquad \text{\color{red}Simplify numerator and denominator.}$$

The new denominator is the prime number 2. Therefore, since the prime factorization of the new denominator consists of only twos and fives, dividing 5 by 2 will yield a terminating decimal:

$$\frac{5}{2} = 2.5$$

19. First reduce the fraction 9/24 to lowest terms:

$$\frac{9}{24} = \frac{3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3} \qquad \text{\color{red}Prime factorization.}$$

$$= \frac{3}{2 \cdot 2 \cdot 2} \qquad \text{\color{red}Cancel common factors.}$$

$$= \frac{3}{8} \qquad \text{\color{red}Simplify numerator and denominator.}$$

The prime factorization of the new denominator is $8 = 2 \cdot 2 \cdot 2$. Since the prime factorization consists of only twos and fives, dividing 3 by 8 will yield a terminating decimal:

$$\frac{3}{8} = 0.375$$

21. First reduce the fraction 256/180 to lowest terms:

$$\frac{256}{180} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5} \qquad \text{\color{red}Prime factorization.}$$

$$= \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 5} \qquad \text{\color{red}Cancel common factors.}$$

$$= \frac{64}{45} \qquad \text{\color{red}Simplify numerator and denominator.}$$

The prime factorization of the denominator is $45 = 3 \cdot 3 \cdot 5$. Since the prime factorization does not consist of only twos and fives, dividing 64 by 45 will yield a repeating decimal:

$$\frac{64}{45} = 1.4\overline{2}$$

23. First reduce the fraction 364/12 to lowest terms:

$$\frac{364}{12} = \frac{2 \cdot 2 \cdot 7 \cdot 13}{2 \cdot 2 \cdot 3} \qquad \textcolor{red}{\text{Prime factorization.}}$$
$$= \frac{7 \cdot 13}{3} \qquad \textcolor{red}{\text{Cancel common factors.}}$$
$$= \frac{91}{3} \qquad \textcolor{red}{\text{Simplify numerator and denominator.}}$$

The prime factorization of the denominator is $3 = 3$. Since the prime factorization does not consist of only twos and fives, dividing 91 by 3 will yield a repeating decimal:

$$\frac{91}{3} = 30.\overline{3}$$

25. The fraction 81/110 is already reduced to lowest terms. The prime factorization of the denominator is $110 = 2 \cdot 5 \cdot 11$. Since the prime factorization does not consist of only twos and fives, dividing 81 by 110 will yield a repeating decimal:

$$\frac{81}{110} = 0.7\overline{36}$$

27. The fraction 76/15 is already reduced to lowest terms. The prime factorization of the denominator is $15 = 3 \cdot 5$. Since the prime factorization does not consist of only twos and fives, dividing 76 by 15 will yield a repeating decimal:

$$\frac{76}{15} = 5.0\overline{6}$$

29. The fraction 50/99 is already reduced to lowest terms. The prime factorization of the denominator is $99 = 3 \cdot 3 \cdot 11$. Since the prime factorization does not consist of only twos and fives, dividing 50 by 99 will yield a repeating decimal:

$$\frac{50}{99} = 0.\overline{50}$$

31. The fraction 61/15 is already reduced to lowest terms. The prime factorization of the denominator is $15 = 3 \cdot 5$. Since the prime factorization does not consist of only twos and fives, dividing 61 by 15 will yield a repeating decimal:

$$\frac{61}{15} = 4.0\overline{6}$$

33. First reduce the fraction 98/66 to lowest terms:

$$
\begin{aligned}
\frac{98}{66} &= \frac{2 \cdot 7 \cdot 7}{2 \cdot 3 \cdot 11} && \text{\color{red}Prime factorization.} \\
&= \frac{7 \cdot 7}{3 \cdot 11} && \text{\color{red}Cancel common factors.} \\
&= \frac{49}{33} && \text{\color{red}Simplify numerator and denominator.}
\end{aligned}
$$

The prime factorization of the denominator is $33 = 3 \cdot 11$. Since the prime factorization does not consist of only twos and fives, dividing 49 by 33 will yield a repeating decimal:

$$\frac{49}{33} = 1.\overline{48}$$

35. First reduce the fraction 190/495 to lowest terms:

$$
\begin{aligned}
\frac{190}{495} &= \frac{2 \cdot 5 \cdot 19}{3 \cdot 3 \cdot 5 \cdot 11} && \text{\color{red}Prime factorization.} \\
&= \frac{2 \cdot 19}{3 \cdot 3 \cdot 11} && \text{\color{red}Cancel common factors.} \\
&= \frac{38}{99} && \text{\color{red}Simplify numerator and denominator.}
\end{aligned}
$$

The prime factorization of the denominator is $99 = 3 \cdot 3 \cdot 11$. Since the prime factorization does not consist of only twos and fives, dividing 38 by 99 will yield a repeating decimal:

$$\frac{38}{99} = 0.\overline{38}$$

37. The fraction 13/15 is already reduced to lowest terms. The prime factorization of the denominator is $15 = 3 \cdot 5$. Since the prime factorization does not consist of only twos and fives, dividing 13 by 15 will yield a repeating decimal:

$$\frac{13}{15} = 0.8\overline{6}$$

39. First reduce the fraction 532/21 to lowest terms:

$$
\begin{aligned}
\frac{532}{21} &= \frac{2 \cdot 2 \cdot 7 \cdot 19}{3 \cdot 7} && \text{\color{red}Prime factorization.} \\
&= \frac{2 \cdot 2 \cdot 19}{3} && \text{\color{red}Cancel common factors.} \\
&= \frac{76}{3} && \text{\color{red}Simplify numerator and denominator.}
\end{aligned}
$$

The prime factorization of the denominator is $3 = 3$. Since the prime factorization does not consist of only twos and fives, dividing 76 by 3 will yield a repeating decimal:

$$\frac{76}{3} = 25.\overline{3}$$

41. First reduce the fraction 26/198 to lowest terms:

$$\frac{26}{198} = \frac{2 \cdot 13}{2 \cdot 3 \cdot 3 \cdot 11} \qquad \textcolor{red}{\text{Prime factorization.}}$$

$$= \frac{13}{3 \cdot 3 \cdot 11} \qquad \textcolor{red}{\text{Cancel common factors.}}$$

$$= \frac{13}{99} \qquad \textcolor{red}{\text{Simplify numerator and denominator.}}$$

The prime factorization of the denominator is $99 = 3 \cdot 3 \cdot 11$. Since the prime factorization does not consist of only twos and fives, dividing 13 by 99 will yield a repeating decimal:

$$\frac{13}{99} = 0.\overline{13}$$

43. The fraction 47/66 is already reduced to lowest terms. The prime factorization of the denominator is $66 = 2 \cdot 3 \cdot 11$. Since the prime factorization does not consist of only twos and fives, dividing 47 by 66 will yield a repeating decimal:

$$\frac{47}{66} = 0.7\overline{12}$$

45. First convert the fraction 7/4 into an equivalent terminating decimal. Then subtract the decimals in the new expression.

$$\frac{7}{4} - 7.4 = 1.75 - 7.4 \qquad \textcolor{red}{\text{Replace 7/4 with 1.75.}}$$

$$= -5.65 \qquad \textcolor{red}{\text{Subtract.}}$$

Thus, $7/4 - 7.4 = -5.65$.

47. First convert the fraction 7/5 into an equivalent terminating decimal. Then add the decimals in the new expression.

$$\frac{7}{5} + 5.31 = 1.4 + 5.31 \qquad \textcolor{red}{\text{Replace 7/5 with 1.4.}}$$

$$= 6.71 \qquad \textcolor{red}{\text{Add.}}$$

Thus, $7/5 + 5.31 = 6.71$.

49. First convert the fraction 9/10 into an equivalent terminating decimal. Then subtract the decimals in the new expression.

$$\frac{9}{10} - 8.61 = 0.9 - 8.61 \qquad \text{Replace 9/10 with 0.9.}$$
$$= -7.71 \qquad \text{Subtract.}$$

Thus, $9/10 - 8.61 = -7.71$.

51. First convert the fraction 6/5 into an equivalent terminating decimal. Then subtract the decimals in the new expression.

$$\frac{6}{5} - 7.65 = 1.2 - 7.65 \qquad \text{Replace 6/5 with 1.2.}$$
$$= -6.45 \qquad \text{Subtract.}$$

Thus, $6/5 - 7.65 = -6.45$.

53. The fraction 7/6 is equivalent to a repeating decimal. Therefore, the strategy is to instead convert the decimal 2.9 into an equivalent fraction, and then subtract the fractions in the new expression.

$$\frac{7}{6} - 2.9 = \frac{7}{6} - \frac{29}{10} \qquad \text{Replace 2.9 with 29/10.}$$
$$= \frac{7 \cdot 5}{6 \cdot 5} - \frac{29 \cdot 3}{10 \cdot 3} \qquad \text{Equivalent fractions with LCD = 30.}$$
$$= \frac{35}{30} - \frac{87}{30} \qquad \text{Simplify numerators and denominators.}$$
$$= -\frac{52}{30} \qquad \text{Subtract.}$$
$$= -\frac{26}{15} \qquad \text{Reduce to lowest terms.}$$

Thus,

$$\frac{7}{6} - 2.9 = -\frac{26}{15}$$

55. The fraction 4/3 is equivalent to a repeating decimal. Therefore, the strategy is to instead convert the decimal 0.32 into an equivalent fraction, and

then subtract the fractions in the new expression.

$$-\frac{4}{3} - 0.32 = -\frac{4}{3} - \frac{8}{25} \qquad \text{Replace 0.32 with 8/25.}$$
$$= -\frac{4 \cdot 25}{3 \cdot 25} - \frac{8 \cdot 3}{25 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 75.$$
$$= -\frac{100}{75} - \frac{24}{75} \qquad \text{Simplify numerators and denominators.}$$
$$= -\frac{124}{75} \qquad \text{Subtract.}$$

Thus,

$$-\frac{4}{3} - 0.32 = -\frac{124}{75}$$

57. The fraction $2/3$ is equivalent to a repeating decimal. Therefore, the strategy is to instead convert the decimal 0.9 into an equivalent fraction, and then add the fractions in the new expression.

$$-\frac{2}{3} + 0.9 = -\frac{2}{3} + \frac{9}{10} \qquad \text{Replace 0.9 with 9/10.}$$
$$= -\frac{2 \cdot 10}{3 \cdot 10} + \frac{9 \cdot 3}{10 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 30.$$
$$= -\frac{20}{30} + \frac{27}{30} \qquad \text{Simplify numerators and denominators.}$$
$$= \frac{7}{30} \qquad \text{Add.}$$

Thus,

$$-\frac{2}{3} + 0.9 = \frac{7}{30}$$

59. The fraction $4/3$ is equivalent to a repeating decimal. Therefore, the strategy is to instead convert the decimal 2.6 into an equivalent fraction, and then subtract the fractions in the new expression.

$$\frac{4}{3} - 2.6 = \frac{4}{3} - \frac{13}{5} \qquad \text{Replace 2.6 with 13/5.}$$
$$= \frac{4 \cdot 5}{3 \cdot 5} - \frac{13 \cdot 3}{5 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 15.$$
$$= \frac{20}{15} - \frac{39}{15} \qquad \text{Simplify numerators and denominators.}$$
$$= -\frac{19}{15} \qquad \text{Subtract.}$$

Thus,

$$\frac{4}{3} - 2.6 = -\frac{19}{15}$$

61. The fraction 5/6 is equivalent to a repeating decimal. Therefore, the strategy is to instead convert the decimal 2.375 into an equivalent fraction, and then add the fractions in the new expression.

$$\frac{5}{6} + 2.375 = \frac{5}{6} + \frac{19}{8} \qquad \text{Replace 2.375 with 19/8.}$$
$$= \frac{5 \cdot 4}{6 \cdot 4} + \frac{19 \cdot 3}{8 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 24.$$
$$= \frac{20}{24} + \frac{57}{24} \qquad \text{Simplify numerators and denominators.}$$
$$= \frac{77}{24} \qquad \text{Add.}$$

Thus,

$$\frac{5}{6} + 2.375 = \frac{77}{24}$$

63. First convert the fraction 11/8 into an equivalent terminating decimal. Then add the decimals in the new expression.

$$\frac{11}{8} + 8.2 = 1.375 + 8.2 \qquad \text{Replace 11/8 with 1.375.}$$
$$= 9.575 \qquad \text{Add.}$$

Thus, $11/8 + 8.2 = 9.575$.

65. First convert the fraction 7/10 into an equivalent terminating decimal. Then add the decimals in the new expression.

$$-\frac{7}{10} + 1.2 = -0.7 + 1.2 \qquad \text{Replace 7/10 with 0.7.}$$
$$= 0.5 \qquad \text{Add.}$$

Thus, $-7/10 + 1.2 = 0.5$.

67. The fraction 11/6 is equivalent to a repeating decimal. Therefore, the strategy is to instead convert the decimal 0.375 into an equivalent fraction, and then add the fractions in the new expression.

$$-\frac{11}{6} + 0.375 = -\frac{11}{6} + \frac{3}{8} \qquad \text{Replace 0.375 with 3/8.}$$
$$= -\frac{11 \cdot 4}{6 \cdot 4} + \frac{3 \cdot 3}{8 \cdot 3} \qquad \text{Equivalent fractions with LCD} = 24.$$
$$= -\frac{44}{24} + \frac{9}{24} \qquad \text{Simplify numerators and denominators.}$$
$$= -\frac{35}{24} \qquad \text{Add.}$$

Thus,

$$-\frac{11}{6} + 0.375 = -\frac{35}{24}$$

5.6   Equations with Decimals

1.

$$
\begin{aligned}
5.57x - 2.45x &= 5.46 &&\text{Original equation.}\\
3.12x &= 5.46 &&\text{Combine like terms on the left side.}\\
x &= 1.75 &&\text{Divide both sides by 3.12.}
\end{aligned}
$$

3.

$$
\begin{aligned}
-5.8x + 0.32 + 0.2x &= -6.96 &&\text{Original equation.}\\
-5.6x + 0.32 &= -6.96 &&\text{Combine like terms on the left side.}\\
-5.6x &= -7.28 &&\text{Add } -0.32 \text{ to both sides.}\\
x &= 1.3 &&\text{Divide both sides by } -5.6.
\end{aligned}
$$

5.

$$
\begin{aligned}
-4.9x + 88.2 &= 24.5 &&\text{Original equation.}\\
-4.9x &= -63.7 &&\text{Add } -88.2 \text{ to both sides.}\\
x &= 13 &&\text{Divide both sides by } -4.9.
\end{aligned}
$$

7.

$$
\begin{aligned}
0.35x - 63.58 &= 55.14 &&\text{Original equation.}\\
0.35x &= 118.72 &&\text{Add 63.58 to both sides.}\\
x &= 339.2 &&\text{Divide both sides by 0.35.}
\end{aligned}
$$

9.

$$
\begin{aligned}
-10.3x + 82.4 &= 0 &&\text{Original equation.}\\
-10.3x &= -82.4 &&\text{Add } -82.4 \text{ to both sides.}\\
x &= 8 &&\text{Divide both sides by } -10.3.
\end{aligned}
$$

11.

$$
\begin{aligned}
-12.5x + 13.5 &= 0 & &\text{Original equation.} \\
-12.5x &= -13.5 & &\text{Add } -13.5 \text{ to both sides.} \\
x &= 1.08 & &\text{Divide both sides by } -12.5.
\end{aligned}
$$

13.

$$
\begin{aligned}
7.3x - 8.9 - 8.34x &= 2.8 & &\text{Original equation.} \\
-1.04x - 8.9 &= 2.8 & &\text{Combine like terms on the left side.} \\
-1.04x &= 11.7 & &\text{Add 8.9 to both sides.} \\
x &= -11.25 & &\text{Divide both sides by } -1.04.
\end{aligned}
$$

15.

$$
\begin{aligned}
-0.2x + 2.2x &= 6.8 & &\text{Original equation.} \\
2x &= 6.8 & &\text{Combine like terms on the left side.} \\
x &= 3.4 & &\text{Divide both sides by 2.}
\end{aligned}
$$

17.

$$
\begin{aligned}
6.24x - 5.2 &= 5.2x & &\text{Original equation.} \\
-5.2 &= 5.2x - 6.24x & &\text{Add } -6.24x \text{ to both sides.} \\
-5.2 &= -1.04x & &\text{Combine like terms on the right side.} \\
5 &= x & &\text{Divide both sides by } -1.04.
\end{aligned}
$$

19.

$$
\begin{aligned}
-0.7x - 2.4 &= -3.7x - 8.91 & &\text{Original equation.} \\
-0.7x + 3.7x - 2.4 &= -8.91 & &\text{Add } 3.7x \text{ to both sides.} \\
3x - 2.4 &= -8.91 & &\text{Combine like terms on the left side.} \\
3x &= -8.91 + 2.4 & &\text{Add 2.4 to both sides.} \\
3x &= -6.51 & &\text{Combine like terms on the right side.} \\
x &= -2.17 & &\text{Divide both sides by 3.}
\end{aligned}
$$

21.

$$
\begin{aligned}
-4.9x &= -5.4x + 8.4 & &\text{Original equation.} \\
-4.9x + 5.4x &= 8.4 & &\text{Add } 5.4x \text{ to both sides.} \\
0.5x &= 8.4 & &\text{Combine like terms on the left side.} \\
x &= 16.8 & &\text{Divide both sides by 0.5.}
\end{aligned}
$$

23.

$$
\begin{array}{ll}
-2.8x = -2.3x - 6.5 & \text{Original equation.} \\
-2.8x + 2.3x = -6.5 & \text{Add } 2.3x \text{ to both sides.} \\
-0.5x = -6.5 & \text{Combine like terms on the left side.} \\
x = 13 & \text{Divide both sides by } -0.5.
\end{array}
$$

25.

$$
\begin{array}{ll}
-2.97x - 2.6 = -3.47x + 7.47 & \text{Original equation.} \\
-2.97x + 3.47x - 2.6 = 7.47 & \text{Add } 3.47x \text{ to both sides.} \\
0.5x - 2.6 = 7.47 & \text{Combine like terms on the left side.} \\
0.5x = 7.47 + 2.6 & \text{Add } 2.6 \text{ to both sides.} \\
0.5x = 10.07 & \text{Combine like terms on the right side.} \\
x = 20.14 & \text{Divide both sides by } 0.5.
\end{array}
$$

27.

$$
\begin{array}{ll}
-1.7x = -0.2x - 0.6 & \text{Original equation.} \\
-1.7x + 0.2x = -0.6 & \text{Add } 0.2x \text{ to both sides.} \\
-1.5x = -0.6 & \text{Combine like terms on the left side.} \\
x = 0.4 & \text{Divide both sides by } -1.5.
\end{array}
$$

29.

$$
\begin{array}{ll}
-1.02x + 7.08 = -2.79x & \text{Original equation.} \\
7.08 = -2.79x + 1.02x & \text{Add } 1.02x \text{ to both sides.} \\
7.08 = -1.77x & \text{Combine like terms on the right side.} \\
-4 = x & \text{Divide both sides by } -1.77.
\end{array}
$$

31.

$$
\begin{array}{ll}
-4.75x - 6.77 = -7.45x + 3.49 & \text{Original equation.} \\
-4.75x + 7.45x - 6.77 = 3.49 & \text{Add } 7.45x \text{ to both sides.} \\
2.7x - 6.77 = 3.49 & \text{Combine like terms on the left side.} \\
2.7x = 3.49 + 6.77 & \text{Add } 6.77 \text{ to both sides.} \\
2.7x = 10.26 & \text{Combine like terms on the right side.} \\
x = 3.8 & \text{Divide both sides by } 2.7.
\end{array}
$$

33.

$$-4.06x - 7.38 = 4.94x$$ Original equation.
$$-7.38 = 4.94x + 4.06x$$ Add $4.06x$ to both sides.
$$-7.38 = 9x$$ Combine like terms on the right side.
$$-0.82 = x$$ Divide both sides by 9.

35.

$$2.3 + 0.1(x + 2.9) = 6.9$$ Original equation.
$$2.3 + 0.1x + 0.29 = 6.9$$ Apply the distributive property.
$$0.1x + 2.59 = 6.9$$ Combine like terms on the left side.
$$0.1x = 4.31$$ Add -2.59 to both sides.
$$x = 43.1$$ Divide both sides by 0.1.

37.

$$0.5(1.5x - 6.58) = 6.88$$ Original equation.
$$0.75x - 3.29 = 6.88$$ Apply the distributive property.
$$0.75x = 10.17$$ Add 3.29 to both sides.
$$x = 13.56$$ Divide both sides by 0.75.

39.

$$-6.3x - 0.4(x - 1.8) = -16.03$$ Original equation.
$$-6.3x - 0.4x + 0.72 = -16.03$$ Apply the distributive property.
$$-6.7x + 0.72 = -16.03$$ Combine like terms on the left side.
$$-6.7x = -16.75$$ Add -0.72 to both sides.
$$x = 2.5$$ Divide both sides by -6.7.

41.

$$2.4(0.3x + 3.2) = -11.4$$ Original equation.
$$0.72x + 7.68 = -11.4$$ Apply the distributive property.
$$0.72x = -19.08$$ Add -7.68 to both sides.
$$x = -26.5$$ Divide both sides by 0.72.

43.

$$-0.8(0.3x + 0.4) = -11.3 \qquad \text{Original equation.}$$
$$-0.24x - 0.32 = -11.3 \qquad \text{Apply the distributive property.}$$
$$-0.24x = -10.98 \qquad \text{Add 0.32 to both sides.}$$
$$x = 45.75 \qquad \text{Divide both sides by } -0.24.$$

45.

$$-7.57 - 2.42(x + 5.54) = 6.95 \qquad \text{Original equation.}$$
$$-7.57 - 2.42x - 13.4068 = 6.95 \qquad \text{Apply the distributive property.}$$
$$-2.42x - 20.9768 = 6.95 \qquad \text{Combine like terms on the left side.}$$
$$-2.42x = 27.9268 \qquad \text{Add 20.9768 to both sides.}$$
$$x = -11.54 \qquad \text{Divide both sides by } -2.42.$$

47.

$$-1.7 - 5.56(x + 6.1) = 12.2 \qquad \text{Original equation.}$$
$$-1.7 - 5.56x - 33.916 = 12.2 \qquad \text{Apply the distributive property.}$$
$$-5.56x - 35.616 = 12.2 \qquad \text{Combine like terms on the left side.}$$
$$-5.56x = 47.816 \qquad \text{Add 35.616 to both sides.}$$
$$x = -8.6 \qquad \text{Divide both sides by } -5.56.$$

49.

$$4.3x - 0.7(x + 2.1) = 8.61 \qquad \text{Original equation.}$$
$$4.3x - 0.7x - 1.47 = 8.61 \qquad \text{Apply the distributive property.}$$
$$3.6x - 1.47 = 8.61 \qquad \text{Combine like terms on the left side.}$$
$$3.6x = 10.08 \qquad \text{Add 1.47 to both sides.}$$
$$x = 2.8 \qquad \text{Divide both sides by 3.6.}$$

51.

$$-4.8x + 3.3(x - 0.4) = -7.05 \qquad \text{Original equation.}$$
$$-4.8x + 3.3x - 1.32 = -7.05 \qquad \text{Apply the distributive property.}$$
$$-1.5x - 1.32 = -7.05 \qquad \text{Combine like terms on the left side.}$$
$$-1.5x = -5.73 \qquad \text{Add 1.32 to both sides.}$$
$$x = 3.82 \qquad \text{Divide both sides by } -1.5.$$

53.

$$0.9(6.2x - 5.9) = 3.4(3.7x + 4.3) - 1.8 \qquad \text{Original equation.}$$

$5.58x - 5.31 = 12.58x + 14.62 - 1.8$	Apply the distributive property on both sides.
$5.58x - 5.31 = 12.58x + 12.82$	Combine like terms on the right side.
$-5.31 = 12.58x - 5.58x + 12.82$	Add $-5.58x$ to both sides.
$-5.31 = 7x + 12.82$	Combine like terms on the right side.
$-5.31 - 12.82 = 7x$	Add -12.82 to both sides.
$-18.13 = 7x$	Combine like terms on the left side.
$-2.59 = x$	Divide both sides by 7.

55.

$$-1.8(-1.6x + 1.7) = -1.8(-3.6x - 4.1) \qquad \text{Original equation.}$$

$2.88x - 3.06 = 6.48x + 7.38$	Apply the distributive property on both sides.
$2.88x - 6.48x - 3.06 = 7.38$	Add $-6.48x$ to both sides.
$-3.6x - 3.06 = 7.38$	Combine like terms on the left side.
$-3.6x = 7.38 + 3.06$	Add 3.06 to both sides.
$-3.6x = 10.44$	Combine like terms on the right side.
$x = -2.9$	Divide both sides by -3.6.

57.

$$0.9(0.4x + 2.5) - 2.5 = -1.9(0.8x + 3.1) \qquad \text{Original equation.}$$

$0.36x + 2.25 - 2.5 = -1.52x - 5.89$	Apply the distributive property on both sides.
$0.36x - 0.25 = -1.52x - 5.89$	Combine like terms on the left side.
$0.36x + 1.52x - 0.25 = -5.89$	Add $1.52x$ to both sides.
$1.88x - 0.25 = -5.89$	Combine like terms on the left side.
$1.88x = -5.89 + 0.25$	Add 0.25 to both sides.
$1.88x = -5.64$	Combine like terms on the right side.
$x = -3$	Divide both sides by 1.88.

59. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let N represent the number of bird houses created.

2. *Set Up an Equation.* Note that

Fixed costs	plus	Number made	times	Unit cost	is	Total cost
200	+	N	$\cdot$	3.00	=	296.00

Hence, our equation is
$$200 + N \cdot 3 = 296.$$

3. *Solve the Equation.* Subtract 200 from each side, then divide the resulting equation by 3.

$200 + 3N = 296$	Original equation.
$200 + 3N - 200 = 296 - 200$	Subtract 200 from both sides.
$3N = 96$	Simplify: $296 - 200 = 96$.
$\dfrac{3N}{3} = \dfrac{96}{3}$	Divide both sides by 3.
$N = 32$	Divide: $96/3 = 32$.

4. *Answer the Question.* Stacy created 32 bird houses.

5. *Look Back.* 32 bird houses at $3.00 apiece cost $96. If we add the fixed costs $200, the total is $296.00.

61. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let N represent the number of staplers purchased.

2. *Set Up an Equation.* Note that

Price per stapler	times	Number of staplers	is	Full Purchase Price
1.50	$\cdot$	N	=	36.00

Hence, our equation is
$$1.50N = 36.00.$$

3. *Solve the Equation.* Divide both sides of the equation by 1.50.

$\dfrac{1.50N}{1.50} = \dfrac{36.00}{1.50}$	Divide both sides by 1.50.
$N = 24$	Divide. $36.00/1.50 = 24$.

4. *Answer the Question.* The business purchased 24 staplers.

5. *Look Back.* Let's calculate the cost of 24 staplers at $1.50 apiece.

$$\text{Total Cost} = \text{Number of staplers} \cdot \text{Unit Price}$$
$$= 24 \cdot 1.50$$
$$= 36.00$$

Thus, at $1.50 apiece, 24 staplers will cost $36.00. Our answer checks.

63. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let N represent the number of table cloths created.

2. *Set Up an Equation.* Note that

Fixed costs	plus	Number made	times	Unit cost	is	Total cost
100	$+$	N	$\cdot$	2.75	$=$	221.00

Hence, our equation is

$$100 + N \cdot 2.75 = 221.$$

3. *Solve the Equation.* Subtract 100 from each side, then divide the resulting equation by 2.75.

$$100 + 2.75N = 221 \qquad \text{Original equation.}$$
$$100 + 2.75N - 100 = 221 - 100 \qquad \text{Subtract 100 from both sides.}$$
$$2.75N = 121 \qquad \text{Simplify: } 221 - 100 = 121.$$
$$\frac{2.75N}{2.75} = \frac{121}{2.75} \qquad \text{Divide both sides by 2.75.}$$
$$N = 44 \qquad \text{Divide: } 121/2.75 = 44.$$

4. *Answer the Question.* Julie created 44 table cloths.

5. *Look Back.* 44 table cloths at $2.75 apiece cost $121. If we add the fixed costs $100, the total is $221.00.

65. The formula governing the relation between the circumference and diameter of a circle is

$$C = \pi d.$$

The 60 feet of decorative fencing will be the circumference of the circular garden. Substitute 60 for C and 3.14 for π.

$$60 = 3.14d$$

Divide both sides of the equation by 3.14.

$$\frac{60}{3.14} = \frac{3.14d}{3.14}$$
$$\frac{60}{3.14} = d$$

We need to round to the nearest hundredth. This requires that we carry the division one additional place to the right of the hundredths place (i.e., to the thousandths place).

$$d \approx 19.108$$

For the final step, we must round 19.108 to the nearest hundredth. In the schematic that follows, we've boxed the hundredths digit (the "rounding digit") and the "test digit" that follows the "rounding digit."

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest hundredth of an foot, the diameter of the circle is approximately $d \approx 19.11$ feet.

67. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let N represent the number of tickets purchased by the YMCA of Sacramento.

2. *Set Up an Equation.* Note that

Price per Ticket	times	Number of Tickets	is	Full Purchase Price
4.25	$\cdot$	N	$=$	1,000

Hence, our equation is

$$4.25N = 1000.$$

3. *Solve the Equation.* Divide both sides of the equation by 4.25.

$$\frac{4.25N}{4.25} = \frac{1000}{4.25}$$ Divide both sides by 4.25.

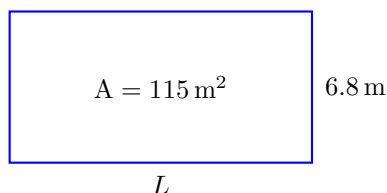$$N \approx 235$$ Divide. Truncate to units place.

4. *Answer the Question.* Because they cannot buy a fractional part of a ticket, we must truncate the answer to the units place. The YMCA of Sacramento can purchase 235 tickets.

5. *Look Back.* Let's calculate the cost of 235 tickets at $4.25 apiece.

$$\text{Cost} = \text{Number of Tickets} \cdot \text{Ticket Price}$$
$$= 235 \cdot 4.25$$
$$= 998.75;$$

Thus, at $4.25 apiece, 235 tickets will cost $998.75. As the YMCA of Sacramento has $1,000 to purchase tickets, note that they don't have enough money left for another ticket.

69. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let N represent the number of mechanical pencils purchased.

2. *Set Up an Equation.* Note that

Price per mechanical pencil	times	Number of mechanical pencils	is	Full Purchase Price
2.25	$\cdot$	N	$=$	65.25

Hence, our equation is
$$2.25N = 65.25.$$

3. *Solve the Equation.* Divide both sides of the equation by 2.25.

$$\frac{2.25N}{2.25} = \frac{65.25}{2.25}$$ Divide both sides by 2.25.

$$N = 29$$ Divide. $65.25/2.25 = 29$.

4. *Answer the Question.* The business purchased 29 mechanical pencils.

5. *Look Back.* Let's calculate the cost of 29 mechanical pencils at $2.25 apiece.

$$\text{Total Cost} = \text{Number of mechanical pencils} \cdot \text{Unit Price}$$
$$= 29 \cdot 2.25$$
$$= 65.25$$

Thus, at $2.25 apiece, 29 mechanical pencils will cost $65.25. Our answer checks.

71. The formula governing the relation between the circumference and diameter of a circle is
$$C = \pi d.$$
The 61 feet of decorative fencing will be the circumference of the circular garden. Substitute 61 for C and 3.14 for π.

$$61 = 3.14d$$

Divide both sides of the equation by 3.14.

$$\frac{61}{3.14} = \frac{3.14d}{3.14}$$
$$\frac{61}{3.14} = d$$

We need to round to the nearest hundredth. This requires that we carry the division one additional place to the right of the hundredths place (i.e., to the thousandths place).

$$d \approx 19.426$$

For the final step, we must round 19.426 to the nearest hundredth. In the schematic that follows, we've boxed the hundredths digit (the "rounding digit") and the "test digit" that follows the "rounding digit."

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest hundredth of an foot, the diameter of the circle is approximately $d \approx 19.43$ feet.

73. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We will use a sketch to define our variables.

A rectangle with area $A = 100\,\text{m}^2$, width $7.5\,\text{m}$ on the right side, and length L along the bottom.

Note that L represents the length of the rectangle.

2. *Set Up an Equation.* The area A of a rectangle is given by the formula

$$A = LW,$$

where L and W represent the length and width of the rectangle, respectively. Substitute 100 for A and 7.5 for W in the formula to obtain

$$100 = L(7.5),$$

or equivalently,
$$100 = 7.5L.$$

3. *Solve the Equation.* Divide both sides of the last equation by 7.5, then round your answer to the nearest tenth.

$$\frac{100}{7.5} = \frac{7.5L}{7.5} \qquad \text{Divide both sides by 7.5.}$$
$$13.33 \approx L \qquad \text{Divide.}$$

To round to the nearest tenth of a meter, identify the rounding and test digits.

Test digit

$$13.\boxed{3}\,\boxed{3}$$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest tenth of a meter, the length of the rectangle is approximately $L \approx 13.3$ meters.

4. *Answer the Question.* To the nearest tenth of a meter, the length of the rectangular plot is $L \approx 13.3$ meters.

5. *Look Back.* We have $L \approx 13.3$ meters and $W = 8.9$ meters. Multiply length and width to find the area.

$$\text{Area} \approx (13.3\,\text{m})(7.5\,\text{m}) \approx 99.75\,\text{m}^2.$$

Note that this is very nearly the exact area of 100 square meters. The discrepancy is due to the fact that we found the length rounded to the nearest tenth of a meter.

75. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let N represent the number of tickets purchased by the Boys and Girls club of Eureka.

2. *Set Up an Equation.* Note that

Price per Ticket	times	Number of Tickets	is	Full Purchase Price
4.50	$\cdot$	N	$=$	1,300

Hence, our equation is
$$4.50N = 1300.$$

3. *Solve the Equation.* Divide both sides of the equation by 4.50.

$$\frac{4.50N}{4.50} = \frac{1300}{4.50} \qquad \textcolor{red}{\text{Divide both sides by 4.50.}}$$
$$N \approx 288 \qquad \textcolor{red}{\text{Divide. Truncate to units place.}}$$

4. *Answer the Question.* Because they cannot buy a fractional part of a ticket, we must truncate the answer to the units place. The Boys and Girls club of Eureka can purchase 288 tickets.

5. *Look Back.* Let's calculate the cost of 288 tickets at $4.50 apiece.

$$\text{Cost} = \text{Number of Tickets} \cdot \text{Ticket Price}$$
$$= 288 \cdot 4.50$$
$$= 1296;$$

Thus, at $4.50 apiece, 288 tickets will cost $1296. As the Boys and Girls club of Eureka has $1,300 to purchase tickets, note that they don't have enough money left for another ticket.

77. We will follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We will use a sketch to define our variables.

   Note that L represents the length of the rectangle.

2. *Set Up an Equation.* The area A of a rectangle is given by the formula

$$A = LW,$$

   where L and W represent the length and width of the rectangle, respectively. Substitute 115 for A and 6.8 for W in the formula to obtain

$$115 = L(6.8),$$

   or equivalently,

$$115 = 6.8L.$$

3. *Solve the Equation.* Divide both sides of the last equation by 6.8, then round your answer to the nearest tenth.

$$\frac{115}{6.8} = \frac{6.8L}{6.8} \qquad \text{Divide both sides by 6.8.}$$
$$16.91 \approx L \qquad \text{Divide.}$$

   To round to the nearest tenth of a meter, identify the rounding and test digits.

   Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest tenth of a meter, the length of the rectangle is approximately $L \approx 16.9$ meters.

4. *Answer the Question.* To the nearest tenth of a meter, the length of the rectangular plot is $L \approx 16.9$ meters.

5. *Look Back.* We have $L \approx 16.9$ meters and $W = 8.9$ meters. Multiply length and width to find the area.

$$\text{Area} \approx (16.9 \,\text{m})(6.8 \,\text{m}) \approx 114.92 \,\text{m}^2.$$

Note that this is very nearly the exact area of 115 square meters. The discrepancy is due to the fact that we found the length rounded to the nearest tenth of a meter.

79. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can satisfy this requirement by simply stating "Let x represent crude oil inventories in millions of barrels for the previous week before the decline."

2. *Set up an Equation.* "Crude oil inventories last week experienced a decline and result in the crude oil inventories now" becomes

Oil inventory last week	minus	decline in inventory	gives	Oil inventory this week
x	$-$	3.8	$=$	353.9

3. *Solve the Equation.* To "undo" the subtraction of 3.8, add both sides of the equation by 3.8.

$$x - 3.8 = 353.9 \qquad \text{Original equation.}$$
$$x - 3.8 + 3.8 = 353.9 + 3.8 \qquad \text{Add both sides of the equation by 3.8.}$$
$$x = 357.7 \qquad \text{On the left, adding by 3.8 "undoes" the effect}$$
of subtracting by 3.8 and returns x. On the right, $353.9 + 3.8 = 357.7$.

4. *Answer the Question.* The previous week, US crude oil inventories were 357.7 million barrels.

5. *Look Back.* Does an inventory figure of 357.7 million barrels satisfy the words in the original problem? We were told that "Crude oil inventories last week experienced a decline and result in the crude oil inventories now." Well, 357.7 decreased by 3.8 gives 353.9.

81. In our solution, we will carefully address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* We can satisfy this requirement by simply stating "Let n represent the index of refraction of zircon."

2. *Set up an Equation.* "Refraction index of diamond is one and one-quarter times the refraction index of zircon." becomes

Refractive index of diamond	=	One and one-quarter	times	Refractive index of zircon
2.4	=	1.25	$\cdot$	n

3. *Solve the Equation.* To "undo" the multiplication by 1.25, divide both sides of the equation by 1.25.

$$2.4 = 1.25 \cdot n \qquad \text{Original equation.}$$

$$\frac{2.4}{1.25} = \frac{1.25 \cdot n}{1.25} \qquad \text{Divide both sides of the equation by 1.25.}$$

$$n = 1.92 \qquad \text{On the right, dividing by 1.25 "undoes" the effect}$$
$$\text{of multiplying by 1.25 and returns } n. \text{ On the left,}$$
$$2.4 \div 1.25 = 1.92.$$

To round to the nearest tenth, identify the rounding and test digits.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest tenth, the refractive index of zircon is approximately $n \approx 1.9$.

4. *Answer the Question.* The index of refraction of a zircon is approximately 1.9.

5. *Look Back.* Does an index of refraction for zircon satisfy the words in the original problem? We were told that "The index of diamond is about one and one-quarter times the index of zircon." Well, 1.25 times 1.9 gives 2.4.

5.7   Introduction to Square Roots

1. Since $16^2 = 256$ and $(-16)^2 = 256$, the two square roots of 256 are 16 and -16.

3. Square roots of negative numbers are undefined, so the number -289 has no square roots.

5. Since $21^2 = 441$ and $(-21)^2 = 441$, the two square roots of 441 are 21 and -21.

7. Since $18^2 = 324$ and $(-18)^2 = 324$, the two square roots of 324 are 18 and -18.

9. Since $12^2 = 144$ and $(-12)^2 = 144$, the two square roots of 144 are 12 and -12.

11. Square roots of negative numbers are undefined, so the number -144 has no square roots.

13. Since $11^2 = 121$ and $(-11)^2 = 121$, the two square roots of 121 are 11 and -11.

15. Since $23^2 = 529$ and $(-23)^2 = 529$, the two square roots of 529 are 23 and -23.

17. Square roots of negative numbers are undefined, so $\sqrt{-9}$ does not exist.

19. The two square roots of 576 are 24 and -24. Since $\sqrt{576}$ represents the positive square root of 576, it follows that $\sqrt{576} = 24$.

21. Square roots of negative numbers are undefined, so $\sqrt{-529}$ does not exist.

23. The two square roots of 25 are 5 and -5. Since $-\sqrt{25}$ represents the negative square root of 25, it follows that $-\sqrt{25} = -5$.

25. The two square roots of 484 are 22 and -22. Since $-\sqrt{484}$ represents the negative square root of 484, it follows that $-\sqrt{484} = -22$.

27. The two square roots of 196 are 14 and -14. Since $-\sqrt{196}$ represents the negative square root of 196, it follows that $-\sqrt{196} = -14$.

29. The two square roots of 441 are 21 and -21. Since $\sqrt{441}$ represents the positive square root of 441, it follows that $\sqrt{441} = 21$.

31. The two square roots of 4 are 2 and -2. Since $-\sqrt{4}$ represents the negative square root of 4, it follows that $-\sqrt{4} = -2$.

33. Since $9^2 = 81$, it follows that $0.9^2 = 0.81$. Therefore, $\sqrt{0.81} = 0.9$.

35. Since $19^2 = 361$, it follows that $1.9^2 = 3.61$. Therefore, $\sqrt{3.61} = 1.9$.

37. Since $\left(\dfrac{15}{4}\right)^2 = \dfrac{225}{16}$, it follows that $\sqrt{\dfrac{225}{16}} = \dfrac{15}{4}$.

39. Since $18^2 = 324$, it follows that $1.8^2 = 3.24$. Therefore, $\sqrt{3.24} = 1.8$.

41. Since $\left(\dfrac{11}{7}\right)^2 = \dfrac{121}{49}$, it follows that $\sqrt{\dfrac{121}{49}} = \dfrac{11}{7}$.

43. Since $\left(\dfrac{23}{11}\right)^2 = \dfrac{529}{121}$, it follows that $\sqrt{\dfrac{529}{121}} = \dfrac{23}{11}$.

45. Since $17^2 = 289$, it follows that $1.7^2 = 2.89$. Therefore, $\sqrt{2.89} = 1.7$.

47. Since $\left(\dfrac{12}{5}\right)^2 = \dfrac{144}{25}$, it follows that $\sqrt{\dfrac{144}{25}} = \dfrac{12}{5}$.

49. Since $\left(\dfrac{16}{19}\right)^2 = \dfrac{256}{361}$, it follows that $\sqrt{\dfrac{256}{361}} = \dfrac{16}{19}$.

51. Since $7^2 = 49$, it follows that $0.7^2 = 0.49$. Therefore, $\sqrt{0.49} = 0.7$.

53.

$$
\begin{aligned}
6 - \sqrt{576} &= 6 - (24) && \text{Evaluate radicals first.} \\
&= 6 - 24 && \text{Multiply.} \\
&= -18 && \text{Subtract.}
\end{aligned}
$$

55. In this case, the radical acts like grouping symbols, so we must evaluate what is inside the radical first.

$$
\begin{aligned}
\sqrt{8^2 + 15^2} &= \sqrt{64 + 225} && \text{Exponents first: } 8^2 = 64 \text{ and } 15^2 = 225. \\
&= \sqrt{289} && \text{Add: } 64 + 225 = 289. \\
&= 17 && \text{Take the nonnegative square root: } \sqrt{289} = 17.
\end{aligned}
$$

57.

$$
\begin{aligned}
6\sqrt{16} - 9\sqrt{49} &= 6(4) - 9(7) && \text{Evaluate radicals first.} \\
&= 24 - 63 && \text{Multiply.} \\
&= -39 && \text{Subtract.}
\end{aligned}
$$

59. In this case, the radical acts like grouping symbols, so we must evaluate what is inside the radical first.

$$
\begin{aligned}
\sqrt{5^2 + 12^2} &= \sqrt{25 + 144} && \text{Exponents first: } 5^2 = 25 \text{ and } 12^2 = 144. \\
&= \sqrt{169} && \text{Add: } 25 + 144 = 169. \\
&= 13 && \text{Take the nonnegative square root: } \sqrt{169} = 13.
\end{aligned}
$$

61. In this case, the radical acts like grouping symbols, so we must evaluate what is inside the radical first.

$$
\begin{aligned}
\sqrt{3^2 + 4^2} &= \sqrt{9 + 16} && \text{Exponents first: } 3^2 = 9 \text{ and } 4^2 = 16. \\
&= \sqrt{25} && \text{Add: } 9 + 16 = 25. \\
&= 5 && \text{Take the nonnegative square root: } \sqrt{25} = 5.
\end{aligned}
$$

63.

$$
\begin{aligned}
-2\sqrt{324} - 6\sqrt{361} &= -2(18) - 6(19) && \text{Evaluate radicals first.} \\
&= -36 - 114 && \text{Multiply.} \\
&= -150 && \text{Subtract.}
\end{aligned}
$$

65.

$$
\begin{aligned}
-4 - 3\sqrt{529} &= -4 - 3(23) && \text{\color{red}Evaluate radicals first.} \\
&= -4 - 69 && \text{\color{red}Multiply.} \\
&= -73 && \text{\color{red}Subtract.}
\end{aligned}
$$

67.

$$
\begin{aligned}
-9\sqrt{484} + 7\sqrt{81} &= -9(22) + 7(9) && \text{\color{red}Evaluate radicals first.} \\
&= -198 + 63 && \text{\color{red}Multiply.} \\
&= -135 && \text{\color{red}Add.}
\end{aligned}
$$

69.

$$
\begin{aligned}
2 - \sqrt{16} &= 2 - (4) && \text{\color{red}Evaluate radicals first.} \\
&= 2 - 4 && \text{\color{red}Multiply.} \\
&= -2 && \text{\color{red}Subtract.}
\end{aligned}
$$

71.

a) $7^2 = 49$, and $8^2 = 64$, so $\sqrt{58}$ lies between 7 and 8.

b) Since 58 is closer to 64 than to 49, it follows that $\sqrt{57}$ is closer to 8 than to 7.

c) By experimentation, $7.6^2 = 57.76$ and $7.7^2 = 59.29$, so 7.6 is the best estimate of $\sqrt{58}$ to the nearest tenth.

73.

a) $8^2 = 64$, and $9^2 = 81$, so $\sqrt{79}$ lies between 8 and 9.

b) Since 79 is closer to 81 than to 64, it follows that $\sqrt{79}$ is closer to 9 than to 8.

c) By experimentation, $8.8^2 = 77.44$ and $8.9^2 = 79.21$, so 8.9 is the best estimate of $\sqrt{79}$ to the nearest tenth.

75.

a) $6^2 = 36$, and $7^2 = 49$, so $\sqrt{44}$ lies between 6 and 7.

b) Since 44 is closer to 49 than to 36, it follows that $\sqrt{44}$ is closer to 7 than to 6.

c) By experimentation, $6.6^2 = 43.56$ and $6.7^2 = 44.89$, so 6.6 is the best estimate of $\sqrt{44}$ to the nearest tenth.

77. Using a 10-digit calculator, $\sqrt{469} \approx 21.65640783$. Rounded to the nearest tenth, $\sqrt{469} \approx 21.7$.

79. Using a 10-digit calculator, $\sqrt{615} \approx 24.79919354$. Rounded to the nearest tenth, $\sqrt{615} \approx 24.8$.

81. Using a 10-digit calculator, $\sqrt{444} \approx 21.07130751$. Rounded to the nearest tenth, $\sqrt{444} \approx 21.1$.

5.8   The Pythagorean Theorem

1. The first step is to sketch a right triangle with one leg measuring 15 meters, and the hypotenuse measuring 25 meters. Let b represent the length of the other leg.

By the Pythagorean Theorem, $15^2 + b^2 = 25^2$. Now solve this equation for b.

$$15^2 + b^2 = 25^2 \qquad \text{The Pythagorean equation.}$$
$$225 + b^2 = 625 \qquad \text{Exponents first: } 15^2 = 225 \text{ and } 25^2 = 625.$$
$$b^2 = 400 \qquad \text{Subtract 225 from both sides.}$$
$$b = 20 \qquad \text{Take the nonnegative square root of 400.}$$

Thus, the length of the other leg is 20 meters.

3. The first step is to sketch a right triangle with legs measuring 12 meters and 16 meters. Let c represent the length of the hypotenuse.

By the Pythagorean Theorem, $12^2 + 16^2 = c^2$. Now solve this equation for c.

$$12^2 + 16^2 = c^2 \qquad \text{The Pythagorean equation.}$$
$$144 + 256 = c^2 \qquad \text{Exponents first: } 12^2 = 144 \text{ and } 16^2 = 256.$$
$$400 = c^2 \qquad \text{Simplify the left side.}$$
$$20 = c \qquad \text{Take the nonnegative square root of 400.}$$

Thus, the length of the hypotenuse is 20 meters.

5. The first step is to sketch a right triangle with one leg measuring 13 meters, and the hypotenuse measuring 22 meters. Let b represent the length of the other leg.

By the Pythagorean Theorem, $13^2 + b^2 = 22^2$. Now solve this equation for b.

$$13^2 + b^2 = 22^2 \qquad \text{The Pythagorean equation.}$$
$$169 + b^2 = 484 \qquad \text{Exponents first: } 13^2 = 169 \text{ and } 22^2 = 484.$$
$$b^2 = 315 \qquad \text{Subtract 169 from both sides.}$$
$$b = \sqrt{315} \qquad \text{Take the nonnegative square root of 315.}$$

Since 315 is not a perfect square, the final answer is $\sqrt{315}$ meters.

7. The first step is to sketch a right triangle with legs measuring 2 meters and 21 meters. Let c represent the length of the hypotenuse.
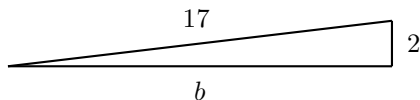

By the Pythagorean Theorem, $2^2 + 21^2 = c^2$. Now solve this equation for c.

$$2^2 + 21^2 = c^2 \qquad \text{The Pythagorean equation.}$$
$$4 + 441 = c^2 \qquad \text{Exponents first: } 2^2 = 4 \text{ and } 21^2 = 441.$$
$$445 = c^2 \qquad \text{Simplify the left side.}$$
$$\sqrt{445} = c \qquad \text{Take the nonnegative square root of 445.}$$

Since 445 is not a perfect square, the final answer is $\sqrt{445}$ meters.

9. The first step is to sketch a right triangle with one leg measuring 12 meters, and the hypotenuse measuring 19 meters. Let b represent the length of the other leg.
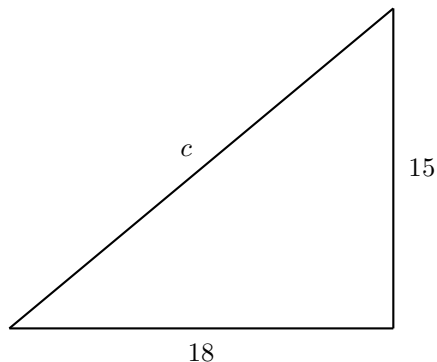
By the Pythagorean Theorem, $12^2 + b^2 = 19^2$. Now solve this equation for b.

$$12^2 + b^2 = 19^2 \qquad \text{The Pythagorean equation.}$$
$$144 + b^2 = 361 \qquad \text{Exponents first: } 12^2 = 144 \text{ and } 19^2 = 361.$$
$$b^2 = 217 \qquad \text{Subtract 144 from both sides.}$$
$$b = \sqrt{217} \qquad \text{Take the nonnegative square root of 217.}$$

Since 217 is not a perfect square, the final answer is $\sqrt{217}$ meters.

11. The first step is to sketch a right triangle with legs measuring 6 meters and 8 meters. Let c represent the length of the hypotenuse.

By the Pythagorean Theorem, $6^2 + 8^2 = c^2$. Now solve this equation for c.

$$6^2 + 8^2 = c^2 \qquad \text{The Pythagorean equation.}$$
$$36 + 64 = c^2 \qquad \text{Exponents first: } 6^2 = 36 \text{ and } 8^2 = 64.$$
$$100 = c^2 \qquad \text{Simplify the left side.}$$
$$10 = c \qquad \text{Take the nonnegative square root of 100.}$$

Thus, the length of the hypotenuse is 10 meters.

13. The first step is to sketch a right triangle with one leg measuring 6 meters, and the hypotenuse measuring 10 meters. Let b represent the length of the other leg.

By the Pythagorean Theorem, $6^2 + b^2 = 10^2$. Now solve this equation for b.

$$6^2 + b^2 = 10^2 \qquad \text{The Pythagorean equation.}$$
$$36 + b^2 = 100 \qquad \text{Exponents first: } 6^2 = 36 \text{ and } 10^2 = 100.$$
$$b^2 = 64 \qquad \text{Subtract 36 from both sides.}$$
$$b = 8 \qquad \text{Take the nonnegative square root of 64.}$$

Thus, the length of the other leg is 8 meters.

15. The first step is to sketch a right triangle with legs measuring 6 meters and 22 meters. Let c represent the length of the hypotenuse.

By the Pythagorean Theorem, $6^2 + 22^2 = c^2$. Now solve this equation for c.

$$6^2 + 22^2 = c^2 \qquad \text{The Pythagorean equation.}$$
$$36 + 484 = c^2 \qquad \text{Exponents first: } 6^2 = 36 \text{ and } 22^2 = 484.$$
$$520 = c^2 \qquad \text{Simplify the left side.}$$
$$\sqrt{520} = c \qquad \text{Take the nonnegative square root of 520.}$$

Since 520 is not a perfect square, the final answer is $\sqrt{520}$ meters.

17. The first step is to sketch a right triangle with legs measuring 3 meters and 18 meters. Let c represent the length of the hypotenuse.

By the Pythagorean Theorem, $3^2 + 18^2 = c^2$. Now solve this equation for c.

$$3^2 + 18^2 = c^2 \qquad \text{The Pythagorean equation.}$$
$$9 + 324 = c^2 \qquad \text{Exponents first: } 3^2 = 9 \text{ and } 18^2 = 324.$$
$$333 = c^2 \qquad \text{Simplify the left side.}$$
$$\sqrt{333} = c \qquad \text{Take the nonnegative square root of 333.}$$

Using a calculator, $\sqrt{333} \approx 18.2482875908947$. Rounded to the nearest hundredth, $\sqrt{333} \approx 18.25$ meters.

19. The first step is to sketch a right triangle with one leg measuring 2 meters, and the hypotenuse measuring 17 meters. Let b represent the length of the other leg.

By the Pythagorean Theorem, $2^2 + b^2 = 17^2$. Now solve this equation for b.

$$2^2 + b^2 = 17^2 \qquad \text{The Pythagorean equation.}$$
$$4 + b^2 = 289 \qquad \text{Exponents first: } 2^2 = 4 \text{ and } 17^2 = 289.$$
$$b^2 = 285 \qquad \text{Subtract 4 from both sides.}$$
$$b = \sqrt{285} \qquad \text{Take the nonnegative square root of 285.}$$

Using a calculator, $\sqrt{285} \approx 16.8819430161341$. Rounded to the nearest tenth, $\sqrt{285} \approx 16.9$ meters.

21. The first step is to sketch a right triangle with legs measuring 15 feet and 18 feet. Let c represent the length of the hypotenuse.

By the Pythagorean Theorem, $15^2 + 18^2 = c^2$. Now solve this equation for c.

$$15^2 + 18^2 = c^2 \qquad \text{The Pythagorean equation.}$$
$$225 + 324 = c^2 \qquad \text{Exponents first: } 15^2 = 225 \text{ and } 18^2 = 324.$$
$$549 = c^2 \qquad \text{Simplify the left side.}$$
$$\sqrt{549} = c \qquad \text{Take the nonnegative square root of 549.}$$

Using a calculator, $\sqrt{549} \approx 23.43074902772$. Rounded to the nearest hundredth, $\sqrt{549} \approx 23.43$ feet.

23. The first step is to sketch a right triangle with one leg measuring 4 meters, and the hypotenuse measuring 8 meters. Let b represent the length of the other leg.

By the Pythagorean Theorem, $4^2 + b^2 = 8^2$. Now solve this equation for b.

$$4^2 + b^2 = 8^2 \qquad \text{The Pythagorean equation.}$$
$$16 + b^2 = 64 \qquad \text{Exponents first: } 4^2 = 16 \text{ and } 8^2 = 64.$$
$$b^2 = 48 \qquad \text{Subtract 16 from both sides.}$$
$$b = \sqrt{48} \qquad \text{Take the nonnegative square root of 48.}$$

Using a calculator, $\sqrt{48} \approx 6.92820323027551$. Rounded to the nearest hundredth, $\sqrt{48} \approx 6.93$ meters.

25. Sketch a rectangle with sides of length 13 and 18, and connect two opposite corners to form a diagonal. A right triangle is formed by two sides and the diagonal. Let c represent the length of the hypotenuse (the diagonal).

By the Pythagorean Theorem, $13^2 + 18^2 = c^2$. Now solve this equation for c.

$$13^2 + 18^2 = c^2 \qquad \text{The Pythagorean equation.}$$
$$169 + 324 = c^2 \qquad \text{Exponents first: } 13^2 = 169 \text{ and } 18^2 = 324.$$
$$493 = c^2 \qquad \text{Simplify the left side.}$$
$$\sqrt{493} = c \qquad \text{Take the nonnegative square root of 493.}$$

Using a calculator, $\sqrt{493} \approx 22.2036033111745$. Rounded to the nearest hundredth, $\sqrt{493} \approx 22.20$ meters.

27. The guy wire, the telephone pole, and the ground form a right triangle with the right angle between the pole and the ground. Therefore, the first step is to sketch a right triangle with the hypotenuse (the guy wire) measuring 24 meters, and one leg (the ground) measuring 10 meters. Let b represent the length of the other leg (the pole).

By the Pythagorean Theorem, $10^2 + b^2 = 24^2$. Now solve this equation for b.

$$10^2 + b^2 = 24^2 \qquad \text{The Pythagorean equation.}$$
$$100 + b^2 = 576 \qquad \text{Exponents first: } 10^2 = 100 \text{ and } 24^2 = 576.$$
$$b^2 = 476 \qquad \text{Subtract 100 from both sides.}$$
$$b = \sqrt{476} \qquad \text{Take the nonnegative square root of 476.}$$

Using a calculator, $\sqrt{476} \approx 21.8174242292714$. Rounded to the nearest hundredth, $\sqrt{476} \approx 21.82$ meters.

29. The trail is approximately in the shape of a right triangle. The part of the trail going south is one leg, the part of the trail going west is another leg. To find the entire length of the trail, we must find the length of the portion of the trail that is represented by the hypoteneuse. So, the first step is to sketch a right triangle with legs measuring 8 kilometers and 15 kilometers. Let c represent the length of the hypotenuse. We must first find the length c, and then sum all the sides together.

By the Pythagorean Theorem, $8^2 + 15^2 = c^2$. Now solve this equation for c.

$$8^2 + 15^2 = c^2 \qquad \text{The Pythagorean equation.}$$
$$64 + 225 = c^2 \qquad \text{Exponents first: } 8^2 = 64 \text{ and } 15^2 = 225.$$
$$289 = c^2 \qquad \text{Simplify the left side.}$$
$$17 = c \qquad \text{Take the nonnegative square root of 289.}$$

The length of the hypotenuse is 17 kilometers. Therefore, the entire trail runs $8 + 15 + 17 = 40$ kilometers.

31. The situation is modeled with a right triangle. Sketch a right triangle with one leg measuring 8 feet, and the hypotenuse measuring 10 feet. Let b represent the length of the other leg.

By the Pythagorean Theorem, $8^2 + b^2 = 10^2$. Now solve this equation for b.

$$8^2 + b^2 = 10^2 \qquad \text{The Pythagorean equation.}$$
$$64 + b^2 = 100 \qquad \text{Exponents first: } 8^2 = 64 \text{ and } 10^2 = 100.$$
$$b^2 = 36 \qquad \text{Subtract 64 from both sides.}$$
$$b = 6 \qquad \text{Take the nonnegative square root of 36.}$$

Leg b has a length of 6 feet. Therefore, the ladder must be 6 feet from the wall to reach the 8-foot window.

Chapter 6

Ratio and Proportion

6.1 Introduction to Ratios and Rates

1. Write 0.14 : 0.44 as a fraction, then multiplying numerator and denominator by 100 moves the decimal point 2 places to the right.

$$
\begin{aligned}
0.14 : 0.44 &= \frac{0.14}{0.44} && \textcolor{red}{\text{Write the ratio as a fraction.}}\\
&= \frac{0.14 \cdot 100}{0.44 \cdot 100} && \textcolor{red}{\text{Multiply numerator and denominator by 100.}}\\
&= \frac{14}{44}
\end{aligned}
$$

Next, factor out the GCD and cancel.

$$
\begin{aligned}
&= \frac{7 \cdot 2}{22 \cdot 2} && \textcolor{red}{\text{Factor out GCD.}}\\
&= \frac{7 \cdot \cancel{2}}{22 \cdot \cancel{2}} && \textcolor{red}{\text{Cancel.}}\\
&= \frac{7}{22}
\end{aligned}
$$

3. Write 0.05 : 0.75 as a fraction, then multiplying numerator and denominator by 100 moves the decimal point 2 places to the right.

$$
\begin{aligned}
0.05 : 0.75 &= \frac{0.05}{0.75} && \textcolor{red}{\text{Write the ratio as a fraction.}}\\
&= \frac{0.05 \cdot 100}{0.75 \cdot 100} && \textcolor{red}{\text{Multiply numerator and denominator by 100.}}\\
&= \frac{5}{75}
\end{aligned}
$$

Next, factor out the GCD and cancel.

$$
\begin{aligned}
&= \frac{1 \cdot 5}{15 \cdot 5} && \text{Factor out GCD.} \\
&= \frac{1 \cdot \cancel{5}}{15 \cdot \cancel{5}} && \text{Cancel.} \\
&= \frac{1}{15}
\end{aligned}
$$

5. Write $0.1 : 0.95$ as a fraction, then multiplying numerator and denominator by 100 moves the decimal point 2 places to the right.

$$
\begin{aligned}
0.1 : 0.95 &= \frac{0.1}{0.95} && \text{Write the ratio as a fraction.} \\
&= \frac{0.1 \cdot 100}{0.95 \cdot 100} && \text{Multiply numerator and denominator by 100.} \\
&= \frac{10}{95}
\end{aligned}
$$

Next, factor out the GCD and cancel.

$$
\begin{aligned}
&= \frac{2 \cdot 5}{19 \cdot 5} && \text{Factor out GCD.} \\
&= \frac{2 \cdot \cancel{5}}{19 \cdot \cancel{5}} && \text{Cancel.} \\
&= \frac{2}{19}
\end{aligned}
$$

7. Write $2\frac{2}{9} : 1\frac{1}{3}$ as a fraction, then change the mixed fractions to improper fractions.

$$
\begin{aligned}
2\frac{2}{9} : 1\frac{1}{3} &= \frac{2\frac{2}{9}}{1\frac{1}{3}} && \text{Write the ratio as a fraction.} \\
&= \frac{\frac{20}{9}}{\frac{4}{3}} && \text{Mixed to improper fractions.}
\end{aligned}
$$

Invert and multiply.

$$
\begin{aligned}
&= \frac{20}{9} \cdot \frac{3}{4} && \text{Invert.} \\
&= \frac{60}{36} && \text{Multiply numerators and denominators.}
\end{aligned}
$$

Factor numerators and denominators, then cancel common factors.

$$= \frac{2 \cdot 2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3} \qquad \text{\textcolor{red}{Factor numerators and denominators.}}$$

$$= \frac{5}{3} \qquad \text{\textcolor{red}{Cancel common factors.}}$$

9. Write $0.36 : 0.6$ as a fraction, then multiplying numerator and denominator by 100 moves the decimal point 2 places to the right.

$$0.36 : 0.6 = \frac{0.36}{0.6} \qquad \text{\textcolor{red}{Write the ratio as a fraction.}}$$

$$= \frac{0.36 \cdot 100}{0.6 \cdot 100} \qquad \text{\textcolor{red}{Multiply numerator and denominator by 100.}}$$

$$= \frac{36}{60}$$

Next, factor out the GCD and cancel.

$$= \frac{3 \cdot 12}{5 \cdot 12} \qquad \text{\textcolor{red}{Factor out GCD.}}$$

$$= \frac{3 \cdot \cancel{12}}{5 \cdot \cancel{12}} \qquad \text{\textcolor{red}{Cancel.}}$$

$$= \frac{3}{5}$$

11. Write $15 : 21$ as a fraction, then reduce to lowest terms.

$$15 : 21 = \frac{15}{21} \qquad \text{\textcolor{red}{Write the ratio as a fraction.}}$$

$$= \frac{5 \cdot 3}{7 \cdot 3} \qquad \text{\textcolor{red}{Factor out greatest common factor.}}$$

$$= \frac{5 \cdot \cancel{3}}{7 \cdot \cancel{3}} \qquad \text{\textcolor{red}{Cancel.}}$$

$$= \frac{5}{7}$$

13. Write $2\frac{8}{9} : 2\frac{2}{3}$ as a fraction, then change the mixed fractions to improper fractions.

$$2\frac{8}{9} : 2\frac{2}{3} = \frac{2\frac{8}{9}}{2\frac{2}{3}} \qquad \text{\textcolor{red}{Write the ratio as a fraction.}}$$

$$= \frac{\frac{26}{9}}{\frac{8}{3}} \qquad \text{\textcolor{red}{Mixed to improper fractions.}}$$

Invert and multiply.

$$= \frac{26}{9} \cdot \frac{3}{8} \qquad \text{Invert.}$$
$$= \frac{78}{72} \qquad \text{Multiply numerators and denominators.}$$

Factor numerators and denominators, then cancel common factors.

$$= \frac{2 \cdot 3 \cdot 13}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} \qquad \text{Factor numerators and denominators.}$$
$$= \frac{13}{2 \cdot 2 \cdot 3} \qquad \text{Cancel common factors.}$$

$$= \frac{13}{12}$$

15. Write $3\frac{8}{9} : 2\frac{1}{3}$ as a fraction, then change the mixed fractions to improper fractions.

$$3\frac{8}{9} : 2\frac{1}{3} = \frac{3\frac{8}{9}}{2\frac{1}{3}} \qquad \text{Write the ratio as a fraction.}$$
$$= \frac{\dfrac{35}{9}}{\dfrac{7}{3}} \qquad \text{Mixed to improper fractions.}$$

Invert and multiply.

$$= \frac{35}{9} \cdot \frac{3}{7} \qquad \text{Invert.}$$
$$= \frac{105}{63} \qquad \text{Multiply numerators and denominators.}$$

Factor numerators and denominators, then cancel common factors.

$$= \frac{3 \cdot 5 \cdot 7}{3 \cdot 3 \cdot 7} \qquad \text{Factor numerators and denominators.}$$
$$= \frac{5}{3} \qquad \text{Cancel common factors.}$$

17. Write $2\frac{5}{8} : 1\frac{3}{4}$ as a fraction, then change the mixed fractions to improper fractions.

$$2\frac{5}{8} : 1\frac{3}{4} = \frac{2\frac{5}{8}}{1\frac{3}{4}} \qquad \text{Write the ratio as a fraction.}$$

$$= \frac{\frac{21}{8}}{\frac{7}{4}} \qquad \text{Mixed to improper fractions.}$$

Invert and multiply.

$$= \frac{21}{8} \cdot \frac{4}{7} \qquad \text{Invert.}$$

$$= \frac{84}{56} \qquad \text{Multiply numerators and denominators.}$$

Factor numerators and denominators, then cancel common factors.

$$= \frac{2 \cdot 2 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 2 \cdot 7} \qquad \text{Factor numerators and denominators.}$$

$$= \frac{3}{2} \qquad \text{Cancel common factors.}$$

19. Write $10 : 35$ as a fraction, then reduce to lowest terms.

$$10 : 35 = \frac{10}{35} \qquad \text{Write the ratio as a fraction.}$$

$$= \frac{2 \cdot 5}{7 \cdot 5} \qquad \text{Factor out greatest common factor.}$$

$$= \frac{2 \cdot \cancel{5}}{7 \cdot \cancel{5}} \qquad \text{Cancel.}$$

$$= \frac{2}{7}$$

21. Write $9 : 33$ as a fraction, then reduce to lowest terms.

$$9 : 33 = \frac{9}{33} \qquad \text{Write the ratio as a fraction.}$$

$$= \frac{3 \cdot 3}{11 \cdot 3} \qquad \text{Factor out greatest common factor.}$$

$$= \frac{3 \cdot \cancel{3}}{11 \cdot \cancel{3}} \qquad \text{Cancel.}$$

$$= \frac{3}{11}$$

23. Write $27 : 99$ as a fraction, then reduce to lowest terms.

$$
\begin{aligned}
27 : 99 &= \frac{27}{99} &&\text{\color{red}Write the ratio as a fraction.}\\
&= \frac{3 \cdot 9}{11 \cdot 9} &&\text{\color{red}Factor out greatest common factor.}\\
&= \frac{3 \cdot \cancel{9}}{11 \cdot \cancel{9}} &&\text{\color{red}Cancel.}\\
&= \frac{3}{11}
\end{aligned}
$$

25. To find the rate for the first automobile, divide its mileage by its gas consumption.

$$
\begin{aligned}
\text{Rate } 1 &= \frac{271.8 \, \text{mi}}{10.1 \, \text{gal}} &&\text{\color{red}Mileage/gallons.}\\
&= \frac{26.9 \, \text{mi}}{1 \, \text{gal}} &&\text{\color{red}Divide: } 271.8/10.1 = 26.9.\\
&= 26.9 \, \text{mi/gal}
\end{aligned}
$$

To find the rate for the second automobile, divide its mileage by its gas consumption.

$$
\begin{aligned}
\text{Rate } 2 &= \frac{257.9 \, \text{mi}}{11.1 \, \text{gal}} &&\text{\color{red}Mileage/gallons.}\\
&= \frac{23.2 \, \text{mi}}{1 \, \text{gal}} &&\text{\color{red}Divide: } 257.9/11.1 = 23.2.\\
&= 23.2 \, \text{mi/gal}
\end{aligned}
$$

Rates are rounded to the nearest tenth of a mile per gallon. The first automobile has the better mileage per gallon.

27. Place 183 over 8.25 hours to get 183 dollars/8.25 hr. Divide 183 by 8.25 until the quotient contains three decimal places, one beyond the pennies (hundredths) place, which will serve as the test digit for rounding.

$$
\begin{aligned}
\frac{183 \, \text{dollars}}{8.25 \, \text{hours}} &\approx \frac{22.182 \, \text{dollars}}{1 \, \text{hour}} &&\text{\color{red}Divide: } 183/8.25 \approx 22.182.\\
&\approx 22.182
\end{aligned}
$$

To round to the nearest penny (hundredth), identify the rounding digit (hundredths place) and the test digit (thousandths place).

Because the test digit is less than 5, leave rounding digit alone, then truncate the test digit. Thus, to the nearest penny, the hourly rate is 22.18 dollars per hour.

29. Place miles traveled over hours traveled and reduce.

$$\frac{140\,\text{miles}}{4\,\text{hours}} = \frac{35\,\text{miles}}{1\,\text{hour}} \qquad \text{Divide: } 140/4 = 35.$$
$$= 35\,\text{miles/hour}$$

Hence, the average speed is 35 miles per hour.

31. Place 187 over 8 hours to get 187 dollars/8 hr. Divide 187 by 8 until the quotient contains three decimal places, one beyond the pennies (hundredths) place, which will serve as the test digit for rounding.

$$\frac{187\,\text{dollars}}{8\,\text{hours}} \approx \frac{23.375\,\text{dollars}}{1\,\text{hour}} \qquad \text{Divide: } 187/8 \approx 23.375.$$
$$\approx 23.375$$

To round to the nearest penny (hundredth), identify the rounding digit (hundredths place) and the test digit (thousandths place).

Because the test digit is greater than or equal to 5, add 1 to rounding digit, then truncate the test digit. Thus, to the nearest penny, the hourly rate is 23.38 dollars per hour.

33. To find the rate for the first automobile, divide its mileage by its gas consumption.

$$\text{Rate 1} = \frac{234.2\,\text{mi}}{10.8\,\text{gal}} \qquad \text{Mileage/gallons.}$$
$$= \frac{21.7\,\text{mi}}{1\,\text{gal}} \qquad \text{Divide: } 234.2/10.8 = 21.7.$$
$$= 21.7\,\text{mi/gal}$$

To find the rate for the second automobile, divide its mileage by its gas consumption.

$$\text{Rate } 2 = \frac{270.5\,\text{mi}}{10.8\,\text{gal}} \qquad\qquad \text{\color{red}{Mileage/gallons.}}$$
$$= \frac{25.0\,\text{mi}}{1\,\text{gal}} \qquad\qquad \text{\color{red}{Divide: } 270.5/10.8 = 25.0.}$$
$$= 25.0\,\text{mi/gal}$$

Rates are rounded to the nearest tenth of a mile per gallon. The second automobile has the better mileage per gallon.

35. Place miles traveled over hours traveled and reduce.

$$\frac{180\,\text{miles}}{5\,\text{hours}} = \frac{36\,\text{miles}}{1\,\text{hour}} \qquad\qquad \text{\color{red}{Divide: } 180/5 = 36.}$$
$$= 36\,\text{miles/hour}$$

Hence, the average speed is 36 miles per hour.

37. Take the total distance traveled in miles over the total time in days and divide to the hundredths place. This will give us the test digit for rounding to the nearest tenth of a mile.

$$\frac{562\,\text{miles}}{38\,\text{days}} \approx \frac{14.78\,\text{miles}}{1\,\text{day}} \qquad\qquad \text{\color{red}{Divide: } 562/38 \approx 14.78.}$$
$$\approx 14.78$$

To round to the nearest tenth, identify the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to rounding digit, then truncate the test digit. Thus, to the nearest tenth of a mile, the daily rate of speed is 14.8 miles per day.

6.2 Introduction to Proportion

1. Consider
$$\frac{9}{7} = \frac{27}{21}$$

Cross multiply and simpilify.

$$9 \cdot 21 = 7 \cdot 27 \qquad \text{Cross multiply.}$$
$$189 = 189 \qquad \text{Simplify both sides.}$$

Because this last statement is true, this is a true proportion. To provide some constrast, consider
$$\frac{7}{2} = \frac{8}{9}$$

Cross multiply and simmplify.

$$7 \cdot 9 = 2 \cdot 8 \qquad \text{Cross multiply.}$$
$$63 = 16 \qquad \text{Simplify both sides.}$$

In this case, the last statement is false. Hence, this is not a true proportion. In similar fashion, readers should check that the remaining two choices are not proportions.

3. Consider
$$\frac{7}{6} = \frac{28}{24}$$

Cross multiply and simpilify.

$$7 \cdot 24 = 6 \cdot 28 \qquad \text{Cross multiply.}$$
$$168 = 168 \qquad \text{Simplify both sides.}$$

Because this last statement is true, this is a true proportion. To provide some constrast, consider
$$\frac{9}{5} = \frac{7}{3}$$

Cross multiply and simmplify.

$$9 \cdot 3 = 5 \cdot 7 \qquad \text{Cross multiply.}$$
$$27 = 35 \qquad \text{Simplify both sides.}$$

In this case, the last statement is false. Hence, this is not a true proportion. In similar fashion, readers should check that the remaining two choices are not proportions.

5. Consider

$$\frac{6}{5} = \frac{24}{20}$$

Cross multiply and simpilify.

$$6 \cdot 20 = 5 \cdot 24 \qquad \text{Cross multiply.}$$
$$120 = 120 \qquad \text{Simplify both sides.}$$

Because this last statement is true, this is a true proportion. To provide some constrast, consider

$$\frac{5}{2} = \frac{2}{8}$$

Cross multiply and simmplify.

$$5 \cdot 8 = 2 \cdot 2 \qquad \text{Cross multiply.}$$
$$40 = 4 \qquad \text{Simplify both sides.}$$

In this case, the last statement is false. Hence, this is not a true proportion. In similar fashion, readers should check that the remaining two choices are not proportions.

7. Consider

$$\frac{3}{7} = \frac{6}{14}$$

Cross multiply and simpilify.

$$3 \cdot 14 = 7 \cdot 6 \qquad \text{Cross multiply.}$$
$$42 = 42 \qquad \text{Simplify both sides.}$$

Because this last statement is true, this is a true proportion. To provide some constrast, consider

$$\frac{7}{4} = \frac{5}{9}$$

Cross multiply and simmplify.

$$7 \cdot 9 = 4 \cdot 5 \qquad \text{Cross multiply.}$$
$$63 = 20 \qquad \text{Simplify both sides.}$$

In this case, the last statement is false. Hence, this is not a true proportion. In similar fashion, readers should check that the remaining two choices are not proportions.

9. Consider

$$\frac{5}{4} = \frac{25}{20}$$

Cross multiply and simpilify.

$$5 \cdot 20 = 4 \cdot 25 \qquad \text{Cross multiply.}$$
$$100 = 100 \qquad \text{Simplify both sides.}$$

Because this last statement is true, this is a true proportion. To provide some constrast, consider

$$\frac{9}{3} = \frac{9}{6}$$

Cross multiply and simmplify.

$$9 \cdot 6 = 3 \cdot 9 \qquad \text{Cross multiply.}$$
$$54 = 27 \qquad \text{Simplify both sides.}$$

In this case, the last statement is false. Hence, this is not a true proportion. In similar fashion, readers should check that the remaining two choices are not proportions.

11. Consider

$$\frac{3}{5} = \frac{6}{10}$$

Cross multiply and simpilify.

$$3 \cdot 10 = 5 \cdot 6 \qquad \text{Cross multiply.}$$
$$30 = 30 \qquad \text{Simplify both sides.}$$

Because this last statement is true, this is a true proportion. To provide some constrast, consider

$$\frac{3}{9} = \frac{9}{5}$$

Cross multiply and simmplify.

$$3 \cdot 5 = 9 \cdot 9 \qquad \text{Cross multiply.}$$
$$15 = 81 \qquad \text{Simplify both sides.}$$

In this case, the last statement is false. Hence, this is not a true proportion. In similar fashion, readers should check that the remaining two choices are not proportions.

13. Cross multiply, then solve the resulting equation.

$$\frac{17}{3} = \frac{x}{18}$$ Orginial Proportion.

$$3 \cdot x = 17 \cdot 18$$ Product of means and extremes are equal.

$$3x = 306$$ Simplify.

$$\frac{3x}{3} = \frac{306}{3}$$ Divide both sides by 3.

$$x = 102$$ Simplify.

15. Cross multiply, then solve the resulting equation.

$$\frac{6x + 10}{6} = \frac{11}{3}$$ Original Proportion.

$$3(6x + 10) = 6(11)$$ Products of means and extremes are equal.

$$18x + 30 = 66$$ On the left, distribute.
                          On the right, multiply.

$$18x + 30 - 30 = 66 - 30$$ Subtract 30 from both sides.

$$18x = 36$$ Simplify.

$$\frac{18x}{18} = \frac{36}{18}$$ Divide both sides by 18.

$$x = 2$$ Simplify both sides.

17. Cross multiply, then solve the resulting equation.

$$\frac{17}{9} = \frac{x}{18}$$ Orginial Proportion.

$$9 \cdot x = 17 \cdot 18$$ Product of means and extremes are equal.

$$9x = 306$$ Simplify.

$$\frac{9x}{9} = \frac{306}{9}$$ Divide both sides by 9.

$$x = 34$$ Simplify.

19. Cross multiply, then solve the resulting equation.

$$\frac{11}{2} = \frac{x}{8}$$ Orginial Proportion.

$$2 \cdot x = 11 \cdot 8$$ Product of means and extremes are equal.

$$2x = 88$$ Simplify.

$$\frac{2x}{2} = \frac{88}{2}$$ Divide both sides by 2.

$$x = 44$$ Simplify.

21. Cross multiply, then solve the resulting equation.

$$\frac{7x+15}{15} = \frac{10}{3}$$ Original Proportion.

$$3(7x+15) = 15(10)$$ Products of means and extremes are equal.

$$21x + 45 = 150$$ On the left, distribute.

On the right, multiply.

$$21x + 45 - 45 = 150 - 45$$ Subtract 45 from both sides.

$$21x = 105$$ Simplify.

$$\frac{21x}{21} = \frac{105}{21}$$ Divide both sides by 21.

$$x = 5$$ Simplify both sides.

23. Cross multiply, then solve the resulting equation.

$$\frac{11}{2} = \frac{x}{10}$$ Orginial Proportion.

$$2 \cdot x = 11 \cdot 10$$ Product of means and extremes are equal.

$$2x = 110$$ Simplify.

$$\frac{2x}{2} = \frac{110}{2}$$ Divide both sides by 2.

$$x = 55$$ Simplify.

25. Cross multiply, then solve the resulting equation.

$$\frac{5x+8}{12} = \frac{2}{3}$$ Original Proportion.

$$3(5x+8) = 12(2)$$ Products of means and extremes are equal.

$$15x + 24 = 24$$ On the left, distribute.

On the right, multiply.

$$15x + 24 - 24 = 24 - 24$$ Subtract 24 from both sides.

$$15x = 0$$ Simplify.

$$\frac{15x}{15} = \frac{0}{15}$$ Divide both sides by 15.

$$x = 0$$ Simplify both sides.

27. Cross multiply, then solve the resulting equation.

$$\frac{2}{15} = \frac{24}{x}$$ Orginial Proportion.

$2 \cdot x = 15 \cdot 24$ Product of means and extremes are equal.

$2x = 360$ Simplify.

$$\frac{2x}{2} = \frac{360}{2}$$ Divide both sides by 2.

$x = 180$ Simplify.

29. Cross multiply, then solve the resulting equation.

$$\frac{3}{16} = \frac{6}{x}$$ Orginial Proportion.

$3 \cdot x = 16 \cdot 6$ Product of means and extremes are equal.

$3x = 96$ Simplify.

$$\frac{3x}{3} = \frac{96}{3}$$ Divide both sides by 3.

$x = 32$ Simplify.

31. Cross multiply, then solve the resulting equation.

$$\frac{5}{22} = \frac{20}{x}$$ Orginial Proportion.

$5 \cdot x = 22 \cdot 20$ Product of means and extremes are equal.

$5x = 440$ Simplify.

$$\frac{5x}{5} = \frac{440}{5}$$ Divide both sides by 5.

$x = 88$ Simplify.

33. Cross multiply, then solve the resulting equation.

$$\frac{2x + 10}{6} = \frac{14}{3}$$ Original Proportion.

$3(2x + 10) = 6(14)$ Products of means and extremes are equal.

$6x + 30 = 84$ On the left, distribute.

 On the right, multiply.

$6x + 30 - 30 = 84 - 30$ Subtract 30 from both sides.

$6x = 54$ Simplify.

$$\frac{6x}{6} = \frac{54}{6}$$ Divide both sides by 6.

$x = 9$ Simplify both sides.

35. Cross multiply, then solve the resulting equation.

$$\frac{7}{2} = \frac{21}{x}$$ Orginial Proportion.

$$7 \cdot x = 2 \cdot 21$$ Product of means and extremes are equal.

$$7x = 42$$ Simplify.

$$\frac{7x}{7} = \frac{42}{7}$$ Divide both sides by 7.

$$x = 6$$ Simplify.

37. Let x represent the cost of 7 dog bones. 13 dog bones cost \$1.97. Assuming the rate for 13 dog bones at \$1.97 equals the rate for 7 dog bones at an unknown cost x, we can set up a proportion.

$$\frac{13 \, \text{dog bones}}{1.97 \, \text{dollars}} = \frac{7 \, \text{dog bones}}{x \, \text{dollars}}$$

Now, let's drop the units, cross multiply and solve.

$$\frac{13}{1.97} = \frac{7}{x}$$ Original Proportion.

$$13 \cdot x = 1.97 \cdot 7$$ Product of means and extremes are equal.

$$13x = 13.79$$ Simplify.

$$\frac{13x}{13} = \frac{13.79}{13}$$ Divide both sides by 13.

$$x = 1.06076923076923$$ Simplify.

$$x = 1.06$$ Round to the nearest penny.

Thus, 7 dog bones costs \$1.06.

39. Let x represent the cost of 14 bananas. 7 bananas cost \$2.55. Assuming the rate for 7 bananas at \$2.55 equals the rate for 14 bananas at an unknown cost x, we can set up a proportion.

$$\frac{7 \, \text{bananas}}{2.55 \, \text{dollars}} = \frac{14 \, \text{bananas}}{x \, \text{dollars}}$$

Now, let's drop the units, cross multiply and solve.

$$\frac{7}{2.55} = \frac{14}{x}$$ Original Proportion.

$$7 \cdot x = 2.55 \cdot 14$$ Product of means and extremes are equal.

$$7x = 35.7$$ Simplify.

$$\frac{7x}{7} = \frac{35.7}{7}$$ Divide both sides by 7.

$$x = 5.1$$ Simplify.

$$x = 5.10$$ Round to the nearest penny.

Thus, 14 bananas costs \$5.10.

41. Let x represent the cost of 11 oranges. 13 oranges cost \$3.61. Assuming the rate for 13 oranges at \$3.61 equals the rate for 11 oranges at an unknown cost x, we can set up a proportion.

$$\frac{13 \text{ oranges}}{3.61 \text{ dollars}} = \frac{11 \text{ oranges}}{x \text{ dollars}}$$

Now, let's drop the units, cross multiply and solve.

$$\frac{13}{3.61} = \frac{11}{x} \qquad \text{Original Proportion.}$$
$$13 \cdot x = 3.61 \cdot 11 \qquad \text{Product of means and extremes are equal.}$$
$$13x = 39.71 \qquad \text{Simplify.}$$
$$\frac{13x}{13} = \frac{39.71}{13} \qquad \text{Divide both sides by 13.}$$
$$x = 3.05461538461538 \qquad \text{Simplify.}$$
$$x = 3.05 \qquad \text{Round to the nearest penny.}$$

Thus, 11 oranges costs \$3.05.

43. Let x represent the cost of 13 dog bones. 3 dog bones cost \$1.23. Assuming the rate for 3 dog bones at \$1.23 equals the rate for 13 dog bones at an unknown cost x, we can set up a proportion.

$$\frac{3 \text{ dog bones}}{1.23 \text{ dollars}} = \frac{13 \text{ dog bones}}{x \text{ dollars}}$$

Now, let's drop the units, cross multiply and solve.

$$\frac{3}{1.23} = \frac{13}{x} \qquad \text{Original Proportion.}$$
$$3 \cdot x = 1.23 \cdot 13 \qquad \text{Product of means and extremes are equal.}$$
$$3x = 15.99 \qquad \text{Simplify.}$$
$$\frac{3x}{3} = \frac{15.99}{3} \qquad \text{Divide both sides by 3.}$$
$$x = 5.33 \qquad \text{Simplify.}$$
$$x = 5.33 \qquad \text{Round to the nearest penny.}$$

Thus, 13 dog bones costs \$5.33.

45. Let x represent the cost of 13 apples. 3 apples cost \$3.24. Assuming the rate for 3 apples at \$3.24 equals the rate for 13 apples at an unknown cost x, we can set up a proportion.

$$\frac{3 \text{ apples}}{3.24 \text{ dollars}} = \frac{13 \text{ apples}}{x \text{ dollars}}$$

Now, let's drop the units, cross multiply and solve.

$$\frac{3}{3.24} = \frac{13}{x} \qquad \text{Original proportion.}$$

$$3 \cdot x = 3.24 \cdot 13 \qquad \text{Product of means and extremes are equal.}$$

$$3x = 42.12 \qquad \text{Simplify.}$$

$$\frac{3x}{3} = \frac{42.12}{3} \qquad \text{Divide both sides by 3.}$$

$$x = 14.04 \qquad \text{Simplify.}$$

$$x = 14.04 \qquad \text{Round to the nearest penny.}$$

Thus, 13 apples costs \$14.04.

47. Let x represent the cost of 8 dog bones. 4 dog bones cost \$1.03. Assuming the rate for 4 dog bones at \$1.03 equals the rate for 8 dog bones at an unknown cost x, we can set up a proportion.

$$\frac{4 \text{ dog bones}}{1.03 \text{ dollars}} = \frac{8 \text{ dog bones}}{x \text{ dollars}}$$

Now, let's drop the units, cross multiply and solve.

$$\frac{4}{1.03} = \frac{8}{x} \qquad \text{Original proportion.}$$

$$4 \cdot x = 1.03 \cdot 8 \qquad \text{Product of means and extremes are equal.}$$

$$4x = 8.24 \qquad \text{Simplify.}$$

$$\frac{4x}{4} = \frac{8.24}{4} \qquad \text{Divide both sides by 4.}$$

$$x = 2.06 \qquad \text{Simplify.}$$

$$x = 2.06 \qquad \text{Round to the nearest penny.}$$

Thus, 8 dog bones costs \$2.06.

49. Let x represent the cost in cents of 20 rolls when 5 rolls cost 12 cents. Now create a proportion using these two ratios.

$$\frac{5 \text{ rolls}}{12 \text{ cents}} = \frac{20 \text{ rolls}}{x \text{ cents}}$$

Now, let's drop the units, cross multiply and solve.

$$\frac{5}{12} = \frac{20}{x} \qquad \text{Original proportion.}$$

$$5 \cdot x = 20 \cdot 12 \qquad \text{Product of means and extremes are equal.}$$

$$\frac{5x}{5} = \frac{20 \cdot 12}{5} \qquad \text{Divide both sides by 5.}$$

$$x = 48 \qquad \text{Simplify.}$$

Thus, 20 rolls would cost 48 cents.

51. Let x represent the number of dumptrucks needed to remove 40,000 cubic yards of material when 3000 cubic yards of material requires 200 dumptrucks for removal. Now create a proportion using these two ratios.

$$\frac{3000 \,\text{cubic yards}}{200 \,\text{dumptrucks}} = \frac{40,000 \,\text{cubic yards}}{x \,\text{dumptrucks}}$$

Now, let's drop the units, cross multiply and solve.

$$\frac{3000}{200} = \frac{40,000}{x} \qquad \text{Original proportion.}$$

$$3000 \cdot x = 40,000 \cdot 200 \qquad \text{Product of means and extremes are equal.}$$

$$3000x = 8,000,000 \qquad \text{Simplify.}$$

$$\frac{3000x}{3000} = \frac{8,000,000}{3000} \qquad \text{Divide both sides by 3000.}$$

$$x \approx 2666.6 \qquad \text{Simplify.}$$

$$x = 2667 \qquad \text{Round to the nearest whole number.}$$

Thus, approximately 2,667 dumptruck loads were required to remove 40,000 cubic yards of material from the landslide.

53. Let x represent the millions of US dollars that converts to 10 million Australian dollars when the exchange rate is such that 1.75 million Australian dollars represents \$1.64 million US dollars. Create a proportion using these two ratios.

$$\frac{1.75 \,\text{million Australian dollars}}{1.64 \,\text{million US dollars}} = \frac{10 \,\text{million Australian dollars}}{x \,\text{million US dollars}}$$

Now, let's drop the units, cross multiply and solve. Your answer will be in the millions of US dollars.

$$\frac{1.75}{1.64} = \frac{10}{x} \qquad \text{Original proportion.}$$

$$1.75 \cdot x = 10 \cdot 1.64 \qquad \text{Product of means and extremes are equal.}$$

$$1.75 \cdot x = 16.4 \qquad \text{Simplify.}$$

$$\frac{1.75x}{1.75} = \frac{16.4}{1.75} \qquad \text{Divide both sides by 1.75.}$$

$$x \approx 9.371 \qquad \text{Divide to three decimal places.}$$

Thus, to the nearest hundredth of a million dollars, 10 million in Australian dollars is equivalent to about \$9.73 million US dollars. Note that currency exchange rates fluctuate daily and these values will change accordingly.

6.3 Unit Conversion: American System

1. Multiply by the appropriate conversion factor.

$$8\,\text{yd} = 8\,\text{yd} \cdot \frac{3\,\text{ft}}{1\,\text{yd}} \qquad\qquad \text{Apply } 3\,\text{ft}/1\,\text{yd.}$$

$$= 8\,\cancel{\text{yd}} \cdot \frac{3\,\text{ft}}{1\,\cancel{\text{yd}}} \qquad\qquad \text{Cancel common unit.}$$

$$= 24\,\text{ft} \qquad\qquad \text{Multiply.}$$

3. multiply by the appropriate conversion factor.

$$261\,\text{ft} = 261\,\text{ft} \cdot \frac{1\,\text{yd}}{3\,\text{ft}} \qquad\qquad \text{Apply } 1\,\text{yd}/3\,\text{ft.}$$

$$= 261\,\cancel{\text{ft}} \cdot \frac{1\,\text{yd}}{3\,\cancel{\text{ft}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \frac{261}{3}\,\text{yd} \qquad\qquad \text{Multiply fractions.}$$

$$= 87\,\text{yd} \qquad\qquad \text{Divide.}$$

5. We multiply by a chain of conversion factors, the first to change inches to feet, the second to change feet to yards.

$$235\,\text{in} = 235\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \cdot \frac{1\,\text{yd}}{3\,\text{ft}} \qquad \text{Multiply by conversion factors.}$$

$$= 235\,\cancel{\text{in}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\,\text{yd}}{3\,\cancel{\text{ft}}} \qquad \text{Cancel common units.}$$

$$= \frac{235 \cdot 1 \cdot 1}{12 \cdot 3}\,\text{yd} \qquad \text{Multiply fractions.}$$

$$= 6.52\,\text{yd} \qquad \text{Simplify.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone and truncate everything to the right of the rounding digit. Hence, to the nearest tenth of a yard, 235 inches is approximately 6.5 yards.
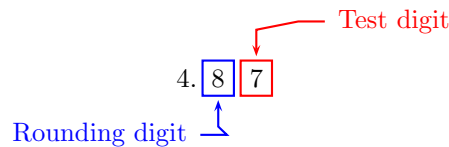
7. multiply by the appropriate conversion factor.

$$141\,\text{ft} = 141\,\text{ft} \cdot \frac{1\,\text{yd}}{3\,\text{ft}} \qquad\qquad \text{Apply } 1\,\text{yd}/3\,\text{ft}.$$

$$= 141\,\cancel{\text{ft}} \cdot \frac{1\,\text{yd}}{3\,\cancel{\text{ft}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \frac{141}{3}\,\text{yd} \qquad\qquad \text{Multiply fractions.}$$

$$= 47\,\text{yd} \qquad\qquad \text{Divide.}$$

9. Multiply by the appropriate conversion factor.

$$2.8\,\text{mi} = 2.8\,\text{mi} \cdot \frac{5280\,\text{ft}}{1\,\text{mi}} \qquad\qquad \text{Apply } 5280\,\text{ft}/1\,\text{mi}.$$

$$= 2.8\,\cancel{\text{mi}} \cdot \frac{5280\,\text{ft}}{1\,\cancel{\text{mi}}} \qquad\qquad \text{Cancel common unit.}$$

$$= 14784\,\text{ft} \qquad\qquad \text{Multiply.}$$

11. We multiply by a chain of conversion factors, the first to change inches to feet, the second to change feet to yards.

$$104\,\text{in} = 104\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \cdot \frac{1\,\text{yd}}{3\,\text{ft}} \qquad\qquad \text{Multiply by conversion factors.}$$

$$= 104\,\cancel{\text{in}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\,\text{yd}}{3\,\cancel{\text{ft}}} \qquad\qquad \text{Cancel common units.}$$

$$= \frac{104 \cdot 1 \cdot 1}{12 \cdot 3}\,\text{yd} \qquad\qquad \text{Multiply fractions.}$$

$$= 2.88\,\text{yd} \qquad\qquad \text{Simplify.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to rounding digit and truncate everything to the right of the rounding digit. Hence, to the nearest tenth of a yard, 104 inches is approximately 2.9 yards.

13. We multiply by a chain of conversion factors, the first to change inches to feet, the second to change feet to miles.

$$
\begin{aligned}
168372\,\text{in} &= 168372\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \cdot \frac{1\,\text{mi}}{5280\,\text{ft}} && \text{Multiply by conversion factors.}\\
&= 168372\,\cancel{\text{in}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\,\text{mi}}{5280\,\cancel{\text{ft}}} && \text{Cancel common units.}\\
&= \frac{168372 \cdot 1 \cdot 1}{12 \cdot 5280}\,\text{mi} && \text{Multiply fractions.}\\
&= 2.65\,\text{mi} && \text{Simplify.}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate everything after the rounding digit. Hence, to the nearest tenth of a mile, 168,372 inches is approximately 2.7 miles.

15. Multiply by the appropriate conversion factor.

$$
\begin{aligned}
82\,\text{ft} &= 82\,\text{ft} \cdot \frac{12\,\text{in}}{1\,\text{ft}} && \text{Apply } 12\,\text{in}/1\,\text{ft.}\\
&= 82\,\cancel{\text{ft}} \cdot \frac{12\,\text{in}}{1\,\cancel{\text{ft}}} && \text{Cancel common unit.}\\
&= 984\,\text{in} && \text{Multiply.}
\end{aligned}
$$

17. We multiply by a chain of conversion factors, the first to change yards to feet, the second to change feet to inches.

$$
\begin{aligned}
2.9\,\text{yd} &= 2.9\,\text{yd} \cdot \frac{3\,\text{ft}}{1\,\text{yd}} \cdot \frac{12\,\text{in}}{1\,\text{ft}} && \text{Multiply by conversion factors.}\\
&= 2.9\,\cancel{\text{yd}} \cdot \frac{3\,\cancel{\text{ft}}}{1\,\cancel{\text{yd}}} \cdot \frac{12\,\text{in}}{1\,\cancel{\text{ft}}} && \text{Cancel common units.}\\
&= \frac{2.9 \cdot 3 \cdot 12}{1 \cdot 1}\,\text{in} && \text{Multiply fractions.}\\
&= 104.4\,\text{in} && \text{Simplify.}
\end{aligned}
$$

Locate the rounding digit in the units place and the test digit in the tenths place.

Because the test digit is less than 5, leave the rounding digit alone and truncate everything to the right of the decimal point. Hence, to the nearest inch, 2.9 yards is approximately 104 inches.

19. Multiply by the appropriate conversion factor.

$$25756\,\text{ft} = 25756\,\text{ft} \cdot \frac{1\,\text{mi}}{5280\,\text{ft}} \qquad \text{Apply } 1\,\text{mi}/5280\,\text{ft.}$$

$$= 25756\,\text{ft} \cdot \frac{1\,\text{mi}}{5280\,\text{ft}} \qquad \text{Cancel common unit.}$$

$$= \frac{25756}{5280}\,\text{mi} \qquad \text{Multiply.}$$

$$= 4.87\,\text{mi} \qquad \text{Carry division to two decimal places.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a mile, 25,756 feet is approximately 4.9 miles.

21. Multiply by the appropriate conversion factor.

$$5\,\text{yd} = 5\,\text{yd} \cdot \frac{3\,\text{ft}}{1\,\text{yd}} \qquad \text{Apply } 3\,\text{ft}/1\,\text{yd.}$$

$$= 5\,\text{yd} \cdot \frac{3\,\text{ft}}{1\,\text{yd}} \qquad \text{Cancel common unit.}$$

$$= 15\,\text{ft} \qquad \text{Multiply.}$$

23. We multiply by a chain of conversion factors, the first to change inches to feet, the second to change feet to miles.

$$
\begin{aligned}
169312 \,\text{in} &= 169312 \,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \cdot \frac{1\,\text{mi}}{5280\,\text{ft}} && \textcolor{red}{\text{Multiply by conversion factors.}}\\
&= 169312 \,\cancel{\text{in}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\,\text{mi}}{5280\,\cancel{\text{ft}}} && \textcolor{red}{\text{Cancel common units.}}\\
&= \frac{169312 \cdot 1 \cdot 1}{12 \cdot 5280}\,\text{mi} && \textcolor{red}{\text{Multiply fractions.}}\\
&= 2.67\,\text{mi} && \textcolor{red}{\text{Simplify.}}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate everything after the rounding digit. Hence, to the nearest tenth of a mile, 169,312 inches is approximately 2.7 miles.

25. We multiply by a chain of conversion factors, the first to change yards to feet, the second to change feet to inches.

$$
\begin{aligned}
1.5 \,\text{yd} &= 1.5 \,\text{yd} \cdot \frac{3\,\text{ft}}{1\,\text{yd}} \cdot \frac{12\,\text{in}}{1\,\text{ft}} && \textcolor{red}{\text{Multiply by conversion factors.}}\\
&= 1.5 \,\cancel{\text{yd}} \cdot \frac{3\,\cancel{\text{ft}}}{1\,\cancel{\text{yd}}} \cdot \frac{12\,\text{in}}{1\,\cancel{\text{ft}}} && \textcolor{red}{\text{Cancel common units.}}\\
&= \frac{1.5 \cdot 3 \cdot 12}{1 \cdot 1}\,\text{in} && \textcolor{red}{\text{Multiply fractions.}}\\
&= 54.0\,\text{in} && \textcolor{red}{\text{Simplify.}}
\end{aligned}
$$

Locate the rounding digit in the units place and the test digit in the tenths place.

Because the test digit is less than 5, leave the rounding digit alone and truncate everything to the right of the decimal point. Hence, to the nearest inch, 1.5 yards is approximately 54 inches.

27. Multiply by the appropriate conversion factor.

$$360\,\text{in} = 360\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \qquad \text{Apply } 1\,\text{ft}/12\,\text{in.}$$

$$= 360\,\cancel{\text{in}} \cdot \frac{1\,\text{ft}}{12\,\cancel{\text{in}}} \qquad \text{Cancel common unit.}$$

$$= \frac{360}{12}\,\text{ft} \qquad \text{Multiply.}$$

$$= 30\,\text{ft} \qquad \text{Divide.}$$

29. Multiply by the appropriate conversion factor.

$$48\,\text{in} = 48\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \qquad \text{Apply } 1\,\text{ft}/12\,\text{in.}$$

$$= 48\,\cancel{\text{in}} \cdot \frac{1\,\text{ft}}{12\,\cancel{\text{in}}} \qquad \text{Cancel common unit.}$$

$$= \frac{48}{12}\,\text{ft} \qquad \text{Multiply.}$$

$$= 4\,\text{ft} \qquad \text{Divide.}$$

31. Multiply by the appropriate conversion factor.

$$15363\,\text{ft} = 15363\,\text{ft} \cdot \frac{1\,\text{mi}}{5280\,\text{ft}} \qquad \text{Apply } 1\,\text{mi}/5280\,\text{ft.}$$

$$= 15363\,\cancel{\text{ft}} \cdot \frac{1\,\text{mi}}{5280\,\cancel{\text{ft}}} \qquad \text{Cancel common unit.}$$

$$= \frac{15363}{5280}\,\text{mi} \qquad \text{Multiply.}$$

$$= 2.90\,\text{mi} \qquad \text{Carry division to two decimal places.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate.. Hence, to the nearest tenth of a mile, 15,363 feet is approximately 2.9 miles.

33. We multiply by a chain of conversion factors, the first to change miles to feet, the second to change feet to inches.

$$1.7 \, \text{mi} = 1.7 \, \text{mi} \cdot \frac{5280 \, \text{ft}}{1 \, \text{mi}} \cdot \frac{12 \, \text{in}}{1 \, \text{ft}} \qquad \text{Multiply by conversion factors.}$$

$$= 1.7 \, \cancel{\text{mi}} \cdot \frac{5280 \, \cancel{\text{ft}}}{1 \, \cancel{\text{mi}}} \cdot \frac{12 \, \text{in}}{1 \, \cancel{\text{ft}}} \qquad \text{Cancel common units.}$$

$$= \frac{1.7 \cdot 5280 \cdot 12}{1 \cdot 1} \, \text{ft} \qquad \text{Multiply fractions.}$$

$$= 107712 \, \text{in} \qquad \text{Simplify.}$$

Hence, 1.7 miles equals 107,712 inches.

35. We multiply by a chain of conversion factors, the first to change miles to feet, the second to change feet to inches.

$$3.1 \, \text{mi} = 3.1 \, \text{mi} \cdot \frac{5280 \, \text{ft}}{1 \, \text{mi}} \cdot \frac{12 \, \text{in}}{1 \, \text{ft}} \qquad \text{Multiply by conversion factors.}$$

$$= 3.1 \, \cancel{\text{mi}} \cdot \frac{5280 \, \cancel{\text{ft}}}{1 \, \cancel{\text{mi}}} \cdot \frac{12 \, \text{in}}{1 \, \cancel{\text{ft}}} \qquad \text{Cancel common units.}$$

$$= \frac{3.1 \cdot 5280 \cdot 12}{1 \cdot 1} \, \text{ft} \qquad \text{Multiply fractions.}$$

$$= 196416 \, \text{in} \qquad \text{Simplify.}$$

Hence, 3.1 miles equals 196,416 inches.

37. Multiply by the appropriate conversion factor.

$$3.6 \, \text{mi} = 3.6 \, \text{mi} \cdot \frac{5280 \, \text{ft}}{1 \, \text{mi}} \qquad \text{Apply } 5280 \, \text{ft}/1 \, \text{mi}.$$

$$= 3.6 \, \cancel{\text{mi}} \cdot \frac{5280 \, \text{ft}}{1 \, \cancel{\text{mi}}} \qquad \text{Cancel common unit.}$$

$$= 19008 \, \text{ft} \qquad \text{Multiply.}$$

39. Multiply by the appropriate conversion factor.

$$18 \, \text{ft} = 18 \, \text{ft} \cdot \frac{12 \, \text{in}}{1 \, \text{ft}} \qquad \text{Apply } 12 \, \text{in}/1 \, \text{ft}.$$

$$= 18 \, \cancel{\text{ft}} \cdot \frac{12 \, \text{in}}{1 \, \cancel{\text{ft}}} \qquad \text{Cancel common unit.}$$

$$= 216 \, \text{in} \qquad \text{Multiply.}$$

41. Multiply by the appropriate conversion factor.

$$5\frac{1}{8}\,\text{lb} = 5\frac{1}{8}\,\text{lb}\cdot\frac{16\,\text{oz}}{1\,\text{lb}} \qquad \text{Apply }16\,\text{oz}/1\,\text{lb}.$$
$$= 5\frac{1}{8}\,\cancel{\text{lb}}\cdot\frac{16\,\text{oz}}{1\,\cancel{\text{lb}}} \qquad \text{Cancel common unit.}$$
$$= \left(\frac{41}{8}\cdot16\right)\text{oz} \qquad \text{Mixed to improper fraction.}$$
$$= 82\,\text{oz}$$

Hence, $5\frac{1}{8}$ pounds equals 82 ounces.

43. This problem requires multiplying by a chain of conversion factors.

$$2.4\,\text{ton} = 2.4\,\text{ton}\cdot\frac{2000\,\text{lb}}{1\,\text{ton}}\cdot\frac{16\,\text{oz}}{1\,\text{lb}} \qquad \text{Multiply by conversion factors.}$$
$$= 2.4\,\cancel{\text{ton}}\cdot\frac{2000\,\cancel{\text{lb}}}{1\,\cancel{\text{ton}}}\cdot\frac{16\,\text{oz}}{1\,\cancel{\text{lb}}} \qquad \text{Cancel common units.}$$
$$= \left(\frac{2.4\cdot2000\cdot16}{1\cdot1}\right)\text{oz} \qquad \text{Multiply.}$$
$$= 76800\,\text{oz}$$

Hence, 2.4 tons equals $76,800$ ounces.

45. Multiply by the appropriate conversion factor.

$$34\,\text{oz} = 34\,\text{oz}\cdot\frac{1\,\text{lb}}{16\,\text{oz}} \qquad \text{Apply }1\,\text{lb}/16\,\text{oz}.$$
$$= 34\,\cancel{\text{oz}}\cdot\frac{1\,\text{lb}}{16\,\cancel{\text{oz}}} \qquad \text{Cancel common unit.}$$
$$= \frac{34}{16}\,\text{lb} \qquad \text{Multiply.}$$
$$= \frac{17}{8}\,\text{lb} \qquad \text{Reduce.}$$
$$= 2\frac{1}{8}\,\text{lb} \qquad \text{Change to mixed fraction.}$$

47. Multiply by the appropriate conversion factor.

$$2.2\,\text{ton} = 2.2\,\text{ton}\cdot\frac{2000\,\text{lb}}{1\,\text{ton}} \qquad \text{Apply }2000\,\text{lb}/1\,\text{ton}.$$
$$= 2.2\,\cancel{\text{ton}}\cdot\frac{2000\,\text{lb}}{1\,\cancel{\text{ton}}} \qquad \text{Cancel common unit.}$$
$$= \left(\frac{2.2\cdot2000}{1}\right)\text{lb} \qquad \text{Multiply.}$$
$$= 4400\,\text{lb}$$

Hence, 2.2 tons equals 4, 400 pounds.

49. Multiply by the appropriate conversion factor.

$$70\,\text{oz} = 70\,\text{oz} \cdot \frac{1\,\text{lb}}{16\,\text{oz}} \qquad\qquad \text{Apply } 1\,\text{lb}/16\,\text{oz.}$$

$$= 70\,\cancel{\text{oz}} \cdot \frac{1\,\text{lb}}{16\,\cancel{\text{oz}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \frac{70}{16}\,\text{lb} \qquad\qquad\qquad \text{Multiply.}$$

$$= \frac{35}{8}\,\text{lb} \qquad\qquad\qquad \text{Reduce.}$$

$$= 4\frac{3}{8}\,\text{lb} \qquad\qquad\qquad \text{Change to mixed fraction.}$$

51. Multiply by the appropriate conversion factor.

$$9560\,\text{lb} = 9560\,\text{lb} \cdot \frac{1\,\text{ton}}{2000\,\text{lb}} \qquad\qquad \text{Apply } 1\,\text{ton}/2000\,\text{lb.}$$

$$= 9560\,\cancel{\text{lb}} \cdot \frac{1\,\text{ton}}{2000\,\cancel{\text{lb}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \left(\frac{9560}{2000}\right)\,\text{ton} \qquad\qquad \text{Multiply.}$$

$$= 4.78\,\text{ton} \qquad\qquad\qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate.. Hence, to the nearest tenth of a ton, 9,560 pounds is approximately 4.8 tons.

53. Multiply by the appropriate conversion factor.

$$2\frac{1}{2}\,\text{lb} = 2\frac{1}{2}\,\text{lb} \cdot \frac{16\,\text{oz}}{1\,\text{lb}} \qquad\qquad \text{Apply } 16\,\text{oz}/1\,\text{lb}.$$

$$= 2\frac{1}{2}\,\cancel{\text{lb}} \cdot \frac{16\,\text{oz}}{1\,\cancel{\text{lb}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \left(\frac{5}{2} \cdot 16\right)\,\text{oz} \qquad\qquad \text{Mixed to improper fraction.}$$

$$= 40\,\text{oz}$$

Hence, $2\frac{1}{2}$ pounds equals 40 ounces.

55. Multiply by the appropriate conversion factor.

$$5.9\,\text{ton} = 5.9\,\text{ton} \cdot \frac{2000\,\text{lb}}{1\,\text{ton}} \qquad\qquad \text{Apply } 2000\,\text{lb}/1\,\text{ton}.$$

$$= 5.9\,\cancel{\text{ton}} \cdot \frac{2000\,\text{lb}}{1\,\cancel{\text{ton}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \left(\frac{5.9 \cdot 2000}{1}\right)\,\text{lb} \qquad\qquad \text{Multiply.}$$

$$= 11800\,\text{lb}$$

Hence, 5.9 tons equals $11,800$ pounds.

57. This problem requires multiplying by a chain of conversion factors.

$$2.5\,\text{ton} = 2.5\,\text{ton} \cdot \frac{2000\,\text{lb}}{1\,\text{ton}} \cdot \frac{16\,\text{oz}}{1\,\text{lb}} \qquad \text{Multiply by conversion factors.}$$

$$= 2.5\,\cancel{\text{ton}} \cdot \frac{2000\,\cancel{\text{lb}}}{1\,\cancel{\text{ton}}} \cdot \frac{16\,\text{oz}}{1\,\cancel{\text{lb}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{2.5 \cdot 2000 \cdot 16}{1 \cdot 1}\right)\,\text{oz} \qquad \text{Multiply.}$$

$$= 80000\,\text{oz}$$

Hence, 2.5 tons equals $80,000$ ounces.

59. Multiply by the appropriate conversion factor.

$$8111\,\text{lb} = 8111\,\text{lb} \cdot \frac{1\,\text{ton}}{2000\,\text{lb}} \qquad\qquad \text{Apply } 1\,\text{ton}/2000\,\text{lb}.$$

$$= 8111\,\cancel{\text{lb}} \cdot \frac{1\,\text{ton}}{2000\,\cancel{\text{lb}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \left(\frac{8111}{2000}\right)\,\text{ton} \qquad\qquad \text{Multiply.}$$

$$= 4.0555\,\text{ton} \qquad\qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate.. Hence, to the nearest tenth of a ton, 8,111 pounds is approximately 4.1 tons.

61. This problem requires multiplying by a chain of conversion factors.

$$4.5625 \, \text{pt} = 4.5625 \, \text{pt} \cdot \frac{2 \, \text{c}}{1 \, \text{pt}} \cdot \frac{8 \, \text{fl oz}}{1 \, \text{c}} \qquad \text{Multiply by conversion factors.}$$

$$= 4.5625 \, \cancel{\text{pt}} \cdot \frac{2 \, \cancel{\text{c}}}{1 \, \cancel{\text{pt}}} \cdot \frac{8 \, \text{fl oz}}{1 \, \cancel{\text{c}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{4.5625 \cdot 2 \cdot 8}{1 \cdot 1} \right) \, \text{fl oz} \qquad \text{Multiply fractions.}$$

$$= 73 \, \text{fl oz} \qquad \text{Multiply.}$$

Hence, 4.5625 pints equals 73 fluid ounces.

63. This problem requires multiplying by a chain of conversion factors.

$$32 \, \text{fl oz} = 32 \, \text{fl oz} \cdot \frac{1 \, \text{c}}{8 \, \text{fl oz}} \cdot \frac{1 \, \text{pt}}{2 \, \text{c}} \qquad \text{Multiply by conversion factors.}$$

$$= 32 \, \cancel{\text{fl oz}} \cdot \frac{1 \, \cancel{\text{c}}}{8 \, \cancel{\text{fl oz}}} \cdot \frac{1 \, \text{pt}}{2 \, \cancel{\text{c}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{32 \cdot 1 \cdot 1}{8 \cdot 2} \right) \, \text{pt} \qquad \text{Multiply fractions.}$$

$$= \frac{32}{16} \, \text{pt} \qquad \text{Multiply.}$$

$$= 2 \, \text{pt} \qquad \text{Divide.}$$

Hence, 32 fluid ounces equals 2 pints.

65. This problem requires multiplying by a chain of conversion factors.

$$3.7\,\text{gal} = 3.7\,\text{gal} \cdot \frac{4\,\text{qt}}{1\,\text{gal}} \cdot \frac{2\,\text{pt}}{1\,\text{qt}} \qquad \text{Multiply by conversion factors.}$$

$$= 3.7\,\cancel{\text{gal}} \cdot \frac{4\,\cancel{\text{qt}}}{1\,\cancel{\text{gal}}} \cdot \frac{2\,\text{pt}}{1\,\cancel{\text{qt}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{3.7 \cdot 4 \cdot 2}{1 \cdot 1}\right)\,\text{pt} \qquad \text{Multiply.}$$

$$= 29.6\,\text{pt}$$

Hence, 3.7 gallons equals 29.6 pints.

67. This problem requires multiplying by a chain of conversion factors.

$$216\,\text{pt} = 216\,\text{pt} \cdot \frac{1\,\text{qt}}{2\,\text{pt}} \cdot \frac{1\,\text{gal}}{4\,\text{qt}} \qquad \text{Multiply by conversion factors.}$$

$$= 216\,\cancel{\text{pt}} \cdot \frac{1\,\cancel{\text{qt}}}{2\,\cancel{\text{pt}}} \cdot \frac{1\,\text{gal}}{4\,\cancel{\text{qt}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{216 \cdot 1 \cdot 1}{2 \cdot 4}\right)\,\text{gal} \qquad \text{Multiply fractions.}$$

$$= \frac{216}{8}\,\text{gal} \qquad \text{Multiply.}$$

$$= 27\,\text{gal} \qquad \text{Divide.}$$

Hence, 216 pints equals 27 gallons.

69. This problem requires multiplying by a chain of conversion factors.

$$544\,\text{fl oz} = 544\,\text{fl oz} \cdot \frac{1\,\text{c}}{8\,\text{fl oz}} \cdot \frac{1\,\text{pt}}{2\,\text{c}} \qquad \text{Multiply by conversion factors.}$$

$$= 544\,\cancel{\text{fl oz}} \cdot \frac{1\,\cancel{\text{c}}}{8\,\cancel{\text{fl oz}}} \cdot \frac{1\,\text{pt}}{2\,\cancel{\text{c}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{544 \cdot 1 \cdot 1}{8 \cdot 2}\right)\,\text{pt} \qquad \text{Multiply fractions.}$$

$$= \frac{544}{16}\,\text{pt} \qquad \text{Multiply.}$$

$$= 34\,\text{pt} \qquad \text{Divide.}$$

Hence, 544 fluid ounces equals 34 pints.

71. This problem requires multiplying by a chain of conversion factors.

$$112\,\text{pt} = 112\,\text{pt} \cdot \frac{1\,\text{qt}}{2\,\text{pt}} \cdot \frac{1\,\text{gal}}{4\,\text{qt}} \qquad \color{red}\text{Multiply by conversion factors.}$$

$$= 112\,\cancel{\text{pt}} \cdot \frac{1\,\cancel{\text{qt}}}{2\,\cancel{\text{pt}}} \cdot \frac{1\,\text{gal}}{4\,\cancel{\text{qt}}} \qquad \color{red}\text{Cancel common units.}$$

$$= \left(\frac{112 \cdot 1 \cdot 1}{2 \cdot 4}\right)\,\text{gal} \qquad \color{red}\text{Multiply fractions.}$$

$$= \frac{112}{8}\,\text{gal} \qquad \color{red}\text{Multiply.}$$

$$= 14\,\text{gal} \qquad \color{red}\text{Divide.}$$

Hence, 112 pints equals 14 gallons.

73. This problem requires multiplying by a chain of conversion factors.

$$7.7\,\text{gal} = 7.7\,\text{gal} \cdot \frac{4\,\text{qt}}{1\,\text{gal}} \cdot \frac{2\,\text{pt}}{1\,\text{qt}} \qquad \color{red}\text{Multiply by conversion factors.}$$

$$= 7.7\,\cancel{\text{gal}} \cdot \frac{4\,\cancel{\text{qt}}}{1\,\cancel{\text{gal}}} \cdot \frac{2\,\text{pt}}{1\,\cancel{\text{qt}}} \qquad \color{red}\text{Cancel common units.}$$

$$= \left(\frac{7.7 \cdot 4 \cdot 2}{1 \cdot 1}\right)\,\text{pt} \qquad \color{red}\text{Multiply.}$$

$$= 61.6\,\text{pt}$$

Hence, 7.7 gallons equals 61.6 pints.

75. This problem requires multiplying by a chain of conversion factors.

$$3.875\,\text{pt} = 3.875\,\text{pt} \cdot \frac{2\,\text{c}}{1\,\text{pt}} \cdot \frac{8\,\text{fl oz}}{1\,\text{c}} \qquad \color{red}\text{Multiply by conversion factors.}$$

$$= 3.875\,\cancel{\text{pt}} \cdot \frac{2\,\cancel{\text{c}}}{1\,\cancel{\text{pt}}} \cdot \frac{8\,\text{fl oz}}{1\,\cancel{\text{c}}} \qquad \color{red}\text{Cancel common units.}$$

$$= \left(\frac{3.875 \cdot 2 \cdot 8}{1 \cdot 1}\right)\,\text{fl oz} \qquad \color{red}\text{Multiply fractions.}$$

$$= 62\,\text{fl oz} \qquad \color{red}\text{Multiply.}$$

Hence, 3.875 pints equals 62 fluid ounces.

77. This problem requires multiplying by a chain of conversion factors.

$$7.8\,\text{yr} = 7.8\,\text{yr} \cdot \frac{365\,\text{day}}{1\,\text{yr}} \cdot \frac{24\,\text{hr}}{1\,\text{day}} \qquad \textcolor{red}{\text{Multiply by conversion factors.}}$$

$$= 7.8\,\cancel{\text{yr}} \cdot \frac{365\,\cancel{\text{day}}}{1\,\cancel{\text{yr}}} \cdot \frac{24\,\text{hr}}{1\,\cancel{\text{day}}} \qquad \textcolor{red}{\text{Cancel common units.}}$$

$$= \left(\frac{7.8 \cdot 365 \cdot 24}{1 \cdot 1} \right)\,\text{hr} \qquad \textcolor{red}{\text{Multiply fractions.}}$$

$$= 68328\,\text{hr} \qquad \textcolor{red}{\text{Multiply.}}$$

Hence, 7.8 years equals 68,328 hours.

79. This problem requires multiplying by a chain of conversion factors.

$$7.6\,\text{yr} = 7.6\,\text{yr} \cdot \frac{365\,\text{day}}{1\,\text{yr}} \cdot \frac{24\,\text{hr}}{1\,\text{day}} \qquad \textcolor{red}{\text{Multiply by conversion factors.}}$$

$$= 7.6\,\cancel{\text{yr}} \cdot \frac{365\,\cancel{\text{day}}}{1\,\cancel{\text{yr}}} \cdot \frac{24\,\text{hr}}{1\,\cancel{\text{day}}} \qquad \textcolor{red}{\text{Cancel common units.}}$$

$$= \left(\frac{7.6 \cdot 365 \cdot 24}{1 \cdot 1} \right)\,\text{hr} \qquad \textcolor{red}{\text{Multiply fractions.}}$$

$$= 66576\,\text{hr} \qquad \textcolor{red}{\text{Multiply.}}$$

Hence, 7.6 years equals 66,576 hours.

81. This problem requires multiplying by a chain of conversion factors.

$$4025005\,\text{s} = 4025005\,\text{s} \cdot \frac{1\,\text{min}}{60\,\text{s}} \cdot \frac{1\,\text{hr}}{60\,\text{min}} \cdot \frac{1\,\text{day}}{24\,\text{hr}} \qquad \textcolor{red}{\text{Multiply by conversion factors.}}$$

$$= 4025005\,\cancel{\text{s}} \cdot \frac{1\,\cancel{\text{min}}}{60\,\cancel{\text{s}}} \cdot \frac{1\,\cancel{\text{hr}}}{60\,\cancel{\text{min}}} \cdot \frac{1\,\text{day}}{24\,\cancel{\text{hr}}} \qquad \textcolor{red}{\text{Cancel common units.}}$$

$$= \left(\frac{4025005 \cdot 1 \cdot 1 \cdot 1}{60 \cdot 60 \cdot 24} \right)\,\text{day} \qquad \textcolor{red}{\text{Multiply fractions.}}$$

$$= \left(\frac{4025005}{86400} \right)\,\text{day} \qquad \textcolor{red}{\text{Multiply fractions.}}$$

$$= 46.5857060185185\,\text{day} \qquad \textcolor{red}{\text{Divide.}}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Test digit

46. 5 8

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a day, 4,025,005 seconds is approximately 46.6 days.

83. This problem requires multiplying by a chain of conversion factors.

$$37668 \, \text{hr} = 37668 \, \text{hr} \cdot \frac{1 \, \text{day}}{24 \, \text{hr}} \cdot \frac{1 \, \text{yr}}{365 \, \text{day}} \qquad \text{Multiply by conversion factors.}$$

$$= 37668 \, \cancel{\text{hr}} \cdot \frac{1 \, \cancel{\text{day}}}{24 \, \cancel{\text{hr}}} \cdot \frac{1 \, \text{yr}}{365 \, \cancel{\text{day}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{37668 \cdot 1 \cdot 1}{24 \cdot 365} \right) \, \text{yr} \qquad \text{Multiply fractions.}$$

$$= 4.3 \, \text{yr} \qquad \text{Multiply.}$$

Hence, 37,668 hours equals 4.3 years.

85. This problem requires multiplying by a chain of conversion factors.

$$22776 \, \text{hr} = 22776 \, \text{hr} \cdot \frac{1 \, \text{day}}{24 \, \text{hr}} \cdot \frac{1 \, \text{yr}}{365 \, \text{day}} \qquad \text{Multiply by conversion factors.}$$

$$= 22776 \, \cancel{\text{hr}} \cdot \frac{1 \, \cancel{\text{day}}}{24 \, \cancel{\text{hr}}} \cdot \frac{1 \, \text{yr}}{365 \, \cancel{\text{day}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{22776 \cdot 1 \cdot 1}{24 \cdot 365} \right) \, \text{yr} \qquad \text{Multiply fractions.}$$

$$= 2.6 \, \text{yr} \qquad \text{Multiply.}$$

Hence, 22,776 hours equals 2.6 years.

87. This problem requires multiplying by a chain of conversion factors.

$$96 \, \text{day} = 96 \, \text{day} \cdot \frac{24 \, \text{hr}}{1 \, \text{day}} \cdot \frac{60 \, \text{min}}{1 \, \text{hr}} \cdot \frac{60 \, \text{s}}{1 \, \text{min}} \qquad \text{Multiply by conversion factors.}$$

$$= 96 \, \cancel{\text{day}} \cdot \frac{24 \, \cancel{\text{hr}}}{1 \, \cancel{\text{day}}} \cdot \frac{60 \, \cancel{\text{min}}}{1 \, \cancel{\text{hr}}} \cdot \frac{60 \, \text{s}}{1 \, \cancel{\text{min}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{96 \cdot 24 \cdot 60 \cdot 60}{1 \cdot 1 \cdot 1} \right) \, \text{s} \qquad \text{Multiply fractions.}$$

$$= 8294400 \, \text{s} \qquad \text{Multiply.}$$

Hence, 96 days equals 8,294,400 seconds.

89. This problem requires multiplying by a chain of conversion factors.

$$40 \, \text{day} = 40 \, \text{day} \cdot \frac{24 \, \text{hr}}{1 \, \text{day}} \cdot \frac{60 \, \text{min}}{1 \, \text{hr}} \cdot \frac{60 \, \text{s}}{1 \, \text{min}} \qquad \text{Multiply by conversion factors.}$$

$$= 40 \, \cancel{\text{day}} \cdot \frac{24 \, \cancel{\text{hr}}}{1 \, \cancel{\text{day}}} \cdot \frac{60 \, \cancel{\text{min}}}{1 \, \cancel{\text{hr}}} \cdot \frac{60 \, \text{s}}{1 \, \cancel{\text{min}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{40 \cdot 24 \cdot 60 \cdot 60}{1 \cdot 1 \cdot 1} \right) \, \text{s} \qquad \text{Multiply fractions.}$$

$$= 3456000 \, \text{s} \qquad \text{Multiply.}$$

Hence, 40 days equals 3,456,000 seconds.

91. This problem requires multiplying by a chain of conversion factors.

$$3750580 \, \text{s} = 3750580 \, \text{s} \cdot \frac{1 \, \text{min}}{60 \, \text{s}} \cdot \frac{1 \, \text{hr}}{60 \, \text{min}} \cdot \frac{1 \, \text{day}}{24 \, \text{hr}} \qquad \text{Multiply by conversion factors.}$$

$$= 3750580 \, \cancel{\text{s}} \cdot \frac{1 \, \cancel{\text{min}}}{60 \, \cancel{\text{s}}} \cdot \frac{1 \, \cancel{\text{hr}}}{60 \, \cancel{\text{min}}} \cdot \frac{1 \, \text{day}}{24 \, \cancel{\text{hr}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{3750580 \cdot 1 \cdot 1 \cdot 1}{60 \cdot 60 \cdot 24} \right) \, \text{day} \qquad \text{Multiply fractions.}$$

$$= \left(\frac{3750580}{86400} \right) \, \text{day} \qquad \text{Multiply fractions.}$$

$$= 43.4094907407407 \, \text{day} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Test digit

$$43.\boxed{4}\,\boxed{0}$$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a day, 3,750,580 seconds is approximately 43.4 days.

93. There are 5280 feet in a mile, 60 minutes in an hour, and 60 seconds in a minute.

$$367\,\frac{\text{ft}}{\text{s}} \approx 367\,\frac{\text{ft}}{\text{s}} \cdot \frac{1\,\text{mi}}{5280\,\text{ft}} \cdot \frac{60\,\text{min}}{1\,\text{h}} \cdot \frac{60\,\text{s}}{1\,\text{min}} \qquad \text{Conversion factors.}$$

$$\approx 367\,\frac{\cancel{\text{ft}}}{\cancel{\text{s}}} \cdot \frac{1\,\text{mi}}{5280\,\cancel{\text{ft}}} \cdot \frac{60\,\cancel{\text{min}}}{1\,\text{h}} \cdot \frac{60\,\cancel{\text{s}}}{1\,\cancel{\text{min}}} \qquad \text{Cancel common units.}$$

$$\approx \frac{367\cdot 1\cdot 60\cdot 60}{5280\cdot 1\cdot 1}\,\frac{\text{mi}}{\text{hr}} \qquad \text{Multiply fractions.}$$

$$\approx 250.2\,\frac{\text{mi}}{\text{hr}} \qquad \text{Multiply and divide.}$$

To round to the nearest mile per hour, identify the rounding and test digits.

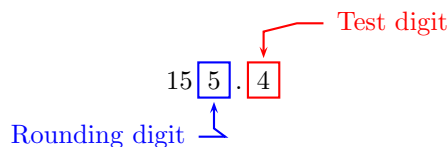Test digit

$$25\,\boxed{0}\,.\,\boxed{2}$$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest mile per hour, the speed is approximately 250 miles per hour.

95. There are 5280 feet in a mile, 60 minutes in an hour, and 60 seconds in a minute.

$$442\,\frac{\text{ft}}{\text{s}} \approx 442\,\frac{\text{ft}}{\text{s}} \cdot \frac{1\,\text{mi}}{5280\,\text{ft}} \cdot \frac{60\,\text{min}}{1\,\text{h}} \cdot \frac{60\,\text{s}}{1\,\text{min}} \qquad \text{Conversion factors.}$$

$$\approx 442\,\frac{\cancel{\text{ft}}}{\cancel{\text{s}}} \cdot \frac{1\,\text{mi}}{5280\,\cancel{\text{ft}}} \cdot \frac{60\,\cancel{\text{min}}}{1\,\text{h}} \cdot \frac{60\,\cancel{\text{s}}}{1\,\cancel{\text{min}}} \qquad \text{Cancel common units.}$$

$$\approx \frac{442\cdot 1\cdot 60\cdot 60}{5280\cdot 1\cdot 1}\,\frac{\text{mi}}{\text{hr}} \qquad \text{Multiply fractions.}$$

$$\approx 301.3\,\frac{\text{mi}}{\text{hr}} \qquad \text{Multiply and divide.}$$

To round to the nearest mile per hour, identify the rounding and test digits.

$$30\boxed{1}.\boxed{3}$$

Test digit

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest mile per hour, the speed is approximately 301 miles per hour.

97. There are 5280 feet in a mile, 60 minutes in an hour, and 60 seconds in a minute.

$$30\,\frac{\text{mi}}{\text{h}} \approx 30\,\frac{\text{mi}}{\text{h}} \cdot \frac{5280\,\text{ft}}{1\,\text{mi}} \cdot \frac{1\,\text{h}}{60\,\text{min}} \cdot \frac{1\,\text{min}}{60\,\text{s}} \qquad \text{Conversion factors.}$$

$$\approx 30\,\frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{5280\,\text{ft}}{1\,\cancel{\text{mi}}} \cdot \frac{1\,\cancel{\text{h}}}{60\,\cancel{\text{min}}} \cdot \frac{1\,\cancel{\text{min}}}{60\,\text{s}} \qquad \text{Cancel common units.}$$

$$\approx \frac{30 \cdot 5280 \cdot 1 \cdot 1}{1 \cdot 60 \cdot 60}\,\frac{\text{ft}}{\text{s}} \qquad \text{Multiply fractions.}$$

$$\approx 44.0\,\frac{\text{ft}}{\text{s}} \qquad \text{Multiply and divide.}$$

To round to the nearest foot per second, identify the rounding and test digits.

$$4\boxed{4}.\boxed{0}$$

Test digit

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest foot per second, the speed is approximately 44 feet per second.

99. There are 5280 feet in a mile, 60 minutes in an hour, and 60 seconds in a minute.

$$106\,\frac{\text{mi}}{\text{h}} \approx 106\,\frac{\text{mi}}{\text{h}} \cdot \frac{5280\,\text{ft}}{1\,\text{mi}} \cdot \frac{1\,\text{h}}{60\,\text{min}} \cdot \frac{1\,\text{min}}{60\,\text{s}} \qquad \text{Conversion factors.}$$

$$\approx 106\,\frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{5280\,\text{ft}}{1\,\cancel{\text{mi}}} \cdot \frac{1\,\cancel{\text{h}}}{60\,\cancel{\text{min}}} \cdot \frac{1\,\cancel{\text{min}}}{60\,\text{s}} \qquad \text{Cancel common units.}$$

$$\approx \frac{106 \cdot 5280 \cdot 1 \cdot 1}{1 \cdot 60 \cdot 60}\,\frac{\text{ft}}{\text{s}} \qquad \text{Multiply fractions.}$$

$$\approx 155.4\,\frac{\text{ft}}{\text{s}} \qquad \text{Multiply and divide.}$$

To round to the nearest foot per second, identify the rounding and test digits.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest foot per second, the speed is approximately 155 feet per second.

101. The relationship between pounds and tons is 1 ton = 2000 pounds. Create a ratio whose numerator is 1 ton and denominator 2000 pounds. Since the numerator and denominator are equivalent, this ratio has a value 1. Multiply by this conversion factor. Cancelling the common units of pounds, your answer will be in tons.

$$3200\,\text{lb} = 3200\,\text{lb} \cdot \frac{1\,\text{ton}}{2000\,\text{lb}} \qquad\qquad \text{Apply } 1\,\text{ton}/2000\,\text{lb.}$$

$$= 3200\,\cancel{\text{lb}} \cdot \frac{1\,\text{ton}}{2000\,\cancel{\text{lb}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \left(\frac{3200}{2000}\right)\text{ton} \qquad\qquad \text{Multiply.}$$

$$= 1.6\,\text{ton} \qquad\qquad \text{Divide.}$$

Hence, Joe lifted 1.6 tons.

103. We want to know how many "two-minutes" are in one year. One way to do this is to form a proportion.

Let x represent the number of pipes that break in one year when one pipe breaks every two minutes. Now create a proportion using these two ratios.

$$\frac{2\,\text{min}}{1\,\text{pipe}} = \frac{1\,\text{year}}{x\,\text{pipes}}$$

To solve this proportion, we need to have consistent units. Convert the year to minutes.

$$1\,\text{yr} = 1\,\text{yr} \cdot \frac{365\,\text{day}}{1\,\text{yr}} \cdot \frac{24\,\text{hr}}{1\,\text{day}} \cdot \frac{60\,\text{min}}{1\,\text{hour}} \qquad \text{Multiply by conversion factors.}$$

$$= 1\,\cancel{\text{yr}} \cdot \frac{365\,\cancel{\text{day}}}{1\,\cancel{\text{yr}}} \cdot \frac{24\,\text{hr}}{1\,\cancel{\text{day}}} \cdot \frac{60\,\text{min}}{1\,\cancel{\text{hour}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{1 \cdot 365 \cdot 24 \cdot 60}{1 \cdot 1 \cdot 1}\right)\text{min} \qquad \text{Multiply fractions.}$$

$$= 525600\,\text{min} \qquad\qquad \text{Multiply.}$$

Thus there are 525,600 minutes in one year. Rewrite the proportion using this value.

$$\frac{2\,\text{min}}{1\,\text{pipe}} = \frac{525600\,\text{min}}{x\,\text{pipes}}$$

Now, we can drop the units, cross multiply and solve.

$$\frac{2}{1} = \frac{525600}{x} \qquad \textcolor{red}{\text{Original proportion.}}$$

$$2 \cdot x = 525600 \cdot 1 \qquad \textcolor{red}{\text{Product of means and extremes are equal.}}$$

$$\frac{2x}{2} = \frac{525600}{2} \qquad \textcolor{red}{\text{Divide both sides by 2.}}$$

$$x = 262800 \qquad \textcolor{red}{\text{Simplify.}}$$

Thus, in the US, 262,800 pipes break each year.

6.4   Unit Conversion: Metric System

1. The metric system prefix centi means "1/100"

3. The metric system prefix hecto means "100"

5. The metric system prefix deci means "1/10"

7. The metric system prefix mg means "milligram."

9. The metric system prefix m means "meter."

11. The metric system prefix kL means "kilolitre" or "kiloliter."

13. The metric system prefix hm means "hectometer."

15. The metric system prefix dam means "dekameter."

17. The metric system prefix dL means "decilitre" or "deciliter."

19. The metric system prefix hg means "hectogram."

21. The metric system prefix dg means "decigram."

23. The metric system prefix hL means "hectolitre" or "hectoliter."

25. To change 5,490 millimeters to meters, apply the appropriate conversion factor.

$$5490 \, \text{cm} = 5490 \, \text{mm} \cdot \frac{1 \, \text{m}}{1000 \, \text{mm}} \qquad \text{Apply conversion factor.}$$

$$= 5490 \, \cancel{\text{mm}} \cdot \frac{1 \, \text{m}}{1000 \, \cancel{\text{mm}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{5490 \cdot 1}{1000} \right) \text{m} \qquad \text{Multiply fractions.}$$

$$= 5.49 \, \text{m} \qquad \text{Simplify.}$$

Hence, 5,490 millimeters equals 5.49 meters.

27. To change 64 meters to millimeters, apply the appropriate conversion factor.

$$64 \, \text{m} = 64 \, \text{m} \cdot \frac{1000 \, \text{mm}}{1 \, \text{m}} \qquad \text{Apply conversion factor.}$$

$$64 \, \cancel{\text{m}} = 64 \, \cancel{\text{m}} \cdot \frac{1000 \, \text{mm}}{1 \, \cancel{\text{m}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{64 \cdot 1000}{1} \right) \text{mm} \qquad \text{Multiply fractions.}$$

$$= 64000 \, \text{mm} \qquad \text{Simplify.}$$

Hence, 64 meters equals 64,000 millimeters.

29. To change 4,571 millimeters to meters, apply the appropriate conversion factor.

$$4571 \, \text{cm} = 4571 \, \text{mm} \cdot \frac{1 \, \text{m}}{1000 \, \text{mm}} \qquad \text{Apply conversion factor.}$$

$$= 4571 \, \cancel{\text{mm}} \cdot \frac{1 \, \text{m}}{1000 \, \cancel{\text{mm}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{4571 \cdot 1}{1000} \right) \text{m} \qquad \text{Multiply fractions.}$$

$$= 4.571 \, \text{m} \qquad \text{Simplify.}$$

Hence, 4,571 millimeters equals 4.571 meters.

31. To change 15 meters to centimeters, apply the appropriate conversion factor.

$$15\,\text{m} = 15\,\text{m} \cdot \frac{100\,\text{cm}}{1\,\text{m}}$$ Apply conversion factor.

$$15\,\cancel{\text{m}} = 15\,\cancel{\text{m}} \cdot \frac{100\,\text{cm}}{1\,\cancel{\text{m}}}$$ Cancel common unit.

$$= \left(\frac{15 \cdot 100}{1}\right)\,\text{cm}$$ Multiply fractions.

$$= 1500\,\text{cm}$$ Simplify.

Hence, 15 meters equals 1,500 centimeters.

33. To change 569 centimeters to meters, apply the appropriate conversion factor.

$$569\,\text{cm} = 569\,\text{cm} \cdot \frac{1\,\text{m}}{100\,\text{cm}}$$ Apply conversion factor.

$$= 569\,\cancel{\text{cm}} \cdot \frac{1\,\text{m}}{100\,\cancel{\text{cm}}}$$ Cancel common unit.

$$= \left(\frac{569 \cdot 1}{100}\right)\,\text{m}$$ Multiply fractions.

$$= 5.69\,\text{m}$$ Simplify.

Hence, 569 centimeters equals 5.69 meters.

35. To change 79 meters to centimeters, apply the appropriate conversion factor.

$$79\,\text{m} = 79\,\text{m} \cdot \frac{100\,\text{cm}}{1\,\text{m}}$$ Apply conversion factor.

$$79\,\cancel{\text{m}} = 79\,\cancel{\text{m}} \cdot \frac{100\,\text{cm}}{1\,\cancel{\text{m}}}$$ Cancel common unit.

$$= \left(\frac{79 \cdot 100}{1}\right)\,\text{cm}$$ Multiply fractions.

$$= 7900\,\text{cm}$$ Simplify.

Hence, 79 meters equals 7,900 centimeters.

37. To change 7.6 kilometers to meters, apply the appropriate conversion factor.

$$7.6\,\text{km} = 7.6\,\text{km} \cdot \frac{1000\,\text{m}}{1\,\text{km}} \qquad \text{Apply conversion factor.}$$

$$= 7.6\,\cancel{\text{km}} \cdot \frac{1000\,\text{m}}{1\,\cancel{\text{km}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{7.6 \cdot 1000}{1}\right)\text{m} \qquad \text{Multiply fractions.}$$

$$= 7600\,\text{m} \qquad \text{Simplify.}$$

Hence, 7.6 kilometers equals 7,600 meters.

39. To change 861 centimeters to meters, apply the appropriate conversion factor.

$$861\,\text{cm} = 861\,\text{cm} \cdot \frac{1\,\text{m}}{100\,\text{cm}} \qquad \text{Apply conversion factor.}$$

$$= 861\,\cancel{\text{cm}} \cdot \frac{1\,\text{m}}{100\,\cancel{\text{cm}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{861 \cdot 1}{100}\right)\text{m} \qquad \text{Multiply fractions.}$$

$$= 8.61\,\text{m} \qquad \text{Simplify.}$$

Hence, 861 centimeters equals 8.61 meters.

41. To change 4,826 meters to kilometers, apply the appropriate conversion factor.

$$4826\,\text{m} = 4826\,\text{m} \cdot \frac{1\,\text{km}}{1000\,\text{m}} \qquad \text{Apply conversion factor.}$$

$$= 4826\,\cancel{\text{m}} \cdot \frac{1\,\text{km}}{1000\,\cancel{\text{m}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{4826 \cdot 1}{1000}\right)\text{km} \qquad \text{Multiply fractions.}$$

$$= 4.826\,\text{km} \qquad \text{Simplify.}$$

Hence, 4,826 meters equals 4.826 kilometers.

43. To change 4,724 meters to kilometers, apply the appropriate conversion factor.

$$
\begin{aligned}
4724\,\text{m} &= 4724\,\text{m} \cdot \frac{1\,\text{km}}{1000\,\text{m}} && \text{\textcolor{red}{Apply conversion factor.}} \\
&= 4724\,\cancel{\text{m}} \cdot \frac{1\,\text{km}}{1000\,\cancel{\text{m}}} && \text{\textcolor{red}{Cancel common unit.}} \\
&= \left(\frac{4724 \cdot 1}{1000}\right)\text{km} && \text{\textcolor{red}{Multiply fractions.}} \\
&= 4.724\,\text{km} && \text{\textcolor{red}{Simplify.}}
\end{aligned}
$$

Hence, 4,724 meters equals 4.724 kilometers.

45. To change 6.5 kilometers to meters, apply the appropriate conversion factor.

$$
\begin{aligned}
6.5\,\text{km} &= 6.5\,\text{km} \cdot \frac{1000\,\text{m}}{1\,\text{km}} && \text{\textcolor{red}{Apply conversion factor.}} \\
&= 6.5\,\cancel{\text{km}} \cdot \frac{1000\,\text{m}}{1\,\cancel{\text{km}}} && \text{\textcolor{red}{Cancel common unit.}} \\
&= \left(\frac{6.5 \cdot 1000}{1}\right)\text{m} && \text{\textcolor{red}{Multiply fractions.}} \\
&= 6500\,\text{m} && \text{\textcolor{red}{Simplify.}}
\end{aligned}
$$

Hence, 6.5 kilometers equals 6,500 meters.

47. To change 17 meters to millimeters, apply the appropriate conversion factor.

$$
\begin{aligned}
17\,\text{m} &= 17\,\text{m} \cdot \frac{1000\,\text{mm}}{1\,\text{m}} && \text{\textcolor{red}{Apply conversion factor.}} \\
17\,\cancel{\text{m}} &= 17\,\cancel{\text{m}} \cdot \frac{1000\,\text{mm}}{1\,\cancel{\text{m}}} && \text{\textcolor{red}{Cancel common unit.}} \\
&= \left(\frac{17 \cdot 1000}{1}\right)\text{mm} && \text{\textcolor{red}{Multiply fractions.}} \\
&= 17000\,\text{mm} && \text{\textcolor{red}{Simplify.}}
\end{aligned}
$$

Hence, 17 meters equals 17,000 millimeters.

49. To change 512 milligrams to centigrams, we need to apply a chain of conversion factors.

$$512\,\text{mg} = 512\,\text{mg} \cdot \frac{1\,\text{g}}{1000\,\text{mg}} \cdot \frac{100\,\text{cg}}{1\,\text{g}}$$ Apply conversion factors.

$$= 512\,\cancel{\text{mg}} \cdot \frac{1\,\cancel{\text{g}}}{1000\,\cancel{\text{mg}}} \cdot \frac{100\,\text{cg}}{1\,\cancel{\text{g}}}$$ Cancel common units.

$$= \left(\frac{512 \cdot 1 \cdot 100}{1000 \cdot 1} \right) \text{cg}$$ Multiply fractions.

$$= 51.2\,\text{cg}$$ Simplify.

Hence, 512 milligrams equals 51.2 centigrams.

51. To change 541 milligrams to centigrams, we need to apply a chain of conversion factors.

$$541\,\text{mg} = 541\,\text{mg} \cdot \frac{1\,\text{g}}{1000\,\text{mg}} \cdot \frac{100\,\text{cg}}{1\,\text{g}}$$ Apply conversion factors.

$$= 541\,\cancel{\text{mg}} \cdot \frac{1\,\cancel{\text{g}}}{1000\,\cancel{\text{mg}}} \cdot \frac{100\,\text{cg}}{1\,\cancel{\text{g}}}$$ Cancel common units.

$$= \left(\frac{541 \cdot 1 \cdot 100}{1000 \cdot 1} \right) \text{cg}$$ Multiply fractions.

$$= 54.1\,\text{cg}$$ Simplify.

Hence, 541 milligrams equals 54.1 centigrams.

53. To change 70 grams to centigrams, apply the appropriate conversion factor.

$$70\,\text{g} = 70\,\text{g} \cdot \frac{100\,\text{cg}}{1\,\text{g}}$$ Apply conversion factor.

$$70\,\cancel{\text{g}} = 70\,\cancel{\text{g}} \cdot \frac{100\,\text{cg}}{1\,\cancel{\text{g}}}$$ Cancel common unit.

$$= \left(\frac{70 \cdot 100}{1} \right) \text{cg}$$ Multiply fractions.

$$= 7000\,\text{cg}$$ Simplify.

Hence, 70 grams equals 7,000 centigrams.

55. To change 53 centigrams to milligrams, we need to apply a chain of conversion factors.

$$53\,\text{cg} = 53\,\text{cg} \cdot \frac{1\,\text{g}}{100\,\text{cg}} \cdot \frac{1000\,\text{mg}}{1\,\text{g}} \qquad \text{Apply conversion factors.}$$

$$= 53\,\cancel{\text{cg}} \cdot \frac{1\,\cancel{\text{g}}}{100\,\cancel{\text{cg}}} \cdot \frac{1000\,\text{mg}}{1\,\cancel{\text{g}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{53 \cdot 1 \cdot 1000}{100 \cdot 1}\right)\,\text{mg} \qquad \text{Multiply fractions.}$$

$$= 530\,\text{mg} \qquad \text{Simplify.}$$

Hence, 53 centigrams equals 530 milligrams.

57. To change 83 kilograms to grams, apply the appropriate conversion factor.

$$83\,\text{kg} = 83\,\text{kg} \cdot \frac{1000\,\text{g}}{1\,\text{kg}} \qquad \text{Apply conversion factor.}$$

$$83\,\cancel{\text{kg}} = 83\,\cancel{\text{kg}} \cdot \frac{1000\,\text{g}}{1\,\cancel{\text{kg}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{83 \cdot 1000}{1}\right)\,\text{g} \qquad \text{Multiply fractions.}$$

$$= 83000\,\text{g} \qquad \text{Simplify.}$$

Hence, 83 kilograms equals 83,000 grams.

59. To change 8,196 grams to kilograms, apply the appropriate conversion factor.

$$8196\,\text{cm} = 8196\,\text{g} \cdot \frac{1\,\text{kg}}{1000\,\text{g}} \qquad \text{Apply conversion factor.}$$

$$= 8196\,\cancel{\text{g}} \cdot \frac{1\,\text{kg}}{1000\,\cancel{\text{g}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{8196 \cdot 1}{1000}\right)\,\text{kg} \qquad \text{Multiply fractions.}$$

$$= 8.196\,\text{kg} \qquad \text{Simplify.}$$

Hence, 8,196 grams equals 8.196 kilograms.

61. To change 564 centigrams to grams, apply the appropriate conversion factor.

$$564\,\text{cg} = 564\,\text{cg} \cdot \frac{1\,\text{g}}{100\,\text{cg}} \qquad\qquad \text{Apply conversion factor.}$$

$$= 564\,\cancel{\text{cg}} \cdot \frac{1\,\text{g}}{100\,\cancel{\text{cg}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \left(\frac{564 \cdot 1}{100}\right)\text{g} \qquad\qquad \text{Multiply fractions.}$$

$$= 5.64\,\text{g} \qquad\qquad \text{Simplify.}$$

Hence, 564 centigrams equals 5.64 grams.

63. To change 38 grams to centigrams, apply the appropriate conversion factor.

$$38\,\text{g} = 38\,\text{g} \cdot \frac{100\,\text{cg}}{1\,\text{g}} \qquad\qquad \text{Apply conversion factor.}$$

$$38\,\cancel{\text{g}} = 38\,\cancel{\text{g}} \cdot \frac{100\,\text{cg}}{1\,\cancel{\text{g}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \left(\frac{38 \cdot 100}{1}\right)\text{cg} \qquad\qquad \text{Multiply fractions.}$$

$$= 3800\,\text{cg} \qquad\qquad \text{Simplify.}$$

Hence, 38 grams equals 3,800 centigrams.

65. To change 77 centigrams to milligrams, we need to apply a chain of conversion factors.

$$77\,\text{cg} = 77\,\text{cg} \cdot \frac{1\,\text{g}}{100\,\text{cg}} \cdot \frac{1000\,\text{mg}}{1\,\text{g}} \qquad\qquad \text{Apply conversion factors.}$$

$$= 77\,\cancel{\text{cg}} \cdot \frac{1\,\cancel{\text{g}}}{100\,\cancel{\text{cg}}} \cdot \frac{1000\,\text{mg}}{1\,\cancel{\text{g}}} \qquad\qquad \text{Cancel common units.}$$

$$= \left(\frac{77 \cdot 1 \cdot 1000}{100 \cdot 1}\right)\text{mg} \qquad\qquad \text{Multiply fractions.}$$

$$= 770\,\text{mg} \qquad\qquad \text{Simplify.}$$

Hence, 77 centigrams equals 770 milligrams.

67. To change 5,337 grams to kilograms, apply the appropriate conversion factor.

$$5337 \, \text{cm} = 5337 \, \text{g} \cdot \frac{1 \, \text{kg}}{1000 \, \text{g}} \qquad \text{Apply conversion factor.}$$

$$= 5337 \, \cancel{\text{g}} \cdot \frac{1 \, \text{kg}}{1000 \, \cancel{\text{g}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{5337 \cdot 1}{1000} \right) \text{kg} \qquad \text{Multiply fractions.}$$

$$= 5.337 \, \text{kg} \qquad \text{Simplify.}$$

Hence, 5,337 grams equals 5.337 kilograms.

69. To change 15 kilograms to grams, apply the appropriate conversion factor.

$$15 \, \text{kg} = 15 \, \text{kg} \cdot \frac{1000 \, \text{g}}{1 \, \text{kg}} \qquad \text{Apply conversion factor.}$$

$$15 \, \cancel{\text{kg}} = 15 \, \cancel{\text{kg}} \cdot \frac{1000 \, \text{g}}{1 \, \cancel{\text{kg}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{15 \cdot 1000}{1} \right) \text{g} \qquad \text{Multiply fractions.}$$

$$= 15000 \, \text{g} \qquad \text{Simplify.}$$

Hence, 15 kilograms equals 15,000 grams.

71. To change 833 centigrams to grams, apply the appropriate conversion factor.

$$833 \, \text{cg} = 833 \, \text{cg} \cdot \frac{1 \, \text{g}}{100 \, \text{cg}} \qquad \text{Apply conversion factor.}$$

$$= 833 \, \cancel{\text{cg}} \cdot \frac{1 \, \text{g}}{100 \, \cancel{\text{cg}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{833 \cdot 1}{100} \right) \text{g} \qquad \text{Multiply fractions.}$$

$$= 8.33 \, \text{g} \qquad \text{Simplify.}$$

Hence, 833 centigrams equals 8.33 grams.

73. To change 619,560 centilitres to kilolitres, we apply a chain of conversion factors.

$$619560 \,\text{cL} = 619560 \,\text{cL} \cdot \frac{1 \,\text{L}}{100 \,\text{cL}} \cdot \frac{1 \,\text{kL}}{1000 \,\text{L}}$$ Apply conversion factors.

$$= 619560 \,\cancel{\text{cL}} \cdot \frac{1 \,\cancel{\text{L}}}{100 \,\cancel{\text{cL}}} \cdot \frac{1 \,\text{kL}}{1000 \,\cancel{\text{L}}}$$ Cancel common units.

$$= \left(\frac{619560 \cdot 1 \cdot 1}{100 \cdot 1000} \right) \,\text{kL}$$ Multiply fractions.

$$= 6.1956 \,\text{kL}$$ Simmplify.

Hence, 619,560 centilitres equals 6.1956 kilolitres.

75. To change 15.2 litres to millilitres, we apply the appropriate conversion factor.

$$15.2 \,\text{L} = 15.2 \,\text{L} \cdot \frac{1000 \,\text{mL}}{1 \,\text{L}}$$ Apply conversion factors.

$$= 15.2 \,\cancel{\text{L}} \cdot \frac{1000 \,\text{mL}}{1 \,\cancel{\text{L}}}$$ Cancel common unit.

$$= 15200 \,\text{mL}$$ Simplify.

Hence, 15.2 litres equals 15,200 millilitres.

77. To change 10,850 centilitres to litres, we apply the appropriate conversion factor.

$$10850 \,\text{cL} = 10850 \,\text{cL} \cdot \frac{1 \,\text{L}}{100 \,\text{cL}}$$ Apply conversion factors.

$$= 10850 \,\cancel{\text{cL}} \cdot \frac{1 \,\text{L}}{100 \,\cancel{\text{cL}}}$$ Cancel common unit.

$$= \left(\frac{10850 \cdot 1}{100} \right) \,\text{L}$$ Multiply fractions.

$$= 108.5 \,\text{L}$$ Simplify.

Hence, 10,850 centilitres equals 108.5 litres.

79. To change 10.7 litres to millilitres, we apply the appropriate conversion factor.

$$10.7 \,\text{L} = 10.7 \,\text{L} \cdot \frac{1000 \,\text{mL}}{1 \,\text{L}}$$ Apply conversion factors.

$$= 10.7 \,\cancel{\text{L}} \cdot \frac{1000 \,\text{mL}}{1 \,\cancel{\text{L}}}$$ Cancel common unit.

$$= 10700 \,\text{mL}$$ Simplify.

Hence, 10.7 litres equals 10,700 millilitres.

81. To change 15,665 millilitres to litres, we apply the appropriate conversion factor.

$$15665\,\text{mL} = 15665\,\text{mL} \cdot \frac{1\,\text{L}}{1000\,\text{mL}} \qquad \text{Apply conversion factors.}$$

$$= 15665\,\cancel{\text{mL}} \cdot \frac{1\,\text{L}}{1000\,\cancel{\text{mL}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{15665 \cdot 1}{1000} \right) \text{L} \qquad \text{Multiply fractions.}$$

$$= 15.665\,\text{L} \qquad \text{Simplify.}$$

Hence, 15,665 millilitres equals 15.665 litres.

83. To change 6.3 kilolitres to centilitres, we need a chain of conversion factors.

$$6.3\,\text{kL} = 6.3\,\text{kL} \cdot \frac{1000\,\text{L}}{1\,\text{kL}} \cdot \frac{100\,\text{cL}}{1\,\text{L}} \qquad \text{Apply conversion factors.}$$

$$= 6.3\,\cancel{\text{kL}} \cdot \frac{1000\,\cancel{\text{L}}}{1\,\cancel{\text{kL}}} \cdot \frac{100\,\text{cL}}{1\,\cancel{\text{L}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{6.3 \cdot 1000 \cdot 100}{1 \cdot 1} \right) \text{cL} \qquad \text{Multiply fractions.}$$

$$= 630000\,\text{cL} \qquad \text{Simplify.}$$

Hence, 6.3 kilolitres equals 630,000 centilitres.

85. To change 4.5 kilolitres to centilitres, we need a chain of conversion factors.

$$4.5\,\text{kL} = 4.5\,\text{kL} \cdot \frac{1000\,\text{L}}{1\,\text{kL}} \cdot \frac{100\,\text{cL}}{1\,\text{L}} \qquad \text{Apply conversion factors.}$$

$$= 4.5\,\cancel{\text{kL}} \cdot \frac{1000\,\cancel{\text{L}}}{1\,\cancel{\text{kL}}} \cdot \frac{100\,\text{cL}}{1\,\cancel{\text{L}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{4.5 \cdot 1000 \cdot 100}{1 \cdot 1} \right) \text{cL} \qquad \text{Multiply fractions.}$$

$$= 450000\,\text{cL} \qquad \text{Simplify.}$$

Hence, 4.5 kilolitres equals 450,000 centilitres.

87. To change 10.6 litres to centilitres, we apply the appropriate conversion factor.

$$10.6\,\text{L} = 10.6\,\text{L} \cdot \frac{100\,\text{cL}}{1\,\text{L}} \qquad \text{Apply conversion factors.}$$

$$= 10.6\,\cancel{\text{L}} \cdot \frac{100\,\text{cL}}{1\,\cancel{\text{L}}} \qquad \text{Cancel common unit.}$$

$$= 1060\,\text{cL} \qquad \text{Simplify.}$$

Hence, 10.6 litres equals 1,060 centilitres.

89. To change 14,383 centilitres to litres, we apply the appropriate conversion factor.

$$14383\,\text{cL} = 14383\,\text{cL} \cdot \frac{1\,\text{L}}{100\,\text{cL}} \qquad \text{Apply conversion factors.}$$

$$= 14383\,\cancel{\text{cL}} \cdot \frac{1\,\text{L}}{100\,\cancel{\text{cL}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{14383 \cdot 1}{100}\right)\,\text{L} \qquad \text{Multiply fractions.}$$

$$= 143.83\,\text{L} \qquad \text{Simplify.}$$

Hence, 14,383 centilitres equals 143.83 litres.

91. To change 9.9 litres to centilitres, we apply the appropriate conversion factor.

$$9.9\,\text{L} = 9.9\,\text{L} \cdot \frac{100\,\text{cL}}{1\,\text{L}} \qquad \text{Apply conversion factors.}$$

$$= 9.9\,\cancel{\text{L}} \cdot \frac{100\,\text{cL}}{1\,\cancel{\text{L}}} \qquad \text{Cancel common unit.}$$

$$= 990\,\text{cL} \qquad \text{Simplify.}$$

Hence, 9.9 litres equals 990 centilitres.

93. To change 407,331 centilitres to kilolitres, we apply a chain of conversion factors.

$$407331\,\text{cL} = 407331\,\text{cL} \cdot \frac{1\,\text{L}}{100\,\text{cL}} \cdot \frac{1\,\text{kL}}{1000\,\text{L}} \qquad \text{Apply conversion factors.}$$

$$= 407331\,\cancel{\text{cL}} \cdot \frac{1\,\cancel{\text{L}}}{100\,\cancel{\text{cL}}} \cdot \frac{1\,\text{kL}}{1000\,\cancel{\text{L}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{407331 \cdot 1 \cdot 1}{100 \cdot 1000}\right)\,\text{kL} \qquad \text{Multiply fractions.}$$

$$= 4.07331\,\text{kL} \qquad \text{Simmplify.}$$

Hence, 407,331 centilitres equals 4.07331 kilolitres.

95. To change 14,968 millilitres to litres, we apply the appropriate conversion factor.

$$14968\,\text{mL} = 14968\,\text{mL} \cdot \frac{1\,\text{L}}{1000\,\text{mL}} \qquad \text{Apply conversion factors.}$$

$$= 14968\,\cancel{\text{mL}} \cdot \frac{1\,\text{L}}{1000\,\cancel{\text{mL}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{14968 \cdot 1}{1000}\right)\,\text{L} \qquad \text{Multiply fractions.}$$

$$= 14.968\,\text{L} \qquad \text{Simplify.}$$

Hence, 14,968 millilitres equals 14.968 litres.

6.5   American Units to Metric Units and Vice-Versa

1. Multiply by the given conversion ratio.

$$68\,\text{in} = 68\,\text{in} \cdot \frac{2.54\,\text{cm}}{1\,\text{in}} \qquad \text{Apply conversion factor.}$$

$$= 68\,\cancel{\text{in}} \cdot \frac{2.54\,\text{cm}}{1\,\cancel{\text{in}}} \qquad \text{Cancel common unit.}$$

$$= 172.72\,\text{cm} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a centimeter, 68 inches is approximately 172.7 centimeters.

3. Multiply by the given conversion ratio.

$$44\,\text{cm} = 44\,\text{cm} \cdot \frac{1\,\text{in}}{2.54\,\text{cm}} \qquad \text{Apply conversion factor.}$$

$$= 44\,\cancel{\text{cm}} \cdot \frac{1\,\text{in}}{2.54\,\cancel{\text{cm}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{44}{2.54}\right)\,\text{in} \qquad \text{Multiply.}$$

$$= 17.3228346456693\,\text{in} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a inch, 44 centimeters is approximately 17.3 inches.

5. Multiply by the given conversion ratio.

$$
\begin{aligned}
79\,\text{mi} &= 79\,\text{mi} \cdot \frac{1.6093\,\text{km}}{1\,\text{mi}} && \text{Apply conversion factor.}\\
&= 79\,\cancel{\text{mi}} \cdot \frac{1.6093\,\text{km}}{1\,\cancel{\text{mi}}} && \text{Cancel common unit.}\\
&= 127.1347\,\text{km} && \text{Multiply.}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a kilometer, 79 miles is approximately 127.1 kilometers.

7. This will require a chain of conversion factors.

$$
\begin{aligned}
1489\,\text{cm} &= 1489\,\text{cm} \cdot \frac{1\,\text{m}}{100\,\text{cm}} \cdot \frac{1\,\text{yd}}{0.9144\,\text{m}} && \text{Apply conversion factors.}\\
&= 1489\,\cancel{\text{cm}} \cdot \frac{1\,\cancel{\text{m}}}{100\,\cancel{\text{cm}}} \cdot \frac{1\,\text{yd}}{0.9144\,\cancel{\text{m}}} && \text{Cancel common units.}\\
&= \left(\frac{1489}{91.44}\right)\,\text{yd} && \text{Multiply.}\\
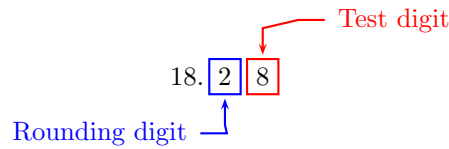&= 16.2839020122485\,\text{yd} && \text{Divide.}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.
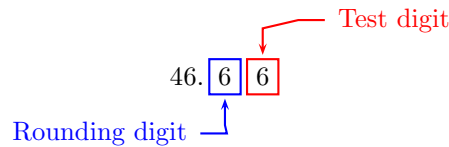
Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a yard, 1489 centimeters is approximately 16.3 yards.

9. This will require a chain of conversion factors.

$$
\begin{aligned}
28\,\mathrm{yd} &= 28\,\mathrm{yd} \cdot \frac{0.9144\,\mathrm{m}}{1\,\mathrm{yd}} \cdot \frac{100\,\mathrm{cm}}{1\,\mathrm{m}} && \text{Apply conversion factors.} \\
&= 28\,\cancel{\mathrm{yd}} \cdot \frac{0.9144\,\cancel{\mathrm{m}}}{1\,\cancel{\mathrm{yd}}} \cdot \frac{100\,\mathrm{cm}}{1\,\cancel{\mathrm{m}}} && \text{Cancel common units.} \\
&= 2560.32\,\mathrm{cm} && \text{Multiply.}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a centimeter, 28 yards is approximately 2560.3 centimeters.

11. This will require a chain of conversion factors.

$$
\begin{aligned}
8.6\,\mathrm{m} &= 8.6\,\mathrm{m} \cdot \frac{3.2808\,\mathrm{ft}}{1\,\mathrm{m}} \cdot \frac{12\,\mathrm{in}}{1\,\mathrm{ft}} && \text{Apply conversion factors.} \\
&= 8.6\,\cancel{\mathrm{m}} \cdot \frac{3.2808\,\cancel{\mathrm{ft}}}{1\,\cancel{\mathrm{m}}} \cdot \frac{12\,\mathrm{in}}{1\,\cancel{\mathrm{ft}}} && \text{Cancel common units.} \\
&= 338.57856\,\mathrm{in} && \text{Multiply.}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a inch, 8.6 meters is approximately 338.6 inches.

13. Multiply by the given conversion ratio.

$$60\,\text{in} = 60\,\text{in} \cdot \frac{2.54\,\text{cm}}{1\,\text{in}} \qquad \text{Apply conversion factor.}$$

$$= 60\,\cancel{\text{in}} \cdot \frac{2.54\,\text{cm}}{1\,\cancel{\text{in}}} \qquad \text{Cancel common unit.}$$

$$= 152.4\,\text{cm} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a centimeter, 60 inches is approximately 152.4 centimeters.

15. This will require a chain of conversion factors.

$$208\,\text{in} = 208\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \cdot \frac{1\,\text{m}}{3.2808\,\text{ft}} \qquad \text{Apply conversion factors.}$$

$$= 208\,\cancel{\text{in}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\,\text{m}}{3.2808\,\cancel{\text{ft}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{208}{12 \cdot 3.2808} \right)\,\text{m} \qquad \text{Multiply.}$$

$$= 5.28326424449321\,\text{m} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a meter, 208 inches is approximately 5.3 meters.

17. Multiply by the given conversion ratio.

$$20\,\text{yd} = 20\,\text{yd} \cdot \frac{1\,\text{m}}{1.0936\,\text{yd}} \qquad\qquad \text{Apply conversion factor.}$$

$$= 20\,\cancel{\text{yd}} \cdot \frac{1\,\text{m}}{1.0936\,\cancel{\text{yd}}} \qquad\qquad \text{Cancel common unit.}$$

$$= \left(\frac{20}{1.0936}\right)\,\text{m} \qquad\qquad \text{Multiply.}$$

$$= 18.2882223847842\,\text{m} \qquad\qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a meter, 20 yards is approximately 18.3 meters.

19. Multiply by the given conversion ratio.

$$29\,\text{mi} = 29\,\text{mi} \cdot \frac{1.6093\,\text{km}}{1\,\text{mi}} \qquad\qquad \text{Apply conversion factor.}$$

$$= 29\,\cancel{\text{mi}} \cdot \frac{1.6093\,\text{km}}{1\,\cancel{\text{mi}}} \qquad\qquad \text{Cancel common unit.}$$

$$= 46.6697\,\text{km} \qquad\qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a kilometer, 29 miles is approximately 46.7 kilometers.

21. Multiply by the given conversion ratio.

$$8.2\,\text{m} = 8.2\,\text{m} \cdot \frac{1.0936\,\text{yd}}{1\,\text{m}} \qquad \text{Apply conversion factor.}$$

$$= 8.2\,\cancel{\text{m}} \cdot \frac{1.0936\,\text{yd}}{1\,\cancel{\text{m}}} \qquad \text{Cancel common unit.}$$

$$= 8.96752\,\text{yd} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Test digit

$$8.\boxed{9}\,\boxed{6}$$

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a yard, 8.2 meters is approximately 9.0 yards.

23. Multiply by the given conversion ratio.

$$4.9\,\text{km} = 4.9\,\text{km} \cdot \frac{1\,\text{mi}}{1.6093\,\text{km}} \qquad \text{Apply conversion factor.}$$

$$= 4.9\,\cancel{\text{km}} \cdot \frac{1\,\text{mi}}{1.6093\,\cancel{\text{km}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{4.9}{1.6093} \right)\,\text{mi} \qquad \text{Divide.}$$

$$= 3.04480208786429\,\text{mi}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Test digit

$$3.\boxed{0}\,\boxed{4}$$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a mile, 4.9 kilometers is approximately 3.0 miles.

25. Multiply by the given conversion ratio.

$$25\,\text{yd} = 25\,\text{yd} \cdot \frac{1\,\text{m}}{1.0936\,\text{yd}} \qquad \textcolor{red}{\text{Apply conversion factor.}}$$

$$= 25\,\cancel{\text{yd}} \cdot \frac{1\,\text{m}}{1.0936\,\cancel{\text{yd}}} \qquad \textcolor{red}{\text{Cancel common unit.}}$$

$$= \left(\frac{25}{1.0936}\right)\text{m} \qquad \textcolor{red}{\text{Multiply.}}$$

$$= 22.8602779809802\,\text{m} \qquad \textcolor{red}{\text{Divide.}}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a meter, 25 yards is approximately 22.9 meters.

27. Multiply by the given conversion ratio.

$$47\,\text{cm} = 47\,\text{cm} \cdot \frac{1\,\text{in}}{2.54\,\text{cm}} \qquad \textcolor{red}{\text{Apply conversion factor.}}$$

$$= 47\,\cancel{\text{cm}} \cdot \frac{1\,\text{in}}{2.54\,\cancel{\text{cm}}} \qquad \textcolor{red}{\text{Cancel common unit.}}$$

$$= \left(\frac{47}{2.54}\right)\text{in} \qquad \textcolor{red}{\text{Multiply.}}$$

$$= 18.503937007874\,\text{in} \qquad \textcolor{red}{\text{Divide.}}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a inch, 47 centimeters is approximately 18.5 inches.

29. Multiply by the given conversion ratio.

$$
\begin{aligned}
8.3 \,\text{km} &= 8.3 \,\text{km} \cdot \frac{1 \,\text{mi}}{1.6093 \,\text{km}} &&\text{\color{red}{Apply conversion factor.}}\\
&= 8.3 \,\cancel{\text{km}} \cdot \frac{1 \,\text{mi}}{1.6093 \,\cancel{\text{km}}} &&\text{\color{red}{Cancel common unit.}}\\
&= \left(\frac{8.3}{1.6093}\right) \text{mi} &&\text{\color{red}{Divide.}}\\
&= 5.15752190393339 \,\text{mi}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a mile, 8.3 kilometers is approximately 5.2 miles.

31. This will require a chain of conversion factors.

$$
\begin{aligned}
41 \,\text{yd} &= 41 \,\text{yd} \cdot \frac{0.9144 \,\text{m}}{1 \,\text{yd}} \cdot \frac{100 \,\text{cm}}{1 \,\text{m}} &&\text{\color{red}{Apply conversion factors.}}\\
&= 41 \,\cancel{\text{yd}} \cdot \frac{0.9144 \,\cancel{\text{m}}}{1 \,\cancel{\text{yd}}} \cdot \frac{100 \,\text{cm}}{1 \,\cancel{\text{m}}} &&\text{\color{red}{Cancel common units.}}\\
&= 3749.04 \,\text{cm} &&\text{\color{red}{Multiply.}}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a centimeter, 41 yards is approximately 3749.0 centimeters.

33. This will require a chain of conversion factors.

$$3.7\,\text{m} = 3.7\,\text{m} \cdot \frac{3.2808\,\text{ft}}{1\,\text{m}} \cdot \frac{12\,\text{in}}{1\,\text{ft}} \qquad \text{Apply conversion factors.}$$

$$= 3.7\,\cancel{\text{m}} \cdot \frac{3.2808\,\cancel{\text{ft}}}{1\,\cancel{\text{m}}} \cdot \frac{12\,\text{in}}{1\,\cancel{\text{ft}}} \qquad \text{Cancel common units.}$$

$$= 145.66752\,\text{in} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a inch, 3.7 meters is approximately 145.7 inches.

35. This will require a chain of conversion factors.

$$1323\,\text{cm} = 1323\,\text{cm} \cdot \frac{1\,\text{m}}{100\,\text{cm}} \cdot \frac{1\,\text{yd}}{0.9144\,\text{m}} \qquad \text{Apply conversion factors.}$$

$$= 1323\,\cancel{\text{cm}} \cdot \frac{1\,\cancel{\text{m}}}{100\,\cancel{\text{cm}}} \cdot \frac{1\,\text{yd}}{0.9144\,\cancel{\text{m}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{1323}{91.44}\right)\,\text{yd} \qquad \text{Multiply.}$$

$$= 14.4685039370079\,\text{yd} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a yard, 1323 centimeters is approximately 14.5 yards.

37. Multiply by the given conversion ratio.

$$8.4\,\text{m} = 8.4\,\text{m} \cdot \frac{1.0936\,\text{yd}}{1\,\text{m}} \qquad \text{Apply conversion factor.}$$

$$= 8.4\,\cancel{\text{m}} \cdot \frac{1.0936\,\text{yd}}{1\,\cancel{\text{m}}} \qquad \text{Cancel common unit.}$$

$$= 9.18624\,\text{yd} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a yard, 8.4 meters is approximately 9.2 yards.

39. This will require a chain of conversion factors.

$$289\,\text{in} = 289\,\text{in} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \cdot \frac{1\,\text{m}}{3.2808\,\text{ft}} \qquad \text{Apply conversion factors.}$$

$$= 289\,\cancel{\text{in}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\,\text{m}}{3.2808\,\cancel{\text{ft}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{289}{12 \cdot 3.2808}\right)\,\text{m} \qquad \text{Multiply.}$$

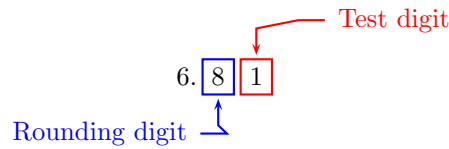$$= 7.34068926278143\,\text{m} \qquad \text{Divide.}$$

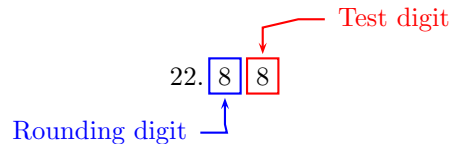Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a meter, 289 inches is approximately 7.3 meters.

41. Multiply by the appropriate conversion factor.

$$15.8\,\text{kg} = 15.8\,\text{kg} \cdot \frac{2.2\,\text{lb}}{1\,\text{kg}} \qquad \text{Apply conversion factor.}$$

$$= 15.8\,\cancel{\text{kg}} \cdot \frac{2.2\,\text{lb}}{1\,\cancel{\text{kg}}} \qquad \text{Cancel common unit.}$$

$$= 34.76\,\text{cm} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

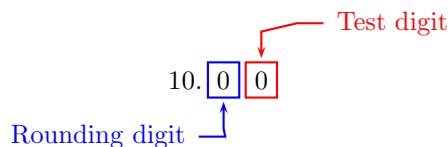$$34.\boxed{7}\,\boxed{6}$$

Test digit

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a pound, 15.8 kilograms is approximately 34.8 pounds.

43. Multiply by the appropriate conversion factor.

$$35\,\text{oz} = 35\,\text{oz} \cdot \frac{28.35\,\text{g}}{1\,\text{oz}} \qquad \text{Apply conversion factor.}$$

$$= 35\,\cancel{\text{oz}} \cdot \frac{28.35\,\text{g}}{1\,\cancel{\text{oz}}} \qquad \text{Cancel common unit.}$$

$$= 992.25\,\text{g} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

$$992.\boxed{2}\,\boxed{5}$$

Test digit

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a gram, 35 ounces is approximately 992.3 grams.

45. Multiply by a chain of conversion factors.

$$2.48\,\text{kg} = 2.48\,\text{kg} \cdot \frac{2.2\,\text{lb}}{1\,\text{kg}} \cdot \frac{16\,\text{oz}}{1\,\text{lb}} \qquad \text{Apply conversion factors.}$$

$$= 2.48\,\cancel{\text{kg}} \cdot \frac{2.2\,\cancel{\text{lb}}}{1\,\cancel{\text{kg}}} \cdot \frac{16\,\text{oz}}{1\,\cancel{\text{lb}}} \qquad \text{Cancel common units.}$$

$$= 87.296\,\text{oz} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of an ounce, 2.48 kilograms is approximately 87.3 ounces.

47. Multiply by a chain of conversion factors.

$$2.35\,\text{kg} = 2.35\,\text{kg} \cdot \frac{2.2\,\text{lb}}{1\,\text{kg}} \cdot \frac{16\,\text{oz}}{1\,\text{lb}} \qquad \text{Apply conversion factors.}$$

$$= 2.35\,\cancel{\text{kg}} \cdot \frac{2.2\,\cancel{\text{lb}}}{1\,\cancel{\text{kg}}} \cdot \frac{16\,\text{oz}}{1\,\cancel{\text{lb}}} \qquad \text{Cancel common units.}$$

$$= 82.72\,\text{oz} \qquad \text{Multiply.}$$

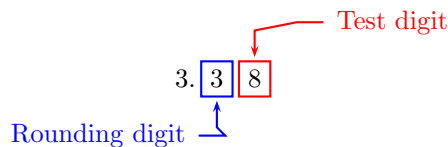Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an ounce, 2.35 kilograms is approximately 82.7 ounces.

49. Multiply by the appropriate conversion factor.

$$15\,\text{lb} = 15\,\text{lb} \cdot \frac{1\,\text{kg}}{2.2\,\text{lb}} \qquad \text{Apply conversion factor.}$$

$$= 15\,\cancel{\text{lb}} \cdot \frac{1\,\text{kg}}{2.2\,\cancel{\text{lb}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{15}{2.2}\right)\,\text{kg} \qquad \text{Multiply.}$$

$$= 6.81818181818182\,\text{kg} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a kilogram, 15 pounds is approximately 6.8 kilograms.

51. Multiply by the appropriate conversion factor.

$$10.4\,\text{kg} = 10.4\,\text{kg} \cdot \frac{2.2\,\text{lb}}{1\,\text{kg}} \qquad \text{Apply conversion factor.}$$

$$= 10.4\,\cancel{\text{kg}} \cdot \frac{2.2\,\text{lb}}{1\,\cancel{\text{kg}}} \qquad \text{Cancel common unit.}$$

$$= 22.88\,\text{cm} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a pound, 10.4 kilograms is approximately 22.9 pounds.

53. Multiply by the appropriate conversion factor.

$$352 \, \text{oz} = 352 \, \text{oz} \cdot \frac{1 \, \text{lb}}{16 \, \text{oz}} \cdot \frac{1 \, \text{kg}}{2.2 \, \text{lb}} \qquad \text{Apply conversion factor.}$$

$$= 352 \, \cancel{\text{oz}} \cdot \frac{1 \, \cancel{\text{lb}}}{16 \, \cancel{\text{oz}}} \cdot \frac{1 \, \text{kg}}{2.2 \, \cancel{\text{lb}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{352}{35.2} \right) \, \text{kg} \qquad \text{Multiply.}$$

$$= 10 \, \text{kg} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an kilogram, 352 ounces is approximately 10.0 kilograms.

55. Multiply by the appropriate conversion factor.

$$96 \, \text{g} = 96 \, \text{g} \cdot \frac{1 \, \text{oz}}{28.35 \, \text{g}} \qquad \text{Apply conversion factor.}$$

$$= 96 \, \cancel{\text{g}} \cdot \frac{1 \, \text{oz}}{28.35 \, \cancel{\text{g}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{96}{28.35} \right) \, \text{oz} \qquad \text{Multiply.}$$

$$= 3.38624338624339 \, \text{oz} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of an ounce, 96 grams is approximately 3.4 ounces.

57. Multiply by the appropriate conversion factor.

$$14\,\text{oz} = 14\,\text{oz} \cdot \frac{28.35\,\text{g}}{1\,\text{oz}} \qquad \text{Apply conversion factor.}$$

$$= 14\,\cancel{\text{oz}} \cdot \frac{28.35\,\text{g}}{1\,\cancel{\text{oz}}} \qquad \text{Cancel common unit.}$$

$$= 396.9\,\text{g} \qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a gram, 14 ounces is approximately 396.9 grams.

59. Multiply by the appropriate conversion factor.

$$54\,\text{lb} = 54\,\text{lb} \cdot \frac{1\,\text{kg}}{2.2\,\text{lb}} \qquad \text{Apply conversion factor.}$$

$$= 54\,\cancel{\text{lb}} \cdot \frac{1\,\text{kg}}{2.2\,\cancel{\text{lb}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{54}{2.2}\right)\,\text{kg} \qquad \text{Multiply.}$$

$$= 24.545454545454\,\text{kg} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a kilogram, 54 pounds is approximately 24.5 kilograms.

61. Multiply by the appropriate conversion factor.

$$92\,\text{g} = 92\,\text{g} \cdot \frac{1\,\text{oz}}{28.35\,\text{g}} \qquad \text{Apply conversion factor.}$$

$$= 92\,\cancel{\text{g}} \cdot \frac{1\,\text{oz}}{28.35\,\cancel{\text{g}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{92}{28.35}\right)\text{oz} \qquad \text{Multiply.}$$

$$= 3.24514991181658\,\text{oz} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an ounce, 92 grams is approximately 3.2 ounces.

63. Multiply by the appropriate conversion factor.

$$388\,\text{oz} = 388\,\text{oz} \cdot \frac{1\,\text{lb}}{16\,\text{oz}} \cdot \frac{1\,\text{kg}}{2.2\,\text{lb}} \qquad \text{Apply conversion factor.}$$

$$= 388\,\cancel{\text{oz}} \cdot \frac{1\,\cancel{\text{lb}}}{16\,\cancel{\text{oz}}} \cdot \frac{1\,\text{kg}}{2.2\,\cancel{\text{lb}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{388}{35.2}\right)\text{kg} \qquad \text{Multiply.}$$

$$= 11.0227272727273\,\text{kg} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an kilogram, 388 ounces is approximately 11.0 kilograms.

65. Multiply by the appropriate conversion factor.

$$
\begin{aligned}
55.1\,\text{L} &= 55.1\,\text{L} \cdot \frac{1\,\text{qt}}{0.946\,\text{L}} && \textcolor{red}{\text{Apply conversion factor.}} \\
&= 55.1\,\cancel{\text{L}} \cdot \frac{1\,\text{qt}}{0.946\,\cancel{\text{L}}} && \textcolor{red}{\text{Cancel common unit.}} \\
&= \left(\frac{55.1}{0.946}\right)\text{qt} && \textcolor{red}{\text{Multiply.}} \\
&= 58.2452431289641\,\text{qt} && \textcolor{red}{\text{Divide.}}
\end{aligned}
$$

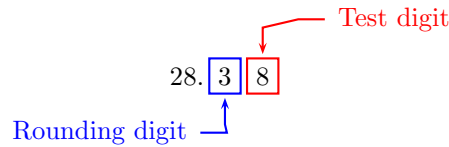Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an quart, 55.1 litres is approximately 58.2 quarts.

67. This requires a chain of conversion factors.

$$
\begin{aligned}
72073\,\text{fl oz} &= 72073\,\text{fl oz} \cdot \frac{1\,\text{L}}{33.8\,\text{fl oz}} \cdot \frac{1\,\text{kL}}{1000\,\text{L}} && \textcolor{red}{\text{Apply conversion factors.}} \\
&= 72073\,\cancel{\text{fl oz}} \cdot \frac{1\,\cancel{\text{L}}}{33.8\,\cancel{\text{fl oz}}} \cdot \frac{1\,\text{kL}}{1000\,\cancel{\text{L}}} && \textcolor{red}{\text{Cancel common unit.}} \\
&= \left(\frac{72073}{33800}\right)\text{kL} && \textcolor{red}{\text{Multiply.}} \\
&= 2.13233727810651\,\text{kL} && \textcolor{red}{\text{Divide.}}
\end{aligned}
$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

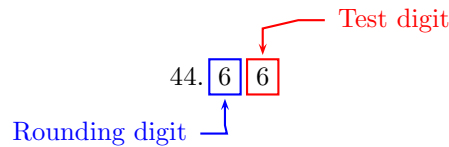Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an kilolitre, 72073 fluiud ounces is approximately 2.1 kilolitres.

69. This requires a chain of conversion factors.

$$2.5\,\text{kL} = 2.5\,\text{kL} \cdot \frac{1000\,\text{L}}{1\,\text{kL}} \cdot \frac{33.8\,\text{fl oz}}{1\,\text{L}} \qquad \text{\color{red}Apply conversion factors.}$$

$$= 2.5\,\cancel{\text{kL}} \cdot \frac{1000\,\cancel{\text{L}}}{1\,\cancel{\text{kL}}} \cdot \frac{33.8\,\text{fl oz}}{1\,\cancel{\text{L}}} \qquad \text{\color{red}Cancel common units.}$$

$$= 84500\,\text{fl oz} \qquad \text{\color{red}Multiply.}$$

Hence, 2.5 kilolitres equals 84500 fluid ounces.

71. Multiply by the appropriate conversion factor.

$$24\,\text{qt} = 24\,\text{qt} \cdot \frac{0.946\,\text{L}}{1\,\text{qt}} \qquad \text{\color{red}Apply conversion factor.}$$

$$= 24\,\cancel{\text{qt}} \cdot \frac{0.946\,\text{L}}{1\,\cancel{\text{qt}}} \qquad \text{\color{red}Cancel common unit.}$$

$$= 22.704\,\text{L} \qquad \text{\color{red}Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a litre, 24 quarts is approximately 22.7 liters.

73. Multiply by the appropriate conversion factor.

$$30\,\text{qt} = 30\,\text{qt} \cdot \frac{0.946\,\text{L}}{1\,\text{qt}} \qquad \text{\color{red}Apply conversion factor.}$$

$$= 30\,\cancel{\text{qt}} \cdot \frac{0.946\,\text{L}}{1\,\cancel{\text{qt}}} \qquad \text{\color{red}Cancel common unit.}$$

$$= 28.38\,\text{L} \qquad \text{\color{red}Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.
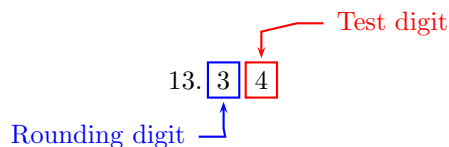
Test digit

28. 3 8

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a litre, 30 quarts is approximately 28.4 liters.

75. Multiply by the appropriate conversion factor.

$$11.8\,\text{gal} = 11.8\,\text{gal} \cdot \frac{3.785\,\text{L}}{1\,\text{gal}} \qquad \text{\color{red}Apply conversion factor.}$$

$$= 11.8\,\cancel{\text{gal}} \cdot \frac{3.785\,\text{L}}{1\,\cancel{\text{gal}}} \qquad \text{\color{red}Cancel common unit.}$$

$$= 44.663\,\text{L} \qquad \text{\color{red}Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Test digit

44. 6 6

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a litre, 11.8 gallons is approximately 44.7 liters.

77. Multiply by the appropriate conversion factor.

$$50.5\,\text{L} = 50.5\,\text{L} \cdot \frac{1\,\text{gal}}{3.785\,\text{L}} \qquad \text{\color{red}Apply conversion factor.}$$

$$= 50.5\,\cancel{\text{L}} \cdot \frac{1\,\text{gal}}{3.785\,\cancel{\text{L}}} \qquad \text{\color{red}Cancel common unit.}$$

$$= \left(\frac{50.5}{3.785}\right)\,\text{gal} \qquad \text{\color{red}Multiply.}$$

$$= 13.3421400264201\,\text{gal} \qquad \text{\color{red}Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Test digit

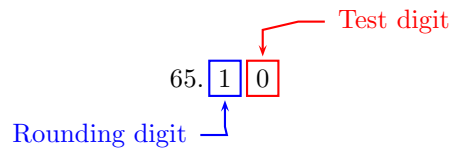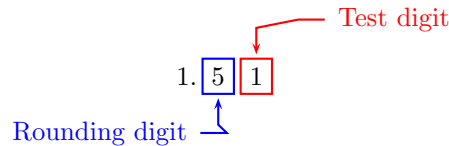13. $\boxed{3}\ \boxed{4}$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an gallon, 50.5 litres is approximately 13.3 gallons.

79. This requires a chain of conversion factors.

$$8.3\,\text{kL} = 8.3\,\text{kL} \cdot \frac{1000\,\text{L}}{1\,\text{kL}} \cdot \frac{33.8\,\text{fl oz}}{1\,\text{L}} \qquad \text{Apply conversion factors.}$$

$$= 8.3\,\cancel{\text{kL}} \cdot \frac{1000\,\cancel{\text{L}}}{1\,\cancel{\text{kL}}} \cdot \frac{33.8\,\text{fl oz}}{1\,\cancel{\text{L}}} \qquad \text{Cancel common units.}$$

$$= 280540\,\text{fl oz} \qquad \text{Multiply.}$$

Hence, 8.3 kilolitres equals 280540 fluid ounces.

81. Multiply by the appropriate conversion factor.

$$42.4\,\text{L} = 42.4\,\text{L} \cdot \frac{1\,\text{qt}}{0.946\,\text{L}} \qquad \text{Apply conversion factor.}$$

$$= 42.4\,\cancel{\text{L}} \cdot \frac{1\,\text{qt}}{0.946\,\cancel{\text{L}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{42.4}{0.946}\right)\,\text{qt} \qquad \text{Multiply.}$$

$$= 44.8202959830867\,\text{qt} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Test digit

44. $\boxed{8}\ \boxed{2}$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an quart, 42.4 litres is approximately 44.8 quarts.

83. Multiply by the appropriate conversion factor.

$$17.2\,\text{gal} = 17.2\,\text{gal} \cdot \frac{3.785\,\text{L}}{1\,\text{gal}} \qquad \text{\color{red}Apply conversion factor.}$$

$$= 17.2\,\cancel{\text{gal}} \cdot \frac{3.785\,\text{L}}{1\,\cancel{\text{gal}}} \qquad \text{\color{red}Cancel common unit.}$$

$$= 65.102\,\text{L} \qquad \text{\color{red}Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a litre, 17.2 gallons is approximately 65.1 liters.

85. This requires a chain of conversion factors.

$$51274\,\text{fl oz} = 51274\,\text{fl oz} \cdot \frac{1\,\text{L}}{33.8\,\text{fl oz}} \cdot \frac{1\,\text{kL}}{1000\,\text{L}} \qquad \text{\color{red}Apply conversion factors.}$$

$$= 51274\,\cancel{\text{fl oz}} \cdot \frac{1\,\cancel{\text{L}}}{33.8\,\cancel{\text{fl oz}}} \cdot \frac{1\,\text{kL}}{1000\,\cancel{\text{L}}} \qquad \text{\color{red}Cancel common unit.}$$

$$= \left(\frac{51274}{33800}\right)\,\text{kL} \qquad \text{\color{red}Multiply.}$$

$$= 1.51698224852071\,\text{kL} \qquad \text{\color{red}Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of an kilolitre, 51274 fluiud ounces is approximately 1.5 kilolitres.

87. Multiply by the appropriate conversion factor.

$$55.6\,\text{L} = 55.6\,\text{L} \cdot \frac{1\,\text{gal}}{3.785\,\text{L}} \qquad \text{Apply conversion factor.}$$

$$= 55.6\,\cancel{\text{L}} \cdot \frac{1\,\text{gal}}{3.785\,\cancel{\text{L}}} \qquad \text{Cancel common unit.}$$

$$= \left(\frac{55.6}{3.785}\right)\text{gal} \qquad \text{Multiply.}$$

$$= 14.6895640686922\,\text{gal} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of an gallon, 55.6 litres is approximately 14.7 gallons.

89. There are 0.6214 miles in a kilometer.

$$60\,\frac{\text{mi}}{\text{hr}} \approx 60\,\frac{\text{mi}}{\text{hr}} \cdot \frac{1\,\text{km}}{0.6214\,\text{mi}} \qquad \text{Apply conversion factor.}$$

$$\approx 60\,\frac{\cancel{\text{mi}}}{\text{hr}} \cdot \frac{1\,\text{km}}{0.6214\,\cancel{\text{mi}}} \qquad \text{Cancel common units.}$$

$$\approx \frac{60 \cdot 1}{0.6214}\,\frac{\text{km}}{\text{hr}} \qquad \text{Multiply fractions.}$$

$$\approx 96.5\,\frac{\text{km}}{\text{hr}} \qquad \text{Multiply and divide.}$$

To round to the nearest kilometer per hour, identify the rounding and test digits.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest kilometer per hour, the speed is approximately 97 kilometers per hour.

91. There are 0.6214 miles in a kilometer.

$$77 \, \frac{\text{mi}}{\text{hr}} \approx 77 \, \frac{\text{mi}}{\text{hr}} \cdot \frac{1 \, \text{km}}{0.6214 \, \text{mi}} \qquad \text{Apply conversion factor.}$$

$$\approx 77 \, \frac{\cancel{\text{mi}}}{\text{hr}} \cdot \frac{1 \, \text{km}}{0.6214 \, \cancel{\text{mi}}} \qquad \text{Cancel common units.}$$

$$\approx \frac{77 \cdot 1}{0.6214} \, \frac{\text{km}}{\text{hr}} \qquad \text{Multiply fractions.}$$

$$\approx 123.9 \, \frac{\text{km}}{\text{hr}} \qquad \text{Multiply and divide.}$$

To round to the nearest kilometer per hour, identify the rounding and test digits.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest kilometer per hour, the speed is approximately 124 kilometers per hour.

93. There are 0.6214 miles in a kilometer.

$$42 \, \frac{\text{km}}{\text{h}} \approx 42 \, \frac{\text{km}}{\text{h}} \cdot \frac{0.6214 \, \text{mi}}{1 \, \text{km}} \qquad \text{Apply conversion factor.}$$

$$\approx 42 \, \frac{\cancel{\text{km}}}{\text{h}} \cdot \frac{0.6214 \, \text{mi}}{1 \, \cancel{\text{km}}} \qquad \text{Cancel common units.}$$

$$\approx \frac{42 \cdot 0.6214}{1} \, \frac{\text{mi}}{\text{h}} \qquad \text{Multiply fractions.}$$

$$\approx 26.0988 \, \frac{\text{mi}}{\text{h}}$$

To round to the nearest mile per hour, identify the rounding and test digits.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest mile per hour, the speed is approximately 26 miles per hour.

95. There are 0.6214 miles in a kilometer.

$$62\,\frac{\text{km}}{\text{h}} \approx 62\,\frac{\text{km}}{\text{h}} \cdot \frac{0.6214\,\text{mi}}{1\,\text{km}} \qquad \text{Apply conversion factor.}$$

$$\approx 62\,\frac{\cancel{\text{km}}}{\text{h}} \cdot \frac{0.6214\,\text{mi}}{1\,\cancel{\text{km}}} \qquad \text{Cancel common units.}$$

$$\approx \frac{62 \cdot 0.6214}{1}\,\frac{\text{mi}}{\text{h}} \qquad \text{Multiply fractions.}$$

$$\approx 38.5268\,\frac{\text{mi}}{\text{h}}$$

To round to the nearest mile per hour, identify the rounding and test digits.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest mile per hour, the speed is approximately 39 miles per hour.

97. Note that 1 meter is approximately 3.2808 feet.

$$2717\,\text{feet} = 2717\,\text{feet} \cdot \frac{1\,\text{m}}{3.2808\,\text{ft}} \qquad \text{Apply conversion factor.}$$

$$= 2717\,\cancel{\text{feet}} \cdot \frac{1\,\text{m}}{3.2808\,\cancel{\text{ft}}} \qquad \text{Cancel common units.}$$

$$= \left(\frac{2717}{3.2808}\right)\,\text{m} \qquad \text{Multiply.}$$

$$\approx 828.15\,\text{m} \qquad \text{Divide.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest tenth of a meter, $2,717$ feet is approximately 828.2 meters.

99. Note that 1 meter is approximately 3.2808 feet.

$$4\,\text{m} = 4\,\text{m} \cdot \frac{3.2808\,\text{ft}}{1\,\text{m}} \qquad\qquad \text{Apply conversion factor.}$$

$$= 4\,\cancel{\text{m}} \cdot \frac{3.2808\,\text{ft}}{1\,\cancel{\text{m}}} \qquad\qquad \text{Cancel common units.}$$

$$= 13.12\,\text{ft} \qquad\qquad\qquad\qquad \text{Multiply.}$$

Locate the rounding digit in the tenths place and the test digit in the hundredths place.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Hence, to the nearest tenth of a foot, 4 meters is approximately 13.1 feet.

101. There are 0.6214 miles in a kilometer.

$$28\,\frac{\text{mi}}{\text{hr}} \approx 28\,\frac{\text{mi}}{\text{hr}} \cdot \frac{1\,\text{km}}{0.6214\,\text{mi}} \qquad\qquad \text{Apply conversion factor.}$$

$$\approx 28\,\frac{\cancel{\text{mi}}}{\text{hr}} \cdot \frac{1\,\text{km}}{0.6214\,\cancel{\text{mi}}} \qquad\qquad \text{Cancel common units.}$$

$$\approx \frac{28 \cdot 1}{0.6214}\,\frac{\text{km}}{\text{hr}} \qquad\qquad\qquad \text{Multiply fractions.}$$

$$\approx 45.0\,\frac{\text{km}}{\text{hr}} \qquad\qquad\qquad\qquad \text{Multiply and divide.}$$

To round to the nearest kilometer per hour, identify the rounding and test digits.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest kilometer per hour, the solar plane lifted off no more than approximately 45 kilometers per hour.

Percent

7.1 Percent, Decimals, Fractions

1.

$$4\frac{7}{10}\% = \frac{4\frac{7}{10}}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{47}{10}}{100} \qquad \text{Mixed to improper fraction.}$$

$$= \frac{47}{10} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$

$$= \frac{47}{1000} \qquad \text{Simplify.}$$

3.

$$7\frac{2}{9}\% = \frac{7\frac{2}{9}}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{65}{9}}{100} \qquad \text{Mixed to improper fraction.}$$

$$= \frac{65}{9} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$

$$= \frac{13}{180} \qquad \text{Simplify.}$$

5.

$$11.76\% = \frac{11.76}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{11.76 \cdot 100}{100 \cdot 100} \qquad \text{Multiply numerator and denominator by 100.}$$

$$= \frac{1176}{10000} \qquad \text{Multiply.}$$

$$= \frac{147}{1250} \qquad \text{Reduce.}$$

7.

$$13.99\% = \frac{13.99}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{13.99 \cdot 100}{100 \cdot 100} \qquad \text{Multiply numerator and denominator by 100.}$$

$$= \frac{1399}{10000} \qquad \text{Multiply.}$$

9.

$$4\frac{1}{2}\% = \frac{4\frac{1}{2}}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{9}{2}}{100} \qquad \text{Mixed to improper fraction.}$$

$$= \frac{9}{2} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$

$$= \frac{9}{200} \qquad \text{Simplify.}$$

11.

$$192\% = \frac{192}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{48}{25} \qquad \text{Reduce.}$$

13.

$$86\% = \frac{86}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{43}{50} \qquad \text{Reduce.}$$

15.

$$130\% = \frac{130}{100} \qquad \textcolor{red}{\text{Percent: parts per hundred.}}$$
$$= \frac{13}{10} \qquad \textcolor{red}{\text{Reduce.}}$$

17.

$$4.07\% = \frac{4.07}{100} \qquad \textcolor{red}{\text{Percent: parts per hundred.}}$$
$$= \frac{4.07 \cdot 100}{100 \cdot 100} \qquad \textcolor{red}{\text{Multiply numerator and denominator by 100.}}$$
$$= \frac{407}{10000} \qquad \textcolor{red}{\text{Multiply.}}$$

19. Move the decimal point two places to the left: $124\% = 1.24$

21. Move the decimal point two places to the left: $0.6379\% = 0.006379$

23. Move the decimal point two places to the left: $28\% = 0.28$

25. Move the decimal point two places to the left: $0.83\% = 0.0083$

27. Move the decimal point two places to the left: $8\% = 0.08$

29. Move the decimal point two places to the left: $59.84\% = 0.5984$

31. Move the decimal point two places to the left: $155\% = 1.55$

33. Move the decimal point two places to the left: $36.5\% = 0.365$

35. Move the decimal point two places to the right: $8.888 = 888.8\%$

37. Move the decimal point two places to the right: $0.85 = 85\%$

39. Move the decimal point two places to the right: $1.681 = 168.1\%$

41. Move the decimal point two places to the right: $0.14 = 14\%$

43. Move the decimal point two places to the right: $8.7 = 870\%$

45. Move the decimal point two places to the right: $0.38 = 38\%$

47. Move the decimal point two places to the right: $0.02 = 2\%$

49. Move the decimal point two places to the right: $0.044 = 4.4\%$

51. Change $1/2$ to a decimal, then change the decimal to a percent:

$$\frac{1}{2} = 0.5 \qquad \text{Divide.}$$
$$\phantom{\frac{1}{2}} = 50\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for $1/2$ with a denominator of 100:
$$\frac{1}{2} = \frac{x}{100}$$

Then solve this proportion for x:

$$2x = 100 \qquad\qquad \text{Cross multiply.}$$
$$\frac{2x}{2} = \frac{100}{2} \qquad\qquad \text{Divide both sides by 2.}$$
$$x = 50 \qquad\qquad \text{Divide.}$$

Thus, $\dfrac{1}{2} = \dfrac{50}{100} = 50\%$.

53. Change $5/2$ to a decimal, then change the decimal to a percent:

$$\frac{5}{2} = 2.5 \qquad \text{Divide.}$$
$$\phantom{\frac{5}{2}} = 250\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for $5/2$ with a denominator of 100:
$$\frac{5}{2} = \frac{x}{100}$$

Then solve this proportion for x:

$$2x = 500 \qquad \text{Cross multiply.}$$
$$\frac{2x}{2} = \frac{500}{2} \qquad \text{Divide both sides by 2.}$$
$$x = 250 \qquad \text{Divide.}$$

Thus, $\dfrac{5}{2} = \dfrac{250}{100} = 250\%$.

55. Change 8/5 to a decimal, then change the decimal to a percent:

$$\frac{8}{5} = 1.6 \qquad \text{Divide.}$$
$$= 160\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 8/5 with a denominator of 100:

$$\frac{8}{5} = \frac{x}{100}$$

Then solve this proportion for x:

$$5x = 800 \qquad \text{Cross multiply.}$$
$$\frac{5x}{5} = \frac{800}{5} \qquad \text{Divide both sides by 5.}$$
$$x = 160 \qquad \text{Divide.}$$

Thus, $\dfrac{8}{5} = \dfrac{160}{100} = 160\%$.

57. Change 14/5 to a decimal, then change the decimal to a percent:

$$\frac{14}{5} = 2.8 \qquad \text{Divide.}$$
$$= 280\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 14/5 with a denominator of 100:

$$\frac{14}{5} = \frac{x}{100}$$

Then solve this proportion for x:

$$5x = 1400 \qquad \text{Cross multiply.}$$
$$\frac{5x}{5} = \frac{1400}{5} \qquad \text{Divide both sides by 5.}$$
$$x = 280 \qquad \text{Divide.}$$

Thus, $\dfrac{14}{5} = \dfrac{280}{100} = 280\%$.

59. Change 9/2 to a decimal, then change the decimal to a percent:

$$\frac{9}{2} = 4.5 \qquad \text{Divide.}$$

$$= 450\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 9/2 with a denominator of 100:
$$\frac{9}{2} = \frac{x}{100}$$

Then solve this proportion for x:

$$2x = 900 \qquad\qquad \text{Cross multiply.}$$

$$\frac{2x}{2} = \frac{900}{2} \qquad\qquad \text{Divide both sides by 2.}$$

$$x = 450 \qquad\qquad \text{Divide.}$$

Thus, $\dfrac{9}{2} = \dfrac{450}{100} = 450\%$.

61. Change 9/4 to a decimal, then change the decimal to a percent:

$$\frac{9}{4} = 2.25 \qquad \text{Divide.}$$

$$= 225\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 9/4 with a denominator of 100:
$$\frac{9}{4} = \frac{x}{100}$$

Then solve this proportion for x:

$$4x = 900 \qquad\qquad \text{Cross multiply.}$$

$$\frac{4x}{4} = \frac{900}{4} \qquad\qquad \text{Divide both sides by 4.}$$

$$x = 225 \qquad\qquad \text{Divide.}$$

Thus, $\dfrac{9}{4} = \dfrac{225}{100} = 225\%$.

63. Change 7/5 to a decimal, then change the decimal to a percent:

$$\frac{7}{5} = 1.4 \qquad \text{Divide.}$$

$$= 140\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 7/5 with a denominator of 100:

$$\frac{7}{5} = \frac{x}{100}$$

Then solve this proportion for x:

$$5x = 700 \qquad \text{Cross multiply.}$$
$$\frac{5x}{5} = \frac{700}{5} \qquad \text{Divide both sides by 5.}$$
$$x = 140 \qquad \text{Divide.}$$

Thus, $\dfrac{7}{5} = \dfrac{140}{100} = 140\%$.

65. Change 6/5 to a decimal, then change the decimal to a percent:

$$\frac{6}{5} = 1.2 \qquad \text{Divide.}$$
$$= 120\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 6/5 with a denominator of 100:

$$\frac{6}{5} = \frac{x}{100}$$

Then solve this proportion for x:

$$5x = 600 \qquad \text{Cross multiply.}$$
$$\frac{5x}{5} = \frac{600}{5} \qquad \text{Divide both sides by 5.}$$
$$x = 120 \qquad \text{Divide.}$$

Thus, $\dfrac{6}{5} = \dfrac{120}{100} = 120\%$.

67. Change 12/5 to a decimal, then change the decimal to a percent:

$$\frac{12}{5} = 2.4 \qquad \text{Divide.}$$
$$= 240\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 12/5 with a denominator of 100:

$$\frac{12}{5} = \frac{x}{100}$$

Then solve this proportion for x:

$$5x = 1200 \qquad \qquad \text{Cross multiply.}$$

$$\frac{5x}{5} = \frac{1200}{5} \qquad \qquad \text{Divide both sides by 5.}$$

$$x = 240 \qquad \qquad \text{Divide.}$$
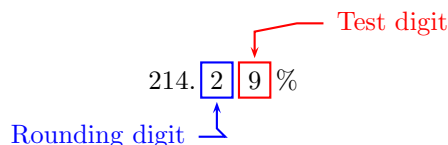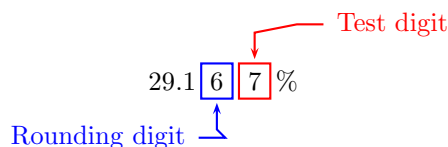
Thus, $\dfrac{12}{5} = \dfrac{240}{100} = 240\%$.

69. First compute an approximate decimal equivalent of the fraction 24/29 by dividing. We will be moving the decimal point two places to the right and then rounding, so the division must be carried out to five decimal places. By division,

$$\frac{24}{29} \approx 0.82759$$

Now move the decimal point two places to the right:

$$0.82759 = 82.759\%$$

To round to the nearest hundredth of a percent, identify the rounding and test digits.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate:

$$82.759\% \approx 82.76\%$$

Thus, $\dfrac{24}{29} \approx 82.76\%$.

71. First compute an approximate decimal equivalent of the fraction 15/7 by dividing. We will be moving the decimal point two places to the right and then rounding, so the division must be carried out to four decimal places. By division,

$$\frac{15}{7} \approx 2.1429$$

Now move the decimal point two places to the right:

$$2.1429 = 214.29\%$$

To round to the nearest tenth of a percent, identify the rounding and test digits.

Test digit

$$214.\boxed{2}\,\boxed{9}\,\%$$

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate:

$$214.29\% \approx 214.3\%$$

Thus, $\dfrac{15}{7} \approx 214.3\%$.

73. First compute an approximate decimal equivalent of the fraction 7/24 by dividing. We will be moving the decimal point two places to the right and then rounding, so the division must be carried out to five decimal places. By division,

$$\frac{7}{24} \approx 0.29167$$

Now move the decimal point two places to the right:

$$0.29167 = 29.167\%$$

To round to the nearest hundredth of a percent, identify the rounding and test digits.

Test digit

$$29.1\,\boxed{6}\,\boxed{7}\,\%$$

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate:

$$29.167\% \approx 29.17\%$$

Thus, $\dfrac{7}{24} \approx 29.17\%$.

75. First compute an approximate decimal equivalent of the fraction 8/3 by dividing. We will be moving the decimal point two places to the right and then rounding, so the division must be carried out to four decimal places. By division,

$$\frac{8}{3} \approx 2.6667$$

Now move the decimal point two places to the right:

$$2.6667 = 266.67\%$$

To round to the nearest tenth of a percent, identify the rounding and test digits.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate:

$$266.67\% \approx 266.7\%$$

Thus, $\dfrac{8}{3} \approx 266.7\%$.

77. First compute an approximate decimal equivalent of the fraction 9/23 by dividing. We will be moving the decimal point two places to the right and then rounding, so the division must be carried out to four decimal places. By division,

$$\frac{9}{23} \approx 0.3913$$

Now move the decimal point two places to the right:

$$0.3913 = 39.13\%$$

To round to the nearest tenth of a percent, identify the rounding and test digits.

Because the test digit is less than 5, leave the rounding digit alone and then truncate:

$$39.13\% \approx 39.1\%$$

Thus, $\dfrac{9}{23} \approx 39.1\%$.

79. First compute an approximate decimal equivalent of the fraction 17/27 by dividing. We will be moving the decimal point two places to the right and then rounding, so the division must be carried out to five decimal places. By division,

$$\frac{17}{27} \approx 0.62963$$

Now move the decimal point two places to the right:

$$0.62963 = 62.963\%$$

To round to the nearest hundredth of a percent, identify the rounding and test digits.

Because the test digit is less than 5, leave the rounding digit alone and then truncate:

$$62.963\% \approx 62.96\%$$

Thus, $\dfrac{17}{27} \approx 62.96\%$.

81.

  i) The negative signs indicate the crime rate has *decreased* from previous measures.

 ii) To find the largest decrease, find the largest magnitude. The largest magnitude will indicate the largest change. The absolute value of percent change in motor vehicle theft at 18.75% is the largest magnitude listed. Therefore, motor vehicle theft decreased the most.

iii) To find the smallest change from previous measures, find the smallest magnitude. The smallest absolute value corresponds to the percent change in burglaries at 2.5%. Therefore, burglary crimes went down the least amount.

83.

  i) Change 1/25 to a decimal, then change the decimal to a percent:

$$\frac{1}{25} = 0.04 \qquad \text{Divide.}$$
$$= 4\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 1/25 with a denominator of 100:
$$\frac{1}{25} = \frac{x}{100}$$
Then solve this proportion for x:

$$25x = 100 \qquad \text{Cross multiply.}$$
$$\frac{25x}{25} = \frac{100}{25} \qquad \text{Divide both sides by 25.}$$
$$x = 4 \qquad \text{Divide.}$$

Thus, $\dfrac{1}{25} = \dfrac{4}{100} = 4\%$.

ii) Change 1/32 to a decimal, then change the decimal to a percent:

$$\frac{1}{32} = 0.03125 \qquad \text{Divide.}$$
$$\approx 3.1\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 1/32 with a denominator of 100:
$$\frac{1}{32} = \frac{x}{100}$$
Then solve this proportion for x:

$$32x = 100 \qquad \text{Cross multiply.}$$
$$\frac{32x}{32} = \frac{100}{32} \qquad \text{Divide both sides by 32.}$$
$$x = 3.125 \qquad \text{Divide.}$$

Thus, $\dfrac{1}{32} = \dfrac{3.125}{100} \approx 3.1\%$ rounded to the nearest tenth of a percent.

iii) Change 1/4 to a decimal, then change the decimal to a percent:

$$\frac{1}{4} = 0.25 \qquad \text{Divide.}$$
$$= 25\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 1/4 with a denominator of 100:
$$\frac{1}{4} = \frac{x}{100}$$
Then solve this proportion for x:

$$4x = 100 \qquad \text{Cross multiply.}$$
$$\frac{4x}{4} = \frac{100}{4} \qquad \text{Divide both sides by 4.}$$
$$x = 4 \qquad \text{Divide.}$$

Thus, $\dfrac{1}{4} = \dfrac{25}{100} = 25\%$.

iv) Change 1/140 to a decimal, then change the decimal to a percent:

$$\dfrac{1}{140} = 0.0071 \qquad \text{Divide.}$$

$$\approx 0.7\% \qquad \text{Move the decimal point two places to the right.}$$

Alternate solution: First create an equivalent fraction for 1/140 with a denominator of 100:

$$\dfrac{1}{140} = \dfrac{x}{100}$$

Then solve this proportion for x:

$$140x = 100 \qquad\qquad \text{Cross multiply.}$$

$$\dfrac{140x}{140} = \dfrac{100}{140} \qquad\qquad \text{Divide both sides by 140.}$$

$$x \approx 0.7 \qquad\qquad \text{Divide.}$$

Thus, $\dfrac{1}{140} \approx \dfrac{0.7}{100} = 0.7\%$.

7.2 Solving Basic Percent Problems

1.

$$x = 22.4\% \cdot 125 \qquad \text{Set up an equation.}$$

$$x = (0.224)(125) \qquad \text{Convert the percent to a decimal.}$$

$$x = 28 \qquad\qquad\quad \text{Multiply.}$$

3.

$$60\% \cdot x = 90 \qquad\qquad \text{Set up an equation.}$$

$$0.6x = 90 \qquad\qquad \text{Convert the percent to a decimal.}$$

$$x = \dfrac{90}{0.6} \qquad\qquad \text{Divide both sides by 0.6.}$$

$$x = 150 \qquad\qquad \text{Simplify.}$$

5.

$$200\% \cdot x = 132 \qquad \text{Set up an equation.}$$
$$2x = 132 \qquad \text{Convert the percent to a decimal.}$$
$$x = \frac{132}{2} \qquad \text{Divide both sides by 2.}$$
$$x = 66 \qquad \text{Simplify.}$$

7.

$$162.5\% \cdot x = 195 \qquad \text{Set up an equation.}$$
$$1.625x = 195 \qquad \text{Convert the percent to a decimal.}$$
$$x = \frac{195}{1.625} \qquad \text{Divide both sides by 1.625.}$$
$$x = 120 \qquad \text{Simplify.}$$

9.

$$126.4\% \cdot x = 158 \qquad \text{Set up an equation.}$$
$$1.264x = 158 \qquad \text{Convert the percent to a decimal.}$$
$$x = \frac{158}{1.264} \qquad \text{Divide both sides by 1.264.}$$
$$x = 125 \qquad \text{Simplify.}$$

11.

$$27 = 45 \cdot x \qquad \text{Set up an equation.}$$
$$x = \frac{27}{45} \qquad \text{Divide both sides by 45.}$$
$$x = 0.6 \qquad \text{Simplify.}$$
$$x = 60\% \qquad \text{Convert the decimal to percent.}$$

13.

$$37.5\% \cdot x = 57 \qquad \text{Set up an equation.}$$
$$0.375x = 57 \qquad \text{Convert the percent to a decimal.}$$
$$x = \frac{57}{0.375} \qquad \text{Divide both sides by 0.375.}$$
$$x = 152 \qquad \text{Simplify.}$$

15.

$$x = 85\% \cdot 100 \qquad \text{Set up an equation.}$$
$$x = (0.85)(100) \qquad \text{Convert the percent to a decimal.}$$
$$x = 85 \qquad \text{Multiply.}$$

17.

$$x = 200\% \cdot 15 \qquad \text{Set up an equation.}$$
$$x = (2)(15) \qquad \text{Convert the percent to a decimal.}$$
$$x = 30 \qquad \text{Multiply.}$$

19.

$$50\% \cdot x = 58 \qquad \text{Set up an equation.}$$
$$0.5x = 58 \qquad \text{Convert the percent to a decimal.}$$
$$x = \frac{58}{0.5} \qquad \text{Divide both sides by 0.5.}$$
$$x = 116 \qquad \text{Simplify.}$$

21.

$$5.6 = 40 \cdot x \qquad \text{Set up an equation.}$$
$$x = \frac{5.6}{40} \qquad \text{Divide both sides by 40.}$$
$$x = 0.14 \qquad \text{Simplify.}$$
$$x = 14\% \qquad \text{Convert the decimal to percent.}$$

23.

$$x = 18.4\% \cdot 125 \qquad \text{Set up an equation.}$$
$$x = (0.184)(125) \qquad \text{Convert the percent to a decimal.}$$
$$x = 23 \qquad \text{Multiply.}$$

25.

$$30.8 = 40 \cdot x \qquad \text{Set up an equation.}$$
$$x = \frac{30.8}{40} \qquad \text{Divide both sides by 40.}$$
$$x = 0.77 \qquad \text{Simplify.}$$
$$x = 77\% \qquad \text{Convert the decimal to percent.}$$

27.

$$7.2 = 16 \cdot x \qquad \text{Set up an equation.}$$
$$x = \frac{7.2}{16} \qquad \text{Divide both sides by 16.}$$
$$x = 0.45 \qquad \text{Simplify.}$$
$$x = 45\% \qquad \text{Convert the decimal to percent.}$$

29.

$$x = 89.6\% \cdot 125 \qquad \text{Set up an equation.}$$
$$x = (0.896)(125) \qquad \text{Convert the percent to a decimal.}$$
$$x = 112 \qquad \text{Multiply.}$$

31.

$$60 = 80 \cdot x \qquad \text{Set up an equation.}$$
$$x = \frac{60}{80} \qquad \text{Divide both sides by 80.}$$
$$x = 0.75 \qquad \text{Simplify.}$$
$$x = 75\% \qquad \text{Convert the decimal to percent.}$$

33.

$$x = 200\% \cdot 11 \qquad \text{Set up an equation.}$$
$$x = (2)(11) \qquad \text{Convert the percent to a decimal.}$$
$$x = 22 \qquad \text{Multiply.}$$

35.

$$27 = 18 \cdot x \qquad \text{Set up an equation.}$$
$$x = \frac{27}{18} \qquad \text{Divide both sides by 18.}$$
$$x = 1.5 \qquad \text{Simplify.}$$
$$x = 150\% \qquad \text{Convert the decimal to percent.}$$

37. First convert the mixed fraction percent to an improper fraction:

$$133\frac{1}{3}\% = \frac{133\frac{1}{3}}{100} \qquad \text{Percent: parts per hundred.}$$
$$= \frac{\frac{400}{3}}{100} \qquad \text{Mixed to improper fraction.}$$
$$= \frac{400}{3} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$
$$= \frac{4}{3} \qquad \text{Simplify.}$$

Then set up and solve the appropriate equation:

$$133\frac{1}{3}\% \cdot x = 80 \qquad \text{Set up an equation.}$$
$$\frac{4}{3}x = 80 \qquad \text{Rewrite using the improper fraction computed above.}$$
$$4x = 80 \cdot 3 \qquad \text{Multiply both sides by 3.}$$
$$4x = 240 \qquad \text{Simplify.}$$
$$x = \frac{240}{4} \qquad \text{Divide both sides by 4.}$$
$$x = 60 \qquad \text{Simplify.}$$

39. First convert the mixed fraction percent to an improper fraction:

$$54\frac{1}{3}\% = \frac{54\frac{1}{3}}{100} \qquad \text{\color{red}Percent: parts per hundred.}$$

$$= \frac{\frac{163}{3}}{100} \qquad \text{\color{red}Mixed to improper fraction.}$$

$$= \frac{163}{3} \cdot \frac{1}{100} \qquad \text{\color{red}Invert and multiply.}$$

$$= \frac{163}{300} \qquad \text{\color{red}Simplify.}$$

Then set up and solve the appropriate equation:

$$x = 54\frac{1}{3}\% \cdot 6 \qquad \text{\color{red}Set up an equation.}$$

$$x = \frac{163}{300} \cdot 6 \qquad \text{\color{red}Rewrite using the improper fraction computed above.}$$

$$x = \frac{978}{300} \qquad \text{\color{red}Multiply numerators and denominators.}$$

$$x = 3.26 \qquad \text{\color{red}Divide.}$$

41. First convert the mixed fraction percent to an improper fraction:

$$62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} \qquad \text{\color{red}Percent: parts per hundred.}$$

$$= \frac{\frac{125}{2}}{100} \qquad \text{\color{red}Mixed to improper fraction.}$$

$$= \frac{125}{2} \cdot \frac{1}{100} \qquad \text{\color{red}Invert and multiply.}$$

$$= \frac{5}{8} \qquad \text{\color{red}Simplify.}$$

Then set up and solve the appropriate equation:

$$x = 62\frac{1}{2}\% \cdot 32 \qquad \text{\color{red}Set up an equation.}$$

$$x = \frac{5}{8} \cdot 32 \qquad \text{\color{red}Rewrite using the improper fraction computed above.}$$

$$x = \frac{160}{8} \qquad \text{\color{red}Multiply numerators and denominators.}$$

$$x = 20 \qquad \text{\color{red}Divide.}$$

43. First convert the mixed fraction percent to an improper fraction:

$$77\frac{1}{7}\% = \frac{77\frac{1}{7}}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{540}{7}}{100} \qquad \text{Mixed to improper fraction.}$$

$$= \frac{540}{7} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$

$$= \frac{27}{35} \qquad \text{Simplify.}$$

Then set up and solve the appropriate equation:

$$77\frac{1}{7}\% \cdot x = 27 \qquad \text{Set up an equation.}$$

$$\frac{27}{35}x = 27 \qquad \text{Rewrite using the improper fraction computed above.}$$

$$27x = 27 \cdot 35 \qquad \text{Multiply both sides by 35.}$$

$$27x = 945 \qquad \text{Simplify.}$$

$$x = \frac{945}{27} \qquad \text{Divide both sides by 27.}$$

$$x = 35 \qquad \text{Simplify.}$$

45. First convert the mixed fraction percent to an improper fraction:

$$142\frac{6}{7}\% = \frac{142\frac{6}{7}}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{1000}{7}}{100} \qquad \text{Mixed to improper fraction.}$$

$$= \frac{1000}{7} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$

$$= \frac{10}{7} \qquad \text{Simplify.}$$

Then set up and solve the appropriate equation:

$$x = 142\frac{6}{7}\% \cdot 77 \qquad \text{Set up an equation.}$$

$$x = \frac{10}{7} \cdot 77 \qquad \text{Rewrite using the improper fraction computed above.}$$

$$x = \frac{770}{7} \qquad \text{Multiply numerators and denominators.}$$

$$x = 110 \qquad \text{Divide.}$$

47.

$$143\frac{1}{2}\% \cdot x = 5.74 \qquad \text{Set up an equation.}$$

$$143.5\% \cdot x = 5.74 \qquad \text{Convert the mixed fraction to an exact decimal.}$$

$$1.435x = 5.74 \qquad \text{Convert the percent to an decimal.}$$

$$x = \frac{5.74}{1.435} \qquad \text{Divide both sides by 1.435.}$$

$$x = 4 \qquad \text{Simplify.}$$

49. First convert the mixed fraction percent to an improper fraction:

$$141\frac{2}{3}\% = \frac{141\frac{2}{3}}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{425}{3}}{100} \qquad \text{Mixed to improper fraction.}$$

$$= \frac{425}{3} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$

$$= \frac{17}{12} \qquad \text{Simplify.}$$

Then set up and solve the appropriate equation:

$$141\frac{2}{3}\% \cdot x = 68 \qquad \text{Set up an equation.}$$

$$\frac{17}{12}x = 68 \qquad \text{Rewrite using the improper fraction computed above.}$$

$$17x = 68 \cdot 12 \qquad \text{Multiply both sides by 12.}$$

$$17x = 816 \qquad \text{Simplify.}$$

$$x = \frac{816}{17} \qquad \text{Divide both sides by 17.}$$

$$x = 48 \qquad \text{Simplify.}$$

51. First convert the mixed fraction percent to an improper fraction:

$$66\frac{2}{3}\% = \frac{66\frac{2}{3}}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{200}{3}}{100} \qquad \text{Mixed to improper fraction.}$$

$$= \frac{200}{3} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$

$$= \frac{2}{3} \qquad \text{Simplify.}$$

Then set up and solve the appropriate equation:

$$x = 66\frac{2}{3}\% \cdot 96 \qquad \text{Set up an equation.}$$

$$x = \frac{2}{3} \cdot 96 \qquad \text{Rewrite using the improper fraction computed above.}$$

$$x = \frac{192}{3} \qquad \text{Multiply numerators and denominators.}$$

$$x = 64 \qquad \text{Divide.}$$

53.

$$59\frac{1}{2}\% \cdot x = 2.38 \qquad \text{Set up an equation.}$$

$$59.5\% \cdot x = 2.38 \qquad \text{Convert the mixed fraction to an exact decimal.}$$

$$0.595x = 2.38 \qquad \text{Convert the percent to an decimal.}$$

$$x = \frac{2.38}{0.595} \qquad \text{Divide both sides by 0.595.}$$

$$x = 4 \qquad \text{Simplify.}$$

55.

$$78\frac{1}{2}\% \cdot x = 7.85 \qquad \text{Set up an equation.}$$

$$78.5\% \cdot x = 7.85 \qquad \text{Convert the mixed fraction to an exact decimal.}$$

$$0.785x = 7.85 \qquad \text{Convert the percent to an decimal.}$$

$$x = \frac{7.85}{0.785} \qquad \text{Divide both sides by 0.785.}$$

$$x = 10 \qquad \text{Simplify.}$$

57. First convert the mixed fraction percent to an improper fraction:

$$56\frac{2}{3}\% = \frac{56\frac{2}{3}}{100} \qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{170}{3}}{100} \qquad \text{Mixed to improper fraction.}$$

$$= \frac{170}{3} \cdot \frac{1}{100} \qquad \text{Invert and multiply.}$$

$$= \frac{17}{30} \qquad \text{Simplify.}$$

Then set up and solve the appropriate equation:

$$x = 56\frac{2}{3}\% \cdot 51 \qquad \text{Set up an equation.}$$

$$x = \frac{17}{30} \cdot 51 \qquad\qquad \text{Rewrite using the improper fraction computed above.}$$

$$x = \frac{867}{30} \qquad\qquad \text{Multiply numerators and denominators.}$$

$$x = 28.9 \qquad\qquad \text{Divide.}$$

59. First convert the mixed fraction percent to an improper fraction:

$$87\frac{1}{2}\% = \frac{87\frac{1}{2}}{100} \qquad\qquad \text{Percent: parts per hundred.}$$

$$= \frac{\frac{175}{2}}{100} \qquad\qquad \text{Mixed to improper fraction.}$$

$$= \frac{175}{2} \cdot \frac{1}{100} \qquad\qquad \text{Invert and multiply.}$$

$$= \frac{7}{8} \qquad\qquad\qquad \text{Simplify.}$$

Then set up and solve the appropriate equation:

$$x = 87\frac{1}{2}\% \cdot 70 \qquad \text{Set up an equation.}$$

$$x = \frac{7}{8} \cdot 70 \qquad\qquad \text{Rewrite using the improper fraction computed above.}$$

$$x = \frac{490}{8} \qquad\qquad \text{Multiply numerators and denominators.}$$

$$x = 61.25 \qquad\qquad \text{Divide.}$$

61. Let r represent the revenue received by the dairy farmer. In words, the "revenue received by the dairy farmer is 20% of the retail price." In symbols, this translates to

$$r = 20\% \cdot 3.80$$

Change 20% to a decimal, then multiply.

$$r = 0.20 \cdot 3.80 \qquad\qquad 20\% = 0.20.$$

$$r = 0.76 \qquad\qquad\qquad 0.20 \cdot 3.80 = 0.76.$$

Hence, the dairy farmer receives $0.76.

7.3 General Applications of Percent

1. Let p represent the percent of the percent of the solution that is sulphuric acid. Then we can translate the problem statement into words and symbols.

Amount of sulphuric acid	is	what percent	of	the total amount of solution
31	=	p	·	250

Because multiplication is commutative, we can write the right-hand side of the last equation as follows.
$$31 = 250p$$

Divide both sides by 250. Carry the division to four decimal places.

$$\frac{31}{250} = \frac{250p}{250} \qquad \text{Divide both sides by 250.}$$
$$0.1240 \approx p \qquad \text{Divide to 4 places.}$$

To change this result to a percent, Move the decimal two places to the right and append a percent symbol.

$$p \approx 12.40\%$$

Now round to the nearest tenth of a percent.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, correct to the nearest tenth of a percent,

$$p \approx 12.4\%.$$

3. Let p represent the percent of the trip already completed. Then we can translate the problem statement into words and symbols.

Miles completed	is	what percent	of	the total trip
186	=	p	·	346

Because multiplication is commutative, we can write the right-hand side of the last equation as follows.

$$186 = 346p$$

Divide both sides by 346. Carry the division to three decimal places.

$$\frac{186}{346} = \frac{346p}{346} \qquad \text{Divide both sides by 346.}$$

$$0.537 \approx p \qquad \text{Divide to 3 places.}$$

To change this result to a percent, move the decimal two places to the right and append a percent symbol.

$$p \approx 53.7\%$$

Now round to the nearest percent.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, correct to the nearest percent,

$$p \approx 54\%.$$

5. Let p represent the percent of the class that is present. Then we can translate the problem statement into words and symbols.

Present	is	what percent	of	class size
19	=	p	$\cdot$	34

Because multiplication is commutative, we can write the right-hand side of the last equation as follows.

$$19 = 34p$$

Divide both sides by 34. Carry the division to three decimal places.

$$\frac{19}{34} = \frac{34p}{34} \qquad \text{Divide both sides by 34.}$$

$$0.558 \approx p \qquad \text{Divide to 3 places.}$$

To change this result to a percent, move the decimal two places to the right and append a percent symbol.

$$p \approx 55.8\%$$

Now round to the nearest percent.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, correct to the nearest percent,

$$p \approx 56\%.$$

7. Let p represent the percent of the total number of questions marked correctly. Then we can translate the problem statement into words and symbols.

Number of correct answers	is	what percent	of	total number of questions
135	=	p	$\cdot$	150

Because multiplication is commutative, we can write the last equation in the form

$$135 = 150p.$$

Solve for p.

$$\frac{135}{150} = \frac{150p}{150} \qquad \text{Divide both sides by 150 .}$$
$$\frac{9}{10} = p \qquad \text{Reduce: } 135/150 = 9/10.$$

We need to change $p = 9/10$ to a percent. Divide 9 by 10 to get

$$p = \frac{9}{10}$$
$$= 0.90$$
$$= 90\%.$$

Thus, Raven got 90% of the questions on the meteorology examination correct.

9. Let T represent the total mileage of the trip. Then we can translate the problem statement into words and symbols.

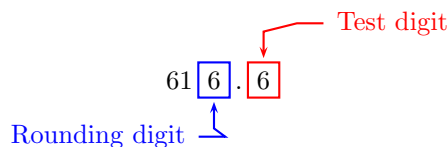| Miles already traveled	is	37%	of	total trip mileage
114	=	37%	$\cdot$	T

Change the percent to a decimal by moving the decimal two places to the left.

$$114 = 0.37 \cdot T$$

Divide both sides by 0.37. Carry the division to one decimal place.

$$\frac{114}{0.37} = \frac{0.37T}{0.37} \qquad \text{Divide both sides by 0.37.}$$
$$308.1 \approx T \qquad \text{Divide to 1 decimal place.}$$

Now round to the nearest mile.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, correct to the nearest mile,

$$T \approx 308$$

Thus, correct to the nearest mile, the total trip mileage is 308 miles.

11. Let S represent the total class size. Then we can translate the problem statement into words and symbols.

Students present	is	50%	of	total class size
65	=	50%	·	S

Change the percent to a decimal by moving the decimal two places to the left.

$$65 = 0.50 \cdot S$$

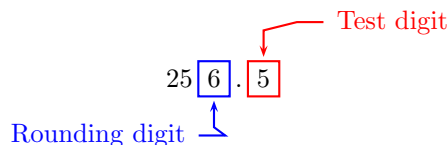Divide both sides by 0.50. Carry the division to one decimal place.

$$\frac{65}{0.50} = \frac{0.50S}{0.50} \qquad \text{Divide both sides by 0.50.}$$
$$130 = S \qquad \text{Divide.}$$

Thus, the total class size is 130 students.

13. Let p represent the commission percentage. Then we can translate the problem statement into words and symbols.

Commission	is	what percent	of	sales price
43	=	p	$\cdot$	591

The commutative property allows us to change the order of multiplication.

$$43 = 591p$$

Divide both sides by 591. Carry the division to four decimal places.

$$\frac{43}{591} = \frac{591p}{591} \qquad \text{Divide both sides by 591.}$$
$$0.0727 \approx p \qquad \text{Divide to 4 places.}$$

Change this a percent by moving the decimal point two places to the right.

$$p \approx 7.27\%$$

Now round to the nearest tenth of a percent.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, correct to the nearest tenth of a percent,

$$p \approx 7.3\%.$$

15. Let N represent the number of questions on the physics examination.

Number of correct answers	is	70%	of	total number of questions
98	=	70%	$\cdot$	N

Because $70\% = 0.70$, this last equation can be written as

$$98 = 0.70N.$$

Solve for N.

$$\frac{98}{0.70} = \frac{0.70N}{0.70} \qquad \text{Divide both sides by 0.70.}$$
$$140 = N \qquad \text{Divide: } 98/0.70 = 140.$$

Hence, there were 140 questions on the physics examination.

17. Let P represent the sales price of the computer. We can now translate the words of the problem statement into mathematical symbols.

Sales tax	is	8%	of	sales price
20	=	8%	·	P

Change the percent to a decimal by moving the decimal point two places to the left.

$$20 = 0.08P$$

Divide both sides by 0.08. Carry the division to one decimal place.

$$\frac{20}{0.08} = \frac{0.08P}{0.08} \qquad \text{Divide both sides by 0.08.}$$
$$250.0 \approx P \qquad \text{Divide to 1 place.}$$

Now we round to the nearest dollar.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, correct to the nearest dollar,

$$P \approx \$250.$$

19. Let P represent the sales price of the computer. We can now translate the words of the problem statement into mathematical symbols.

Commission	is	6%	of	sales price
37	=	6%	·	P

Change the percent to a decimal by moving the decimal point two places to the left.

$$37 = 0.06P$$

Divide both sides by 0.06. Carry the division to one decimal place.

$$\frac{37}{0.06} = \frac{0.06P}{0.06} \qquad \text{Divide both sides by 0.06.}$$
$$616.6 \approx P \qquad \text{Divide to 1 place.}$$

Now we round to the nearest dollar.

Test digit

$$61\boxed{6}.\boxed{6}$$

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, correct to the nearest dollar,

$$P \approx \$617.$$

21. Let T represent the total number of millilitres of solution. Then we can translate the problem statement into words and symbols.

Amount of nitric acid	is	23%	of	the total amount of solution
59	=	23%	·	T

Change the percent to a decimal by moving the decimal two places to the left.

$$59 = 0.23 \cdot T$$

Divide both sides by 0.23. Carry the division to one decimal place.

$$\frac{59}{0.23} = \frac{0.23T}{0.23} \qquad \text{Divide both sides by 0.23.}$$
$$256.5 \approx T \qquad \text{Divide to 1 decimal place.}$$

Now round to the nearest millilitre.

Test digit

$$25\boxed{6}.\boxed{5}$$

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, correct to the nearest millilitre,

$$T \approx 257.$$

Thus, correct to the nearest millilitre, the total solution measures 257 millilitres.

23. Let p represent the state sales tax rate. Then we can translate the problem statement into words and symbols.

Sales tax	is	what percent	of	sales price
45	=	p	$\cdot$	428

The commutative property allows us to change the order of multiplication.

$$45 = 428p$$

Divide both sides by 428. Carry the division to four decimal places.

$$\frac{45}{428} = \frac{428p}{428} \qquad \text{\color{red}Divide both sides by 428.}$$
$$0.1051 \approx p \qquad \text{\color{red}Divide to 4 places.}$$

Change this a percent by moving the decimal point two places to the right.

$$p \approx 10.51\%$$

Now round to the nearest tenth of a percent.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, correct to the nearest tenth of a percent,

$$p \approx 10.5\%.$$

25. Let w represent your weight on Mars.

Weight on Mars	is	38%	of	weight on Earth
w	=	38%	$\cdot$	150

Because $38\% = 0.38$, this last equation can be written as

$$w = 0.38 \cdot 150$$
$$= 57$$

Hence, you would weigh only 57 pounds on Mars.

27. Let s represent the number of seniors in Humboldt County.

Senior population in Humboldt	is	13%	of	total population in Humboldt
s	$=$	13%	$\cdot$	$130,000$

Because $13\% = 0.13$, this last equation can be written as

$$s = 0.13 \cdot 130000$$
$$= 16900$$

There are about 16,900 people aged 65 and older in Humboldt County.

29. Let g represent the amount of antibiotics going to farm animals to make them grow faster, in the millions of pounds. Then we can translate the problem statement into words and symbols.

Amount of antibiotics to make them grow faster	is	13%	of	the total amount of antibiotics to farm animals
g	$=$	13%	$\cdot$	28

Writing the percent as a decimal and solving, we have

$$g = 0.13 \cdot 28 \qquad \text{\textcolor{red}{Change percent to decimal.}}$$
$$g = 3.64 \qquad \text{\textcolor{red}{Multiply: } 0.13 \cdot 28 = 3.64.}$$

Thus, 3.64 million pounds of antibiotics were used on farm animals last year to make them grow faster.

31. Let w represent the total number of millions of gallons drinking water supplied daily to San Diego County.

Gallons of water supplied by the new plant	is	10%	of	the total gallons of water supplied
50	$=$	10%	$\cdot$	w

Since $10\% = 0.10$, this last equation can be written as

$$50 = 0.10w$$

Divide both sides by 0.10.

$$\frac{50}{0.10} = \frac{0.10 \cdot w}{0.10} \qquad \text{\textcolor{red}{Divide both sides by 0.10.}}$$
$$500 = w \qquad \text{\textcolor{red}{Divide.}}$$

The total gallons of drinking water supplied to San Diego County on a daily basis is 500 million gallons.

33. Let w represent the average water content for this time of year.

Current water content	is	92%	of	the average water content
25.9	$=$	92%	$\cdot$	w

Since $92\% = 0.92$, this last equation can be written as

$$25.9 = 0.92w$$

Divide both sides by 0.92.

$$\frac{25.9}{0.92} = \frac{0.92 \cdot w}{0.92} \qquad \text{\color{red}{Divide both sides by 0.92.}}$$
$$28.15 \approx w \qquad\qquad \text{\color{red}{Divide.}}$$

Now round to the nearest tenth.

Test digit

$$28.\boxed{1}\boxed{5}$$

Rounding digit

Because the test digit is equal to 5, add one to the rounding digit, then truncate. Thus, correct to a tenth of an inch,

$$w \approx 28.2$$

Thus, the average water content this time of year is 28.2 inches.

35. Let r represent the percent of stolen vehicles recovered. Then we can translate the problem statement into words and symbols.

Number of recovered vehicles	is	what percent	of	the number of stolen vehicles
427	$=$	r	$\cdot$	499

Because multiplication is commutative, we can write the right-hand side of the last equation as follows.
$$427 = 499r$$

Divide both sides by 499. Carry the division to four decimal places.

$$\frac{427}{499} = \frac{499p}{499} \qquad \text{\color{red}{Divide both sides by 499.}}$$
$$0.8557 \approx r \qquad\qquad \text{\color{red}{Divide to 4 places.}}$$

To change this result to a percent, Move the decimal two places to the right and append a percent symbol.

$$r \approx 85.57\%$$

Now round to the nearest tenth of a percent.

Because the test digit is greater than 5, add one to the rounding digit, then truncate. Thus, correct to the nearest tenth of a percent,

$$r \approx 85.6\%.$$

In Humboldt County, California, approximately 85.6% of stolen vehicles are recovered.

37. Let w represent the percentage of waste Mr. Winkler uses of the average American's waste per person. Then we can translate the problem statement into words and symbols.

Pounds of waste Mr. Winkler produces	is	what percent	of	the pounds of waste an average American produces
40	=	w	·	1600

Because multiplication is commutative, we can write the right-hand side of the last equation as follows.

$$40 = 1600w$$

Divide both sides by 1600.

$$\frac{40}{1600} = \frac{1600w}{1600} \qquad \text{Divide both sides by 1600.}$$
$$0.025 = w \qquad \text{Divide to three decimal places.}$$

To change this result to a percent, move the decimal two places to the right and append a percent symbol.

$$w = 2.5\%$$

Thus, Mr. Winkler throws away only 2.5% of the average American's waste.

7.4   Percent Increase or Decrease

1. Let D represent the discount. We can translate words into symbols as follows.

Discount	is	20.5%	of	marked price
D	$=$	20.5%	$\cdot$	447

Change the percent into a decimal by moving the decimal point two places to the left. Solve the resulting equation for D.

$$D = 0.205 \cdot 447 \qquad \textcolor{red}{20.5\% = 0.205.}$$
$$D = 91.635 \qquad \textcolor{red}{\text{Multiply: } 0.205 \cdot 447 = 91.635.}$$

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Therefore, to the nearest penny $D \approx \$91.64$. To find the new selling price, we must subtract this discount from the original price.

$$\begin{aligned}
\text{Selling price} &= \text{Marked price} - \text{Discount} \\
&= 447 - 91.64 \\
&= 355.36
\end{aligned}$$

Thus, the sales price is \$355.36.

3. To find the decrease in population, first subtract the current population from the original population.

$$\begin{aligned}
\text{Population decrease} &= \text{original population} - \text{current population} \\
&= 10794 - 8925 \\
&= 1869
\end{aligned}$$

Hence, the population has decreased by 1869 people.

Next, let p represent the percent population decrease. Then we can translate the problem into words and symbols.

Population decrease	is	what percent	of	original population
1869	$=$	p	$\cdot$	10794

Use the distributive property to switch the order of multiplication, then solve for p.

$$1869 = 10794p \qquad \text{Commutative property.}$$

$$\frac{1869}{10794} = \frac{10794p}{10794} \qquad \text{Divide both sides by 10794.}$$

$$0.17315 \approx p \qquad \text{Carry division to 5 places.}$$

To change p to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 17.315\%$$

To round to the nearest hundredth of a percent, identify the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Therefore, to the nearest hundredth of a percent, $p \approx 17.32\%$.

5. If the car rack is offered at 3.5% discount, then it will be sold at the following percentage of its marked price.

$$\text{Percentage of marked price} = 100\% - \text{Percent discount}$$
$$= 100\% - 3.5\%$$
$$= 96.5\%$$

Let S represent the sales price after discount. We can now translate words into symbols as follows.

Sales price	is	96.5%	of	Marked price
S	$=$	96.5%	$\cdot$	500

Change the percent to a decimal and solve for S.

$$S = 0.965 \cdot 500 \qquad 96.5\% = 0.965.$$
$$S = 482.500 \qquad \text{Multiply: } 0.965 \cdot 500 = 482.500.$$

To round to the nearest penny, identify the rounding digit and the test digit.

Test digit

$$482.5 \boxed{0} \boxed{0}$$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Therefore, to the nearest penny $S \approx \$482.50$.

7. If Silvertown's population decreases by 4.1%, then we can find the percent that remains by subtracting 4.1% from 100%.

$$\begin{aligned} \text{Percent remaining} &= 100\% - \text{Percent Decrease} \\ &= 100\% - 4.1\% \\ &= 95.9\% \end{aligned}$$

Let N represent the new population. We can then translate the words into symbols as follows.

New population	is	95.9%	of	original population
N	$=$	95.9%	$\cdot$	14678

Change the percent to a decimal by moving the decimal point two places to the left, then solve the resulting equation for N.

$$N = 0.959 \cdot 14678 \qquad \textcolor{red}{95.9\% = 0.959.}$$
$$N = 14076.202 \qquad \textcolor{red}{\text{Multiply: } 0.959 \cdot 14678 = 14076.202.}$$

Now we round to the nearest person. Identify the rounding and test digits.

Test digit

$$1407 \boxed{6} . \boxed{2} 02$$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, correct to the nearest person, the new population is 14,076.

9. Let x represent the salesperson's salary increase. Then we can translate the problem into words and symbols.

Salary increase	is	4.6%	of	current salary
x	$=$	4.6%	$\cdot$	2500

Change the percent to a decimal by moving the decimal point two places to the left, then solve for x.

$$x = 0.046 \cdot 2500 \qquad \text{4.6\% = 0.046.}$$
$$x = 115.000 \qquad \text{Multiply: } 0.046 \cdot 2500 = 115.000.$$

To round to the nearest dollar, identify the rounding digit and the test digit.

$$\text{Test digit}$$
$$11\boxed{5}.\boxed{0}\,00$$
$$\text{Rounding digit}$$

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Therefore, to the nearest dollar, the salary increase is \$115. To compute the new salary N, we must add this increase to the original salary.

New salary	is	current salary	plus	salary increase
N	$=$	2500	$+$	115

Thus, the new salary is $N = \$2,615$ per month.

11. We ask the following question.

Sales Price	is	What Percent	of	Marked Price
292	$=$	p	$\cdot$	358

Use the commutative property to change the order of multiplication. Solve the resulting equation for p.

$$292 = 358p \qquad \text{Commutative property.}$$
$$\frac{292}{358} = \frac{358p}{358} \qquad \text{Divide both sides by 358.}$$
$$0.8156 \approx p \qquad \text{Divide: } 66/358 = 0.8156.$$

We change this result to a percent by moving the decimal point two places to the right.

$$p \approx 81.56\%$$

To round to the nearest tenth of a percent, identify the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Therefore, to the nearest tenth of a percent, $p \approx 81.6\%$. To find the percent discount, we need to subtract this percentage from 100%.

$$\text{Percent Discount} = 100\% - 81.6\%$$
$$= 18.4\%$$

Hence, the percent discount is 18.4%.

13. We ask the following question.

New population	is	what percent	of	original population
12623	=	p	$\cdot$	14393

Use the distributive property to switch the order of multiplication, then solve for p.

$$12623 = 14393p \qquad \text{Commutative property.}$$
$$\frac{12623}{14393} = \frac{14393p}{14393} \qquad \text{Divide both sides by 14393.}$$
$$0.87702 \approx p \qquad \text{Carry division to 5 places.}$$

To change p to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 87.702\%$$

To round to the nearest hundreadth of a percent, identify the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Therefore, to the nearest hundredth of a percent, $p \approx 87.70\%$. But this is the percent of the population that remains in Carlytown. To find the percent decrease, we must subtract this last result from 100%.

$$\text{Percent decrease} = 100\% - \text{percent remaining}$$
$$= 100\% - 87.70\%$$
$$= 12.30\%$$

15. Let x represent the population decrease. We can then translate the words into symbols as follows.

Population decrease	is	2.4%	of	original population
x	$=$	2.4%	$\cdot$	8780

Change the percent to a decimal by moving the decimal point two places to the left, then solve the resulting equation for x.

$$x = 0.024 \cdot 8780 \qquad \color{red}{2.4\% = 0.024.}$$
$$x = 210.720 \qquad \color{red}{\text{Multiply: } 0.024 \cdot 8780 = 210.720.}$$

Now we round to the nearest person. Identify the rounding and test digits.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, correct to the nearest person, the population decrease is

$$p \approx 211.$$

To find the new population, subtract the decrease from the original population.

$$\text{New population} = \text{Original population} - \text{Population Decrease}$$
$$= 8780 - 211$$
$$= 8569$$

Thus, the new population is 8,569.

17. We ask the following question.

New salary	is	what percent	of	original salary
2950	$=$	p	$\cdot$	2600

Use the commutative property to change the order, then solve the equation for p.

$$2950 = 2600p \qquad \color{red}{\text{Commutative property.}}$$
$$\frac{2950}{2600} = \frac{2600p}{2600} \qquad \color{red}{\text{Divide both sides by 2600.}}$$
$$1.1346 \approx p \qquad \color{red}{\text{Divide to 4 places.}}$$

Change the result to a percent by moving the decimal two places to the right.

$$p \approx 113.46\%$$

Identify the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, correct to the nearest tenth of a percent,

$$p \approx 113.5\%.$$

To find the percent increase, we need to find how much more than 100% of our salary is returned. Subtract 100% from the last result.

$$\text{Percent increase} = 113.5\% - 100\%$$
$$= 13.5\%$$

Hence, the percent increase is 13.5%.

19. To find the increase in salary, first subtract the original salary from the new salary.

$$\text{Salary increase} = \text{new salary} - \text{original salary}$$
$$= 4300 - 4200$$
$$= 100$$

Hence, the bartender sees an increase in salary of $100. Now we can translate words into symbols. Let p represent the percent increase.

Salary Increase	is	what percent	of	original salary
100	=	p	·	4200

Use the commutative property to change the order, then solve the equation for p.

$$100 = 4200p \qquad \text{Commutative property.}$$
$$\frac{100}{4200} = \frac{4200p}{4200} \qquad \text{Divide both sides by 4200.}$$
$$0.0238 \approx p \qquad \text{Divide to 4 places.}$$

Change the result to a percent by moving the decimal two places to the right.

$$p \approx 2.38\%$$

Identify the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, correct to the nearest tenth of a percent,

$$p \approx 2.4\%.$$

21. Let N represent the salesperson's new salary. If the salesperson's salary is increased by 5.1%, then his new salary will be 105.1% of his original salary. We can now translate the problem into words and symbols.

New salary	is	105.1%	of	original salary
N	$=$	1.051%	$\cdot$	3200

Change the percent to a decimal by moving the decimal point two places to the left, then solve for N.

$$N = 1.051 \cdot 3200 \qquad \textcolor{red}{105.1\% = 1.051.}$$
$$N = 3363.200 \qquad \textcolor{red}{\text{Multiply: } 1.051 \cdot 3200 = 3363.200.}$$

To round to the nearest dollar, identify the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Therefore, to the nearest dollar, the new salary is $N \approx \$3,363$ per month.

23. We compute the discount D with the following calculation.

$$\text{Discount} = \text{Marked Price} - \text{Sales Price}$$
$$= 437 - 347$$
$$= 90.$$

Hence, the discount is $D = \$90$. We can find the percent discount with the following argument. Let p represent the percent discount.

Discount	is	Percent Discount	of	Marked Price
90	=	p	·	437

Use the commutative property to change the order of multiplication. Solve the resulting equation for p.

$$90 = 437p \qquad \text{Commutative property.}$$
$$\frac{90}{437} = \frac{437p}{437} \qquad \text{Divide both sides by 437.}$$
$$0.2059 \approx p \qquad \text{Divide: } 90/437 = 0.2059.$$

We change this result to a percent by moving the decimal point two places to the right.
$$p \approx 20.59\%$$

To round to the nearest tenth of a percent, identify the rounding digit and the test digit.
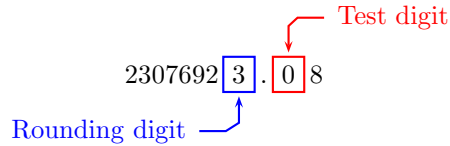

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Therefore, to the nearest tenth of a percent, $p \approx 20.6\%$.

25.

a) To find the increase in price, subtract the original price from the new price.

$$\text{Price increase} = \text{New price} - \text{Original price}$$
$$= 0.28 - 0.20$$
$$= 0.08$$

The increase is price is \$0.08, or 8 cents.

b) Let p represent the percent increase. Then,

Price increase	is	what percent	of	original price
0.08	=	p	·	0.20

Solve the equation for p.

$$0.08 = 0.20p \qquad \text{Change order of multiplication.}$$

$$\frac{0.08}{0.20} = \frac{0.20p}{0.20} \qquad \text{Divide both sides by 0.20.}$$

$$0.40 = p \qquad \text{Divide: } 0.08/0.20 = 0.40.$$

Finally, change $p = 0.40$ to a percent by moving the decimal two places to the right; i.e., $p = 40\%$.

27. First calculate the increase in price.

$$\text{Increase} = \text{later price} - \text{original price}$$
$$= 3.28 - 2.57$$
$$= 0.71$$

Hence, the increase in price is \$0.71.

Let p represent the percent increase.

Price increase	is	what percent	of	original price
0.71	=	p	$\cdot$	2.57

Solve the equation for p.

$$0.71 = 2.57p \qquad \text{Change order of multiplication.}$$

$$\frac{0.71}{2.57} = \frac{2.57p}{2.57} \qquad \text{Divide both sides by 2.57.}$$

$$0.276 \approx p \qquad \text{Divide: } 0.71/2.57 \approx 0.276.$$

Change $p \approx 0.276$ to a percent by moving the decimal two places to the right; i.e., $p = 27.6\%$. Now we will round to the nearest percent.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate. Hence, to the nearest percent, the percent increase is approximately 28%.

29. Let x represent the amount of the insurance increase. Then we can translate the problem into words and symbols.

Insurance increase	is	39%	of	current insurance cost
x	$=$	39%	$\cdot$	858

Change the percent to a decimal by moving the decimal point two places to the left, then solve for x.

$$x = 0.39 \cdot 858 \qquad 39\% = 0.39.$$
$$x = 334.62 \qquad \text{Multiply: } 0.39 \cdot 858 = 334.62.$$

To round to the nearest dollar, identify the rounding digit and the test digit.

Because the test digit is greater than 5, add one to the rounding digit, then truncate. Therefore, to the nearest dollar, the salary increase is \$335. To compute the new insurance cost C, we must add this increase to the original insurance cost.

New cost	is	current cost	plus	insurance increase
C	$=$	858	$+$	335

Thus, the new monthly cost will be $C = \$1,193$ per month.

31. We ask the following question.

New number dairy cows	is	what percent	of	original number dairy cows
$530,000$	$=$	p	$\cdot$	$180,000$

Use the commutative property to change the order, then solve the equation for p.

$$530,000 = 180,000p \qquad \text{Commutative property.}$$
$$\frac{530,000}{180,000} = \frac{180,000p}{180,000} \qquad \text{Divide both sides by 180,000.}$$
$$2.944 \approx p \qquad \text{Divide to 3 places.}$$

Change the result to a percent by moving the decimal two places to the right.

$$p \approx 294.4\%$$

Identify the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone and truncate. Thus, correct to the nearest percent,

$$p \approx 294\%.$$

To find the percent increase, we need to find how much more than 100% of dairy cows Idaho has currently. Subtract 100% from the last result.

$$\text{Percent increase} = 294\% - 100\%$$
$$= 194\%$$

Hence, the number of industrial dairy cows has increased 194% over previous numbers.

33.

a) We ask the following question.

New send time	is	what percent	of	current send time
88	=	p	$\cdot$	140

Use the distributive property to switch the order of multiplication, then solve for p.

$$88 = 140p \qquad \text{Commutative property.}$$
$$\frac{88}{140} = \frac{140p}{140} \qquad \text{Divide both sides by 140.}$$
$$0.628 \approx p \qquad \text{Carry division to 3 places.}$$

To change p to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 62.8\%$$

To round to the nearest percent, identify the rounding digit and the test digit.

Test digit

$$6\boxed{2}.\boxed{8}$$

Rounding digit

Because the test digit is greater than or equal to 5, add one to the rounding digit and truncate. Therefore, to the nearest percent, $p \approx 63\%$. But this is the percent of the new time that the old time was. To find the percent decrease, we must subtract this last result from 100%.

$$\text{Percent decrease} = 100\% - 63\%$$
$$= 37\%$$

b) The new send time is $140 - 88 = 52$ milliseconds. To find the cost per millisecond of time savings, divide the total cost \$1,200,000,000 by 52.

$$c = \frac{1,200,000,000}{52}$$
$$= 23,076,923.08$$

To round to the nearest dollar, identify the rounding digit and the test digit.

Test digit

$$2307692\boxed{3}.\boxed{0}8$$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone and truncate. Therefore, the cost per millisecond of time savings is approximately \$23,076,923.

35. First calculate the increase in hours of daylight from winter to summer.

$$\text{Increase} = \text{hours of daylight in summer} - \text{hours of daylight in winter}$$
$$= 14 - 10$$
$$= 4$$

Thus, there are 4 hours more of daylight in summer than in winter.

Let h represent the percent increase in hours of daylight.

Increase in daylight hours in summer	is	what percent	of	hours of daylight in winter
4	=	h	$\cdot$	10

Solve the equation for h.

$$4 = 10h \qquad \text{Change order of multiplication.}$$
$$\frac{4}{10} = \frac{10h}{10} \qquad \text{Divide both sides by 10.}$$
$$0.4 = h \qquad \text{Divide: } 4/10 = 0.4.$$

Change $h = 0.4$ to a percent by moving the decimal two places to the right; i.e., $h = 40\%$. Therefore, there are 40% more hours of daylight in summer than in winter.

37.

a) We ask the following question.

Home price in 2009	is	what percent	of	home price in 2000
$285,000$	$=$	p	$\cdot$	$152,257$

Use the distributive property to switch the order of multiplication, then solve for p.

$$285,000 = 152,257p \qquad \text{Commutative property.}$$
$$\frac{285,000}{152,257} = \frac{152,257p}{152,257} \qquad \text{Divide both sides by 152,257.}$$
$$1.871 \approx p \qquad \text{Carry division to 3 places.}$$

To change p to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 187.1\%$$

To round to the nearest percent, identify the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone and truncate. Therefore, to the nearest percent, $p \approx 187\%$. But this is the

percent of average home price in 2009. How much more than 100% is this price?

$$\text{Percent increase} = 187\% - 100\%$$
$$= 87\%$$

Thus, there has been a 87% increase in average home price from the year 2000 to 2009.

b) We ask the following question.

Number of homes sold in 2009	is	what percent	of	number of homes sold in 2000
833	=	p	$\cdot$	1358

Use the distributive property to switch the order of multiplication, then solve for p.

$$833 = 1358p \qquad \text{Commutative property.}$$
$$\frac{833}{1358} = \frac{1358p}{1358} \qquad \text{Divide both sides by 1358.}$$
$$0.613 \approx p \qquad \text{Carry division to 3 places.}$$

To change p to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 61.3\%$$

To round to the nearest percent, identify the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone and truncate. Therefore, to the nearest percent, $p \approx 61\%$. But this is the percent of homes sold in 2009. To find the percent change, subtract this from 100%.

$$\text{Percent decrease} = 100\% - 61\%$$
$$= 39\%$$

Thus, there has been a 39% decrease in the number of homes sold from the year 2000 to 2009.

7.5 Interest

1. The formula for simple interest is $I = Prt$. The principal is $P = 7600$ and the interest rate is $r = 8\% = 0.08$. The duration of the investment is $t = 7$ years. Substitute these values into the simple interest formula:

$$\begin{aligned} I &= (7600)(0.08)(7) \qquad && \text{Substitute.} \\ &= 4256 \qquad && \text{Multiply.} \end{aligned}$$

Hence, the interest earned in 7 years is \$4,256.

3. The formula for simple interest is $I = Prt$. The principal is $P = 5800$ and the interest rate is $r = 3.25\% = 0.0325$. The duration of the investment is $t = 4$ years. Substitute these values into the simple interest formula:

$$\begin{aligned} I &= (5800)(0.0325)(4) \qquad && \text{Substitute.} \\ &= 754 \qquad && \text{Multiply.} \end{aligned}$$

Hence, the interest earned in 4 years is \$754.

5. The formula for simple interest is $I = Prt$. The principal is $P = 2400$ and the interest rate is $r = 8.25\% = 0.0825$. The duration of the investment is $t = 5$ years. Substitute these values into the simple interest formula:

$$\begin{aligned} I &= (2400)(0.0825)(5) \qquad && \text{Substitute.} \\ &= 990 \qquad && \text{Multiply.} \end{aligned}$$

Hence, the interest earned in 5 years is \$990.

7. The formula for simple interest is $I = Prt$. The principal is $P = 4000$ and the interest rate is $r = 7.25\% = 0.0725$. The duration of the investment is $t = 6$ years. Substitute these values into the simple interest formula:

$$\begin{aligned} I &= (4000)(0.0725)(6) \qquad && \text{Substitute.} \\ &= 1740 \qquad && \text{Multiply.} \end{aligned}$$

Hence, the interest earned in 6 years is \$1,740.

9. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 2 months must be converted to years:

$$2\,\text{months} = 2\,\text{months} \cdot \frac{1\,\text{yr}}{12\,\text{months}} = 2\,\cancel{\text{months}} \cdot \frac{1\,\text{yr}}{12\,\cancel{\text{months}}} = \frac{2}{12}\,\text{yr} = \frac{1}{6}\,\text{yr}$$

The principal is $P = 3600$ and the interest rate is $r = 4.5\% = 0.045$. The duration of the investment is $1/6$ year. Substitute these values into the simple interest formula:

$$I = (3600)(0.045)\left(\frac{1}{6}\right) \qquad \text{Substitute.}$$

$$= \frac{162}{6} \qquad \text{Multiply numerators.}$$

$$= 27 \qquad \text{Divide.}$$

Hence, the interest owed is $27.

11. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 6 months must be converted to years:

$$6\,\text{months} = 6\,\text{months} \cdot \frac{1\,\text{yr}}{12\,\text{months}} = 6\,\cancel{\text{months}} \cdot \frac{1\,\text{yr}}{12\,\cancel{\text{months}}} = \frac{6}{12}\,\text{yr} = \frac{1}{2}\,\text{yr}$$

The principal is $P = 2400$ and the interest rate is $r = 2\% = 0.02$. The duration of the investment is $1/2$ year. Substitute these values into the simple interest formula:

$$I = (2400)(0.02)\left(\frac{1}{2}\right) \qquad \text{Substitute.}$$

$$= \frac{48}{2} \qquad \text{Multiply numerators.}$$

$$= 24 \qquad \text{Divide.}$$

Hence, the interest owed is $24.

13. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 6 months must be converted to years:

$$6\,\text{months} = 6\,\text{months} \cdot \frac{1\,\text{yr}}{12\,\text{months}} = 6\,\cancel{\text{months}} \cdot \frac{1\,\text{yr}}{12\,\cancel{\text{months}}} = \frac{6}{12}\,\text{yr} = \frac{1}{2}\,\text{yr}$$

The interest owed is $I = 68$ and the interest rate is $r = 8\% = 0.08$. The duration of the investment is $1/2$ year. Substitute these values into the simple interest formula:

$$68 = P(0.08)\left(\frac{1}{2}\right) \qquad \text{Substitute.}$$

$$68 = \frac{0.08}{2}P \qquad \text{Multiply numerators and rearrange the right side.}$$

$$68 \cdot 2 = 0.08P \qquad \text{Multiply both sides by 2.}$$

$$136 = 0.08P \qquad \text{Simplify.}$$

$$\frac{136}{0.08} = P \qquad \text{Divide both sides by 0.08.}$$

$$1700 = P \qquad \text{Simplify.}$$

Hence, the principal was $1,700.

15. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 3 months must be converted to years:

$$3 \, \text{months} = 3 \, \text{months} \cdot \frac{1 \, \text{yr}}{12 \, \text{months}} = 3 \, \cancel{\text{months}} \cdot \frac{1 \, \text{yr}}{12 \, \cancel{\text{months}}} = \frac{3}{12} \, \text{yr} = \frac{1}{4} \, \text{yr}$$

The principal is $P = 3600$ and the interest rate is $r = 8\% = 0.08$. The duration of the investment is $1/4$ year. Substitute these values into the simple interest formula:

$$I = (3600)(0.08)\left(\frac{1}{4}\right) \qquad \text{Substitute.}$$
$$= \frac{288}{4} \qquad \text{Multiply numerators.}$$
$$= 72 \qquad \text{Divide.}$$

Hence, the interest owed is $72.

17. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 2 months must be converted to years:

$$2 \, \text{months} = 2 \, \text{months} \cdot \frac{1 \, \text{yr}}{12 \, \text{months}} = 2 \, \cancel{\text{months}} \cdot \frac{1 \, \text{yr}}{12 \, \cancel{\text{months}}} = \frac{2}{12} \, \text{yr} = \frac{1}{6} \, \text{yr}$$

The interest owed is $I = 85$ and the interest rate is $r = 8.5\% = 0.085$. The duration of the investment is $1/6$ year. Substitute these values into the simple interest formula:

$$85 = P(0.085)\left(\frac{1}{6}\right) \qquad \text{Substitute.}$$
$$85 = \frac{0.085}{6}P \qquad \text{Multiply numerators and rearrange the right side.}$$
$$85 \cdot 6 = 0.085P \qquad \text{Multiply both sides by 6.}$$
$$510 = 0.085P \qquad \text{Simplify.}$$
$$\frac{510}{0.085} = P \qquad \text{Divide both sides by 0.085.}$$
$$6000 = P \qquad \text{Simplify.}$$

Hence, the principal was $6,000.

19. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 3 months must be converted to years:

$$3\,\text{months} = 3\,\text{months} \cdot \frac{1\,\text{yr}}{12\,\text{months}} = 3\,\cancel{\text{months}} \cdot \frac{1\,\text{yr}}{12\,\cancel{\text{months}}} = \frac{3}{12}\,\text{yr} = \frac{1}{4}\,\text{yr}$$

The principal is $P = 4000$ and the interest owed is $I = 35$. The duration of the investment is $1/4$ year. Substitute these values into the simple interest formula:

$$35 = (4000)(r)\left(\frac{1}{4}\right) \qquad \text{Substitute.}$$

$$35 = \frac{4000}{4}r \qquad\qquad \text{Multiply numerators and rearrange the right side.}$$

$$35 \cdot 4 = 4000r \qquad\qquad \text{Multiply both sides by 4.}$$

$$140 = 4000r \qquad\qquad \text{Simplify.}$$

$$\frac{140}{4000} = r \qquad\qquad \text{Divide both sides by 4000.}$$

$$0.035 = r \qquad\qquad \text{Simplify.}$$

Hence, the interest rate was 3.5%.

21. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 6 months must be converted to years:

$$6\,\text{months} = 6\,\text{months} \cdot \frac{1\,\text{yr}}{12\,\text{months}} = 6\,\cancel{\text{months}} \cdot \frac{1\,\text{yr}}{12\,\cancel{\text{months}}} = \frac{6}{12}\,\text{yr} = \frac{1}{2}\,\text{yr}$$

The interest owed is $I = 287$ and the interest rate is $r = 7\% = 0.07$. The duration of the investment is $1/2$ year. Substitute these values into the simple interest formula:

$$287 = P(0.07)\left(\frac{1}{2}\right) \qquad \text{Substitute.}$$

$$287 = \frac{0.07}{2}P \qquad\qquad \text{Multiply numerators and rearrange the right side.}$$

$$287 \cdot 2 = 0.07P \qquad\qquad \text{Multiply both sides by 2.}$$

$$574 = 0.07P \qquad\qquad \text{Simplify.}$$

$$\frac{574}{0.07} = P \qquad\qquad \text{Divide both sides by 0.07.}$$

$$8200 = P \qquad\qquad \text{Simplify.}$$

Hence, the principal was $8,200.

7.5. INTEREST 509

23. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 2 months must be converted to years:

$$2\,\text{months} = 2\,\text{months} \cdot \frac{1\,\text{yr}}{12\,\text{months}} = 2\,\cancel{\text{months}} \cdot \frac{1\,\text{yr}}{12\,\cancel{\text{months}}} = \frac{2}{12}\,\text{yr} = \frac{1}{6}\,\text{yr}$$

The principal is $P = 7300$ and the interest owed is $I = 73$. The duration of the investment is $1/6$ year. Substitute these values into the simple interest formula:

$$73 = (7300)(r)\left(\frac{1}{6}\right) \quad \text{Substitute.}$$

$$73 = \frac{7300}{6}r \qquad \text{Multiply numerators and rearrange the right side.}$$

$$73 \cdot 6 = 7300r \qquad \text{Multiply both sides by 6.}$$

$$438 = 7300r \qquad \text{Simplify.}$$

$$\frac{438}{7300} = r \qquad \text{Divide both sides by 7300.}$$

$$0.06 = r \qquad \text{Simplify.}$$

Hence, the interest rate was 6%.

25. The formula for simple interest is $I = Prt$. Because the interest rate is per year, the given time period of 6 months must be converted to years:

$$6\,\text{months} = 6\,\text{months} \cdot \frac{1\,\text{yr}}{12\,\text{months}} = 6\,\cancel{\text{months}} \cdot \frac{1\,\text{yr}}{12\,\cancel{\text{months}}} = \frac{6}{12}\,\text{yr} = \frac{1}{2}\,\text{yr}$$

The principal is $P = 3200$ and the interest owed is $I = 96$. The duration of the investment is $1/2$ year. Substitute these values into the simple interest formula:

$$96 = (3200)(r)\left(\frac{1}{2}\right) \quad \text{Substitute.}$$

$$96 = \frac{3200}{2}r \qquad \text{Multiply numerators and rearrange the right side.}$$

$$96 \cdot 2 = 3200r \qquad \text{Multiply both sides by 2.}$$

$$192 = 3200r \qquad \text{Simplify.}$$

$$\frac{192}{3200} = r \qquad \text{Divide both sides by 3200.}$$

$$0.06 = r \qquad \text{Simplify.}$$

Hence, the interest rate was 6%.

*Second Edition: 2012-2013*ment>

27. The balance is given by the formula $A = P(1 + rt)$. The principal is $P = 6700$ and the interest rate is $r = 9\% = 0.09$. The duration of the investment is $t = 4$ years. Substitute these values into the balance formula:

$$
\begin{aligned}
A &= (6700)(1 + 0.09 \cdot 4) &&\text{Substitute.} \\
&= (6700)(1 + 0.36) &&\text{Multiply.} \\
&= (6700)(1.36) &&\text{Add.} \\
&= 9112 &&\text{Multiply.}
\end{aligned}
$$

Hence, the balance after 4 years will be $9,112.

29. The balance is given by the formula $A = P(1 + rt)$. The principal is $P = 1600$ and the interest rate is $r = 2\% = 0.02$. The duration of the investment is $t = 3$ years. Substitute these values into the balance formula:

$$
\begin{aligned}
A &= (1600)(1 + 0.02 \cdot 3) &&\text{Substitute.} \\
&= (1600)(1 + 0.06) &&\text{Multiply.} \\
&= (1600)(1.06) &&\text{Add.} \\
&= 1696 &&\text{Multiply.}
\end{aligned}
$$

Hence, the balance after 3 years will be $1,696.

31. The balance is given by the formula $A = P(1 + rt)$. The principal is $P = 8900$ and the interest rate is $r = 2.5\% = 0.025$. The duration of the investment is $t = 2$ years. Substitute these values into the balance formula:

$$
\begin{aligned}
A &= (8900)(1 + 0.025 \cdot 2) &&\text{Substitute.} \\
&= (8900)(1 + 0.05) &&\text{Multiply.} \\
&= (8900)(1.05) &&\text{Add.} \\
&= 9345 &&\text{Multiply.}
\end{aligned}
$$

Hence, the balance after 2 years will be $9,345.

33. The balance is given by the formula $A = P(1 + rt)$. The principal is $P = 5400$ and the interest rate is $r = 4.25\% = 0.0425$. The duration of the investment is $t = 2$ years. Substitute these values into the balance formula:

$$
\begin{aligned}
A &= (5400)(1 + 0.0425 \cdot 2) &&\text{Substitute.} \\
&= (5400)(1 + 0.085) &&\text{Multiply.} \\
&= (5400)(1.085) &&\text{Add.} \\
&= 5859 &&\text{Multiply.}
\end{aligned}
$$

Hence, the balance after 2 years will be $5,859.

35. The balance is given by the formula $A = P(1 + rt)$. The balance is $A = 10222$ and the principal is $P = 7600$. The duration of the investment is $t = 6$ years. Substitute these values into the balance formula and solve the resulting equation:

$$10222 = 7600(1 + r(6)) \qquad \text{Substitute.}$$
$$10222 = 7600 + 45600r \qquad \text{Distribute.}$$
$$2622 = 45600r \qquad \text{Subtract 7600 from both sides.}$$
$$\frac{2622}{45600} = r \qquad \text{Divide both sides by 45600.}$$
$$0.0575 = r \qquad \text{Simplify.}$$

Hence, the interest rate was 5.75%.

37. The balance is given by the formula $A = P(1+rt)$. The balance is $A = 4640$ and the interest rate is $r = 9\% = 0.09$. The duration of the investment is $t = 5$ years. Substitute these values into the balance formula and solve the resulting equation:

$$4640 = P(1 + 0.09 \cdot 5) \qquad \text{Substitute.}$$
$$4640 = P(1 + 0.45) \qquad \text{Multiply.}$$
$$4640 = P(1.45) \qquad \text{Add.}$$
$$\frac{4640}{1.45} = P \qquad \text{Divide both sides by 1.45.}$$
$$3200 = P \qquad \text{Simplify.}$$

Hence, the amount borrowed was $3,200.

39. The balance is given by the formula $A = P(1+rt)$. The balance is $A = 9593$ and the interest rate is $r = 9\% = 0.09$. The duration of the investment is $t = 9$ years. Substitute these values into the balance formula and solve the resulting equation:

$$9593 = P(1 + 0.09 \cdot 9) \qquad \text{Substitute.}$$
$$9593 = P(1 + 0.81) \qquad \text{Multiply.}$$
$$9593 = P(1.81) \qquad \text{Add.}$$
$$\frac{9593}{1.81} = P \qquad \text{Divide both sides by 1.81.}$$
$$5300 = P \qquad \text{Simplify.}$$

Hence, the amount borrowed was $5,300.

41. The balance is given by the formula $A = P(1 + rt)$. The balance is $A = 5941$ and the principal is $P = 5200$. The duration of the investment is $t = 3$ years. Substitute these values into the balance formula and solve the resulting equation:

$$5941 = 5200(1 + r(3)) \qquad \text{Substitute.}$$
$$5941 = 5200 + 15600r \qquad \text{Distribute.}$$
$$741 = 15600r \qquad \text{Subtract 5200 from both sides.}$$
$$\frac{741}{15600} = r \qquad \text{Divide both sides by 15600.}$$
$$0.0475 = r \qquad \text{Simplify.}$$

Hence, the interest rate was 4.75%.

43. The balance is given by the formula $A = P(1 + rt)$. The balance is $A = 5400$ and the principal is $P = 4000$. The duration of the investment is $t = 5$ years. Substitute these values into the balance formula and solve the resulting equation:

$$5400 = 4000(1 + r(5)) \qquad \text{Substitute.}$$
$$5400 = 4000 + 20000r \qquad \text{Distribute.}$$
$$1400 = 20000r \qquad \text{Subtract 4000 from both sides.}$$
$$\frac{1400}{20000} = r \qquad \text{Divide both sides by 20000.}$$
$$0.07 = r \qquad \text{Simplify.}$$

Hence, the interest rate was 7%.

45. The balance is given by the formula $A = P(1 + rt)$. The balance is $A = 11550$ and the interest rate is $r = 7.5\% = 0.075$. The duration of the investment is $t = 5$ years. Substitute these values into the balance formula and solve the resulting equation:

$$11550 = P(1 + 0.075 \cdot 5) \qquad \text{Substitute.}$$
$$11550 = P(1 + 0.375) \qquad \text{Multiply.}$$
$$11550 = P(1.375) \qquad \text{Add.}$$
$$\frac{11550}{1.375} = P \qquad \text{Divide both sides by 1.375.}$$
$$8400 = P \qquad \text{Simplify.}$$

Hence, the amount borrowed was $8,400.

47. The balance is given by the formula $A = P(1 + rt)$. The balance is $A = 5720$ and the principal is $P = 4400$. The duration of the investment is $t = 4$ years. Substitute these values into the balance formula and solve the resulting equation:

$$5720 = 4400(1 + r(4)) \qquad \text{Substitute.}$$
$$5720 = 4400 + 17600r \qquad \text{Distribute.}$$
$$1320 = 17600r \qquad \text{Subtract 4400 from both sides.}$$
$$\frac{1320}{17600} = r \qquad \text{Divide both sides by 17600.}$$
$$0.075 = r \qquad \text{Simplify.}$$

Hence, the interest rate was 7.5%.

49. The balance is given by the formula $A = P(1+rt)$. The balance is $A = 9768$ and the interest rate is $r = 4\% = 0.04$. The duration of the investment is $t = 8$ years. Substitute these values into the balance formula and solve the resulting equation:

$$9768 = P(1 + 0.04 \cdot 8) \qquad \text{Substitute.}$$
$$9768 = P(1 + 0.32) \qquad \text{Multiply.}$$
$$9768 = P(1.32) \qquad \text{Add.}$$
$$\frac{9768}{1.32} = P \qquad \text{Divide both sides by 1.32.}$$
$$7400 = P \qquad \text{Simplify.}$$

Hence, the amount borrowed was \$7,400.

7.6 Pie Charts

1. Raven received 21% of the total votes. We must take 21% of $360°$ to determine the central angle of the sector representing Raven's portion of the votes.

$$21\% \cdot 360° = 0.21 \cdot 360°$$
$$= 75.60°$$

To round to the nearest degree, locate the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest degree, the central angle of the sector representing Raven's portion of the vote is $76°$.
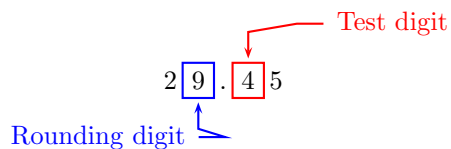
3. Akbar received 23% of the total votes. We must take 23% of $360°$ to determine the central angle of the sector representing Akbar's portion of the votes.

$$23\% \cdot 360° = 0.23 \cdot 360°$$
$$= 82.80°$$

To round to the nearest degree, locate the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest degree, the central angle of the sector representing Akbar's portion of the vote is $83°$.

5. Jamal received 30% of the total votes. We must take 30% of $360°$ to determine the central angle of the sector representing Jamal's portion of the votes.

$$30\% \cdot 360° = 0.3 \cdot 360°$$
$$= 108.00°$$

To round to the nearest degree, locate the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest degree, the central angle of the sector representing Jamal's portion of the vote is $108°$.

7. Chin received 5 out of 50 votes cast.

Chin's votes	are	what percent	of	total votes cast
5	=	p	$\cdot$	50

Use the commutative property to change the order, then solve for p.

$$5 = 50p \qquad \text{Commutative property.}$$
$$\frac{5}{50} = \frac{50p}{50} \qquad \text{Divide both sides by 50.}$$
$$0.10 = p \qquad \text{Divide: } 5/50 = 0.10.$$

Moving the decimal two places to the right, Chin captured 10% of the total vote. We must take 10% of 360° to determine the central angle of the sector representing Chin's portion of the votes.

$$10\% \cdot 360° = 0.10 \cdot 360°$$
$$= 36.00°$$

To round to the nearest degree, locate the rounding digit and the test digit.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest degree, the central angle of the sector representing Chin's portion of the vote is 36°.

9. Kamili received 14 out of 50 votes cast.

Kamili's votes	are	what percent	of	total votes cast
14	=	p	$\cdot$	50

Use the commutative property to change the order, then solve for p.

$$14 = 50p \qquad \text{Commutative property.}$$
$$\frac{14}{50} = \frac{50p}{50} \qquad \text{Divide both sides by 50.}$$
$$0.28 = p \qquad \text{Divide: } 14/50 = 0.28.$$

Moving the decimal two places to the right, Kamili captured 28% of the total vote. We must take 28% of 360° to determine the central angle of the sector representing Kamili's portion of the votes.

$$28\% \cdot 360° = 0.28 \cdot 360°$$
$$= 100.80°$$

To round to the nearest degree, locate the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest degree, the central angle of the sector representing Kamili's portion of the vote is 101°.

11. Hue received 13 out of 50 votes cast.

Hue's votes	are	what percent	of	total votes cast
13	=	p	$\cdot$	50

Use the commutative property to change the order, then solve for p.

$$13 = 50p \qquad \text{Commutative property.}$$
$$\frac{13}{50} = \frac{50p}{50} \qquad \text{Divide both sides by 50.}$$
$$0.26 = p \qquad \text{Divide: } 13/50 = 0.26.$$

Moving the decimal two places to the right, Hue captured 26% of the total vote. We must take 26% of 360° to determine the central angle of the sector representing Hue's portion of the votes.

$$26\% \cdot 360° = 0.26 \cdot 360°$$
$$= 93.60°$$

To round to the nearest degree, locate the rounding digit and the test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest degree, the central angle of the sector representing Hue's portion of the vote is 94°.

13. In the figure, note that Raven received 31% of the total votes.

Because there were 95 votes cast, we must take 31% of 95 to determine the number of votes won by Raven.

$$\begin{aligned} \text{Votes for Raven} &= 31\% \cdot 95 \\ &= 0.31 \cdot 95 \\ &= 29.45 \end{aligned}$$

To round this number to the nearest vote, we must first identify the rounding digit and test digit.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest vote, the number of votes awarded Raven is 29.

15. In the figure, note that Ali received 39% of the total votes.

Because there were 58 votes cast, we must take 39% of 58 to determine the number of votes won by Ali.

$$\begin{aligned} \text{Votes for Ali} &= 39\% \cdot 58 \\ &= 0.39 \cdot 58 \\ &= 22.62 \end{aligned}$$

To round this number to the nearest vote, we must first identify the rounding digit and test digit.

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Thus, to the nearest vote, the number of votes awarded Ali is 23.

17. In the figure, note that Hue received 34% of the total votes.

Because there were 95 votes cast, we must take 34% of 95 to determine the number of votes won by Hue.

$$\begin{aligned} \text{Votes for Hue} &= 34\% \cdot 95 \\ &= 0.34 \cdot 95 \\ &= 32.30 \end{aligned}$$

To round this number to the nearest vote, we must first identify the rounding digit and test digit.

Because the test digit is less than 5, leave the rounding digit alone, then truncate. Thus, to the nearest vote, the number of votes awarded Hue is 32.

19. The first task is to total the votes.

Candidate	Votes
Ali	45
Jamal	34
Jun	52
Totals	131

The next task is to express each quantity of votes as a percentage of the total votes. For example, Ali received 45 out of 131 votes.

Ali's votes	is	what percent	of	total votes
45	=	p	$\cdot$	131

Solve for p.

$$45 = 131p \qquad \text{Commutative property.}$$

$$\frac{45}{131} = \frac{131p}{131} \qquad \text{Divide both sides by 131.}$$

$$0.3435 \approx p \qquad \text{Divide: } 45/131 = 0.3435.$$

To the nearest tenth of a percent, Ali received 34.4% of the total votes. In similar fashion, calculate the percentage of votes earned by the remaining two candidates.

Candidate	Votes	Percent
Ali	45	34.4%
Jamal	34	26.0%
Jun	52	39.7%

Next, calculate the degree measure of the central angle of the "pie" wedge for each candidate. In the case of Ali:

$$\text{Ali's degrees} = 34.4\% \cdot 360°$$
$$= 0.344 \cdot 360°$$
$$= 123.84°$$

In similar fashion, calculate the degree measure of the central angle for each of the remaining candidates.

Candidate	Votes	Percent	Degrees
Ali	45	34.4%	123.84°
Jamal	34	26.0%	93.6°
Jun	52	39.7%	142.92°

Finally, take a protractor and measure three sectors with central angles from the Degrees column of the last table. Annotate each sector with the candidate's name and the percentage of votes received.

21. The first task is to total the votes.

Candidate	Votes
Bernardo	44
Rosa	40
Abdul	58
Totals	142

The next task is to express each quantity of votes as a percentage of the total votes. For example, Bernardo received 44 out of 142 votes.

Bernardo's votes	is	what percent	of	total votes
44	=	p	$\cdot$	142

Solve for p.

$$44 = 142p \qquad \text{Commutative property.}$$
$$\frac{44}{142} = \frac{142p}{142} \qquad \text{Divide both sides by 142.}$$
$$0.3098 \approx p \qquad \text{Divide: } 44/142 = 0.3098.$$

To the nearest tenth of a percent, Bernardo received 31.0% of the total votes. In similar fashion, calculate the percentage of votes earned by the remaining two candidates.

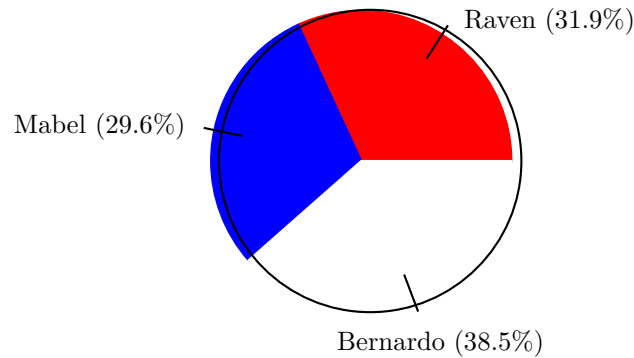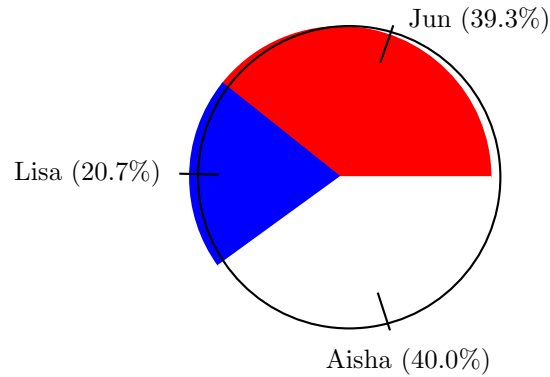| Candidate	Votes	Percent
Bernardo	44	31.0%
Rosa	40	28.2%
Abdul	58	40.8%

Next, calculate the degree measure of the central angle of the "pie" wedge for each candidate. In the case of Bernardo:

$$\text{Bernardo's degrees} = 31.0\% \cdot 360°$$
$$= 0.310 \cdot 360°$$
$$= 111.6°$$

In similar fashion, calculate the degree measure of the central angle for each of the remaining candidates.

Candidate	Votes	Percent	Degrees
Bernardo	44	31.0%	111.6°
Rosa	40	28.2%	101.52°
Abdul	58	40.8%	146.88°

Finally, take a protractor and measure three sectors with central angles from the Degrees column of the last table. Annotate each sector with the candidate's name and the percentage of votes received.

23. The first task is to total the votes.

Candidate	Votes
Mercy	56
Hans	53
Lisa	41
Totals	150

The next task is to express each quantity of votes as a percentage of the total votes. For example, Mercy received 56 out of 150 votes.

Mercy's votes	is	what percent	of	total votes
56	=	p	·	150

Solve for p.

$$56 = 150p \qquad \text{Commutative property.}$$
$$\frac{56}{150} = \frac{150p}{150} \qquad \text{Divide both sides by 150.}$$
$$0.3733 \approx p \qquad \text{Divide: } 56/150 = 0.3733.$$

To the nearest tenth of a percent, Mercy received 37.3% of the total votes. In similar fashion, calculate the percentage of votes earned by the remaining two candidates.

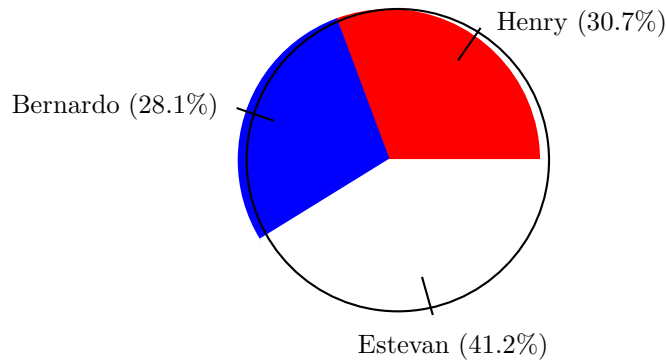| Candidate	Votes	Percent
Mercy	56	37.3%
Hans	53	35.3%
Lisa	41	27.3%

Next, calculate the degree measure of the central angle of the "pie" wedge for each candidate. In the case of Mercy:

$$\begin{aligned}
\text{Mercy's degrees} &= 37.3\% \cdot 360° \\
&= 0.373 \cdot 360° \\
&= 134.28°
\end{aligned}$$

In similar fashion, calculate the degree measure of the central angle for each of the remaining candidates.

Candidate	Votes	Percent	Degrees
Mercy	56	37.3%	134.28°
Hans	53	35.3%	127.08°
Lisa	41	27.3%	98.28°

Finally, take a protractor and measure three sectors with central angles from the Degrees column of the last table. Annotate each sector with the candidate's name and the percentage of votes received.

25. The first task is to total the votes.

Candidate	Votes
Raven	43
Mabel	40
Bernardo	52
Totals	135

The next task is to express each quantity of votes as a percentage of the total votes. For example, Raven received 43 out of 135 votes.

Raven's votes	is	what percent	of	total votes
43	=	p	·	135

Solve for p.

$$43 = 135p \qquad \text{Commutative property.}$$
$$\frac{43}{135} = \frac{135p}{135} \qquad \text{Divide both sides by 135.}$$
$$0.3185 \approx p \qquad \text{Divide: } 43/135 = 0.3185.$$

To the nearest tenth of a percent, Raven received 31.9% of the total votes. In similar fashion, calculate the percentage of votes earned by the remaining two candidates.

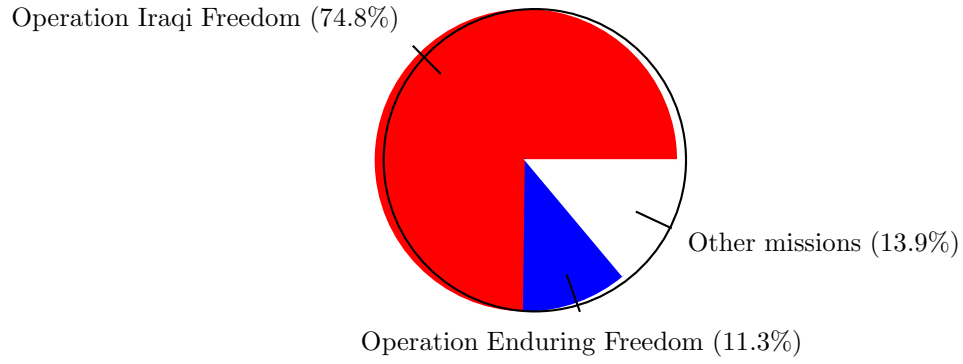| Candidate	Votes	Percent
Raven	43	31.9%
Mabel	40	29.6%
Bernardo	52	38.5%

Next, calculate the degree measure of the central angle of the "pie" wedge for each candidate. In the case of Raven:

$$\text{Raven's degrees} = 31.9\% \cdot 360°$$
$$= 0.319 \cdot 360°$$
$$= 114.84°$$

In similar fashion, calculate the degree measure of the central angle for each of the remaining candidates.

Candidate	Votes	Percent	Degrees
Raven	43	31.9%	114.84°
Mabel	40	29.6%	106.56°
Bernardo	52	38.5%	138.6°

Finally, take a protractor and measure three sectors with central angles from the Degrees column of the last table. Annotate each sector with the candidate's name and the percentage of votes received.

27. The first task is to total the votes.

Candidate	Votes
Jun	57
Lisa	30
Aisha	58
Totals	145

The next task is to express each quantity of votes as a percentage of the total votes. For example, Jun received 57 out of 145 votes.

Jun's votes	is	what percent	of	total votes
57	=	p	·	145

Solve for p.

$$57 = 145p \qquad \text{Commutative property.}$$
$$\frac{57}{145} = \frac{145p}{145} \qquad \text{Divide both sides by 145.}$$
$$0.3931 \approx p \qquad \text{Divide: } 57/145 = 0.3931.$$

To the nearest tenth of a percent, Jun received 39.3% of the total votes. In similar fashion, calculate the percentage of votes earned by the remaining two candidates.

Candidate	Votes	Percent
Jun	57	39.3%
Lisa	30	20.7%
Aisha	58	40.0%

Next, calculate the degree measure of the central angle of the "pie" wedge for each candidate. In the case of Jun:

$$\text{Jun's degrees} = 39.3\% \cdot 360°$$
$$= 0.393 \cdot 360°$$
$$= 141.48°$$

In similar fashion, calculate the degree measure of the central angle for each of the remaining candidates.

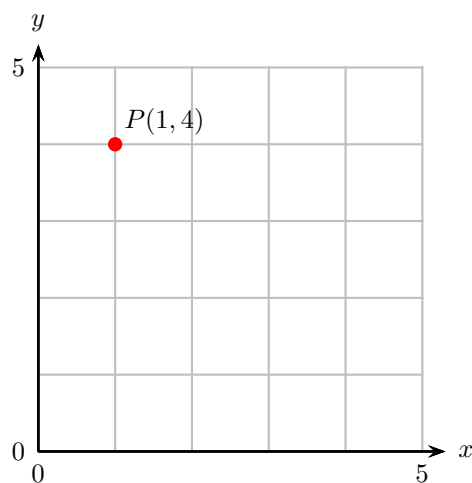| Candidate	Votes	Percent	Degrees
Jun	57	39.3%	141.48°
Lisa	30	20.7%	74.52°
Aisha	58	40.0%	144°

Finally, take a protractor and measure three sectors with central angles from the Degrees column of the last table. Annotate each sector with the candidate's name and the percentage of votes received.
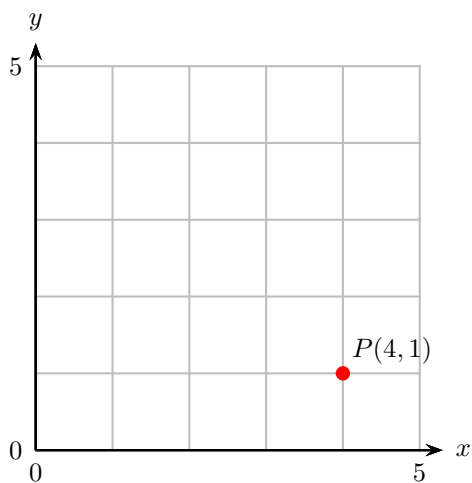
29. The first task is to total the votes.

Candidate	Votes
Henry	35
Bernardo	32
Estevan	47
Totals	114

The next task is to express each quantity of votes as a percentage of the total votes. For example, Henry received 35 out of 114 votes.

Henry's votes	is	what percent	of	total votes
35	$=$	p	$\cdot$	114

Solve for p.

$$35 = 114p \qquad \text{Commutative property.}$$
$$\frac{35}{114} = \frac{114p}{114} \qquad \text{Divide both sides by 114.}$$
$$0.3070 \approx p \qquad \text{Divide: } 35/114 = 0.3070.$$

To the nearest tenth of a percent, Henry received 30.7% of the total votes. In similar fashion, calculate the percentage of votes earned by the remaining two candidates.

Candidate	Votes	Percent
Henry	35	30.7%
Bernardo	32	28.1%
Estevan	47	41.2%

Next, calculate the degree measure of the central angle of the "pie" wedge for each candidate. In the case of Henry:

$$\text{Henry's degrees} = 30.7\% \cdot 360°$$
$$= 0.307 \cdot 360°$$
$$= 110.52°$$

In similar fashion, calculate the degree measure of the central angle for each of the remaining candidates.

Candidate	Votes	Percent	Degrees
Henry	35	30.7%	110.52°
Bernardo	32	28.1%	101.16°
Estevan	47	41.2%	148.32°

Finally, take a protractor and measure three sectors with central angles from the Degrees column of the last table. Annotate each sector with the candidate's name and the percentage of votes received.

31. The first task is to find the total number of troops deployed.

Mission	Troops
Operation Iraqi Freedom	193,598
Operation Enduring Freedom (Afghanistan)	29,212
Other missions	35,849
Totals	258,659

The next task is to express the number of troops deployed for each mission as a percentage of the total number of troops. For example, Operation Iraqi Freedom deployed 193,598 out of the total 258,659 troops.

Iraq mission troops	is	what percent	of	total mission troops
193,598	=	p	·	258,659

Solve for p.

$$193,598 = 258,659p \qquad \text{Commutative property.}$$

$$\frac{193,598}{258,659} = \frac{258,659p}{258,659} \qquad \text{Divide both sides by 258,659.}$$

$$0.7484 \approx p \qquad \text{Divide: } 193,598/258,659 = 0.7484.$$

To the nearest tenth of a percent, 74.8% of the total troops were deployed to the Iraq mission. In similar fashion, calculate the percentage of troops distributed among the remaining missions.

Candidate	Votes	Percent
Operation Iraqi Freedom	193,598	74.8%
Operation Enduring Freedom (Afghanistan)	29,212	11.3%
Other missions	35,849	13.9%

Next, calculate the degree measure of the central angle of the "pie" wedge for each candidate. In the case of troops deployed to Iraq:

$$\begin{aligned} \text{Iraq troops degrees} &= 74.8\% \cdot 360° \\ &= 0.748 \cdot 360° \\ &= 269.28° \end{aligned}$$

In similar fashion, calculate the degree measure of the central angle for each of the remaining missions.

Mission	Troops	Percent	Degrees
Operation Iraqi Freedom	193,598	74.8%	269.28°
Operation Enduring Freedom (Afghanistan)	29,212	11.3%	40.68°
Other missions	35,849	13.9%	50.04°

Finally, take a protractor and measure three sectors with central angles from the Degrees column of the last table. Annotate each sector with the mission's name and the percentage of troops deployed.

Operation Iraqi Freedom (74.8%)

Other missions (13.9%)

Operation Enduring Freedom (11.3%)

Chapter 8

Graphing

8.1   The Cartesian Coordinate System

1. Starting from the origin (the point $(0,0)$), the point P is located 1 unit to the right and 4 units up. Hence, the coordinates of point P are $(1,4)$.

3. Starting from the origin (the point $(0,0)$), the point P is located 4 units to the right and 1 unit up. Hence, the coordinates of point P are $(4,1)$.

5. Starting from the origin (the point $(0,0)$), the point P is located 3 units to the left and 1 unit down. Hence, the coordinates of point P are $(-3,-1)$.

7. Starting from the origin (the point $(0,0)$), the point P is located 1 unit to the left and 1 unit up. Hence, the coordinates of point P are $(-1,1)$.

9. Plot the points $A(-1,1)$, $B(1,1)$, $C(1,2)$, and $D(-1,2)$ and draw the rectangle.

Note that the length of the rectangle is 2 units and the width is 1 unit. Hence, the area is

$$A = LW \qquad \text{Area formula for a rectangle.}$$
$$A = (2)(1) \qquad \text{Substitute 2 for } L \text{ and 1 for } W.$$
$$A = 2. \qquad \text{Multiply.}$$

Hence, the area is $A = 2$ square units.

Alternate solution. Note that each square in the grid represents one square unit. Hence, you can also find the area of the square $ABCD$ by counting the number of square units inside the rectangle. There is 1 row of 2 squares. Hence, the area of the square is $A = 2$ square units.

11. Plot the points $A(-2,-1)$, $B(3,-1)$, $C(3,3)$, and $D(-2,3)$ and draw the rectangle.

Note that the length of the rectangle is 5 units and the width is 4 units. Hence, the area is

$$A = LW \qquad \text{Area formula for a rectangle.}$$
$$A = (5)(4) \qquad \text{Substitute 5 for } L \text{ and 4 for } W.$$
$$A = 20. \qquad \text{Multiply.}$$

Hence, the area is $A = 20$ square units.

Alternate solution. Note that each square in the grid represents one square unit. Hence, you can also find the area of the square $ABCD$ by counting the number of square units inside the rectangle. There are 4 rows of 5 squares. Hence, the area of the square is $A = 20$ square units.

13. Plot the points $A(-4, -2)$, $B(1, -2)$, $C(1, 1)$, and $D(-4, 1)$ and draw the rectangle.

Note that the length of the rectangle is 5 units and the width is 3 units. Hence, the perimeter is

$$
\begin{array}{ll}
P = 2L + 2W & \text{Perimeter formula for a rectangle.} \\
P = 2(5) + 2(3) & \text{Substitute 5 for } L \text{ and 3 for } W. \\
P = 10 + 6 & \text{Multiply.} \\
P = 16. & \text{Add.}
\end{array}
$$

Hence, the perimeter is $P = 16$ units.

15. Plot the points $A(-1,2)$, $B(3,2)$, $C(3,3)$, and $D(-1,3)$ and draw the rectangle.

Note that the length of the rectangle is 4 units and the width is 1 unit. Hence, the perimeter is

$$P = 2L + 2W \qquad \text{Perimeter formula for a rectangle.}$$
$$P = 2(4) + 2(1) \qquad \text{Substitute 4 for } L \text{ and 1 for } W.$$
$$P = 8 + 2 \qquad \text{Multiply.}$$
$$P = 10. \qquad \text{Add.}$$

Hence, the perimeter is $P = 10$ units.

17. Plot the points $A(-3, -1)$, $B(1, -1)$, and $C(-3, 0)$ and draw the triangle.

Note that the triangle is a right triangle, with base $b = 4$ units and hieght $h = 1$ unit. . Hence, the area is

$$A = \frac{1}{2}bh \qquad \text{Area formula for a triangle.}$$

$$A = \frac{1}{2} \cdot 4 \cdot 1 \qquad \text{Substitute 4 for } b \text{ and 1 for } h.$$

$$A = 2 \qquad \text{Multiply.}$$

Therefore, the area is $A = 2$ square units.

19. Plot the points $A(-1, -2)$, $B(0, -2)$, and $C(-1, 0)$ and draw the triangle.

Note that the triangle is a right triangle, with base $b = 1$ unit and hieght $h = 2$ units. . Hence, the area is

$$A = \frac{1}{2}bh \qquad\qquad \text{Area formula for a triangle.}$$

$$A = \frac{1}{2} \cdot 1 \cdot 2 \qquad\qquad \text{Substitute 1 for } b \text{ and 2 for } h.$$

$$A = 1 \qquad\qquad \text{Multiply.}$$

Therefore, the area is $A = 1$ square unit.

21. Plot the points $A(-3, -3)$, $B(0, 0)$. We'll use the Pythagorean Theorem to find the distance between these two points. First, draw a right triangle having legs parallel to the coordinate axes. The hypotenuse is the requested distance between the points A and B.

Note that the triangle is a right triangle, with one leg having length 3 and the second leg having length 3. Let d represent the length of the hypotenuse and the distance between the points A and B. Then, by the Pythagorean Theorem,

$$d^2 = 3^2 + 3^2 \qquad \text{Pythagorean Theorem.}$$
$$d^2 = 9 + 9 \qquad \text{Square first.}$$
$$d^2 = 18 \qquad \text{Add.}$$
$$d = \sqrt{18} \qquad \text{Take the square root.}$$

Therefore, the distance between points $A(-3, -3)$ and $B(0, 0)$ is $\sqrt{18}$.

23. Plot the points $A(-2, -3)$, $B(0, 0)$. We'll use the Pythagorean Theorem to find the distance between these two points. First, draw a right triangle having legs parallel to the coordinate axes. The hypotenuse is the requested distance between the points A and B.

Note that the triangle is a right triangle, with one leg having length 2 and the second leg having length 3. Let d represent the length of the hypotenuse and the distance between the points A and B. Then, by the Pythagorean Theorem,

$$
\begin{aligned}
d^2 &= 2^2 + 3^2 && \text{Pythagorean Theorem.} \\
d^2 &= 4 + 9 && \text{Square first.} \\
d^2 &= 13 && \text{Add.} \\
d &= \sqrt{13} && \text{Take the square root.}
\end{aligned}
$$

Therefore, the distance between points $A(-2, -3)$ and $B(0, 0)$ is $\sqrt{13}$.

25. Finding the area directly would be difficult. Instead, let's try a "backdoor" and note that we can find the area of the shaded triangle in two steps: (1) Find the area of the three right triangles that bound the the shaded triangle, then (2) subtract the result from the area of the rectangle.

Thus:

i) The area of $\triangle ABG$ is:

$$\text{Area}\triangle ABG = \frac{1}{2}(5)(2)$$
$$= 5$$

(ii) The area of $\triangle BCF$ is

$$\text{Area}\triangle BCF = \frac{1}{2}(4)(4)$$
$$= 8$$

(iii) The area of $\triangle FDG$ is

$$\text{Area}\triangle FDG = \frac{1}{2}(1)(2)$$
$$= 1$$

The sum of the areas of these three triangles is:

$$\text{sum of triangles} = 5 + 8 + 1$$
$$= 14$$

The area of the rectangle $ABCD$ is:

$$\text{Area}ABCD = (5)(4)$$
$$= 20.$$

Hence, the area of the shaded triangle is

$$\text{Area Shaded Triangle} = \text{Area of Rectangle} - \text{Sum of Three Triangles}$$
$$= 20 - 14$$
$$= 6.$$

Hence, the area of the shaded triangle is 6.

27. Finding the area directly would be difficult. Instead, let's try a "backdoor" and note that we can find the area of the shaded triangle in two steps: (1) Find the area of the three right triangles that bound the the shaded triangle, then (2) subtract the result from the area of the rectangle.

Thus:

i) The area of $\triangle ABG$ is:

$$\text{Area}\triangle ABG = \frac{1}{2}(5)(3)$$
$$= \frac{15}{2}$$

(ii) The area of $\triangle BCF$ is

$$\text{Area}\triangle BCF = \frac{1}{2}(4)(2)$$
$$= 4$$

(iii) The area of $\triangle FDG$ is

$$\text{Area}\triangle FDG = \frac{1}{2}(3)(1)$$
$$= \frac{3}{2}$$

The sum of the areas of these three triangles is:

$$\text{sum of triangles} = \frac{15}{2} + 4 + \frac{3}{2}$$
$$= 13$$

The area of the rectangle $ABCD$ is:

$$\text{Area}ABCD = (5)(4)$$
$$= 20.$$

Hence, the area of the shaded triangle is

$$\text{Area Shaded Triangle} = \text{Area of Rectangle} - \text{Sum of Three Triangles}$$
$$= 20 - 13$$
$$= 7.$$

Hence, the area of the shaded triangle is 7.

29.   Draw the triangle $\triangle ABC$ with vertices at $A(-4,-1)$, $B(4,-2)$, and $C(1,3)$.

Surround the triangle with a rectangle like the one shown in the following figure.

Calculate the areas of the three bounding right triangles.

i) Area of triangle $\triangle ADB$.

$$\triangle ADB = \frac{1}{2} \cdot 8 \cdot 1$$
$$= 4$$

ii) Area of triangle $\triangle BFC$.

$$\triangle BFC = \frac{1}{2} \cdot 5 \cdot 3$$
$$= \frac{15}{2}$$

iii) Area of triangle $\triangle CGA$.

$$\triangle CGA = \frac{1}{2} \cdot 5 \cdot 4$$
$$= 10$$

The sum of the three bounding right triangles is

$$\text{Sum of triangles} = 4 + \frac{15}{2} + 10$$
$$= \frac{43}{2}$$

The area of the containing rectangle is 40.

$$\text{Area of rectangle} = 8 \cdot 5$$
$$= 40$$

Therefore, the area of the triangle through the points $A(-4, -1)$, $B(4, -2)$, and $C(1, 3)$ is found by subtracting the sum of the three bounding right triangles from the containing rectangle.

$$\text{Area } \triangle ABC = \text{Area of Rectangle} - \text{Sum of Areas of Bounding Triangles}$$
$$= 40 - \frac{43}{2}$$
$$= \frac{37}{2}$$

31. Draw the triangle $\triangle ABC$ with vertices at $A(-3, 1)$, $B(3, -3)$, and $C(1, 4)$.

Surround the triangle with a rectangle like the one shown in the following figure.

Calculate the areas of the three bounding right triangles.

i)   Area of triangle $\triangle ADB$.

$$\triangle ADB = \frac{1}{2} \cdot 6 \cdot 4$$
$$= 12$$

ii) Area of triangle $\triangle BFC$.

$$\triangle BFC = \frac{1}{2} \cdot 7 \cdot 2$$
$$= 7$$

iii) Area of triangle $\triangle CGA$.

$$\triangle CGA = \frac{1}{2} \cdot 4 \cdot 3$$
$$= 6$$

The sum of the three bounding right triangles is

$$\text{Sum of triangles} = 12 + 7 + 6$$
$$= 25$$

The area of the containing rectangle is 42.

$$\text{Area of rectangle} = 6 \cdot 7$$
$$= 42$$

Therefore, the area of the triangle through the points $A(-3, 1)$, $B(3, -3)$, and $C(1, 4)$ is found by subtracting the sum of the three bounding right triangles from the containing rectangle.

$$\text{Area } \triangle ABC = \text{Area of Rectangle} - \text{Sum of Areas of Bounding Triangles}$$
$$= 42 - 25$$
$$= 17$$

8.2   Graphing Linear Equations

1. Substitute $(x, y) = (-1, -6)$ into the equation $y = -2x - 8$.

$$
\begin{aligned}
y &= -2x - 8 && \text{Original equation.}\\
-6 &= -2(-1) - 8 && \text{Substitute: } -1 \text{ for } x, -6 \text{ for } y.\\
-6 &= 2 - 8 && \text{Multiply: } -2(-1) = 2.\\
-6 &= -6 && \text{Subtract: } 2 - 8 = -6.
\end{aligned}
$$

Because this last statement is a true statement, $(-1, -6)$ satisfies (is a solution of) the equation $y = -2x - 8$.

For contrast, consider the point $(3, -13)$.

$$
\begin{aligned}
y &= -2x - 8 && \text{Original equation.} \\
-13 &= -2(3) - 8 && \text{Substitute: } 3 \text{ for } x, -13 \text{ for } y. \\
-13 &= -6 - 8 && \text{Multiply: } -2(3) = -6. \\
-13 &= -14 && \text{Subtract: } -6 - 8 = -14.
\end{aligned}
$$

Note that this last statement is false. Hence, the pair $(3, -13)$ is **not** a solution of $y = -2x - 8$. In similar fashion, readers should also check that the remaining two points are **not** solutions.

3. Substitute $(x, y) = (-4, 31)$ into the equation $y = -6x + 7$.

$$
\begin{aligned}
y &= -6x + 7 && \text{Original equation.} \\
31 &= -6(-4) + 7 && \text{Substitute: } -4 \text{ for } x, 31 \text{ for } y. \\
31 &= 24 + 7 && \text{Multiply: } -6(-4) = 24. \\
31 &= 31 && \text{Add: } 24 + 7 = 31.
\end{aligned}
$$

Because this last statement is a true statement, $(-4, 31)$ satisfies (is a solution of) the equation $y = -6x + 7$.

For contrast, consider the point $(-2, 20)$.

$$
\begin{aligned}
y &= -6x + 7 && \text{Original equation.} \\
20 &= -6(-2) + 7 && \text{Substitute: } -2 \text{ for } x, 20 \text{ for } y. \\
20 &= 12 + 7 && \text{Multiply: } -6(-2) = 12. \\
20 &= 19 && \text{Add: } 12 + 7 = 19.
\end{aligned}
$$

Note that this last statement is false. Hence, the pair $(-2, 20)$ is **not** a solution of $y = -6x + 7$. In similar fashion, readers should also check that the remaining two points are **not** solutions.

5. Substitute $(x, y) = (2, 15)$ into the equation $y = 9x - 3$.

$$
\begin{aligned}
y &= 9x - 3 && \text{Original equation.} \\
15 &= 9(2) - 3 && \text{Substitute: } 2 \text{ for } x, 15 \text{ for } y. \\
15 &= 18 - 3 && \text{Multiply: } 9(2) = 18. \\
15 &= 15 && \text{Subtract: } 18 - 3 = 15.
\end{aligned}
$$

Because this last statement is a true statement, $(2, 15)$ satisfies (is a solution of) the equation $y = 9x - 3$.

For contrast, consider the point $(-8, -74)$.

$$
\begin{aligned}
y &= 9x - 3 && \text{Original equation.} \\
-74 &= 9(-8) - 3 && \text{Substitute: } -8 \text{ for } x, -74 \text{ for } y. \\
-74 &= -72 - 3 && \text{Multiply: } 9(-8) = -72. \\
-74 &= -75 && \text{Subtract: } -72 - 3 = -75.
\end{aligned}
$$

Note that this last statement is false. Hence, the pair $(-8, -74)$ is **not** a solution of $y = 9x - 3$. In similar fashion, readers should also check that the remaining two points are **not** solutions.

7. Substitute $(x, y) = (-2, 14)$ into the equation $y = -5x + 4$.

$$
\begin{aligned}
y &= -5x + 4 && \text{Original equation.} \\
14 &= -5(-2) + 4 && \text{Substitute: } -2 \text{ for } x, 14 \text{ for } y. \\
14 &= 10 + 4 && \text{Multiply: } -5(-2) = 10. \\
14 &= 14 && \text{Add: } 10 + 4 = 14.
\end{aligned}
$$

Because this last statement is a true statement, $(-2, 14)$ satisfies (is a solution of) the equation $y = -5x + 4$.

For contrast, consider the point $(3, -10)$.

$$
\begin{aligned}
y &= -5x + 4 && \text{Original equation.} \\
-10 &= -5(3) + 4 && \text{Substitute: } 3 \text{ for } x, -10 \text{ for } y. \\
-10 &= -15 + 4 && \text{Multiply: } -5(3) = -15. \\
-10 &= -11 && \text{Add: } -15 + 4 = -11.
\end{aligned}
$$

Note that this last statement is false. Hence, the pair $(3, -10)$ is **not** a solution of $y = -5x + 4$. In similar fashion, readers should also check that the remaining two points are **not** solutions.

9. Substitute $(x, y) = (9, k)$ into the equation $y = -6x + 1$.

$$
\begin{aligned}
y &= -6x + 1 && \text{Original equation.} \\
k &= -6(9) + 1 && \text{Substitute: } 9 \text{ for } x, k \text{ for } y. \\
k &= -54 + 1 && \text{Multiply: } -6(9) = -54. \\
k &= -53 && \text{Add: } -54 + 1 = -53.
\end{aligned}
$$

Thus, $k = -53$.

11. Substitute $(x, y) = (k, 7)$ into the equation $y = -4x + 1$.

$$
\begin{aligned}
y &= -4x + 1 && \text{Original equation.} \\
7 &= -4(k) + 1 && \text{Substitute: } k \text{ for } x,\ 7 \text{ for } y. \\
7 - 1 &= -4k + 1 - 1 && \text{Subtract 1 from both sides.} \\
6 &= -4k && \text{Simplify: } 7 - 1 = 6. \\
\frac{6}{-4} &= \frac{-4k}{-4} && \text{Divide both sides by } -4. \\
-\frac{3}{2} &= k && \text{Reduce.}
\end{aligned}
$$

Thus, $k = -3/2$.

13. Substitute $(x, y) = (k, 1)$ into the equation $y = 4x + 8$.

$$
\begin{aligned}
y &= 4x + 8 && \text{Original equation.} \\
1 &= 4(k) + 8 && \text{Substitute: } k \text{ for } x,\ 1 \text{ for } y. \\
1 - 8 &= 4k + 8 - 8 && \text{Subtract 8 from both sides.} \\
-7 &= 4k && \text{Simplify: } 1 - 8 = -7. \\
\frac{-7}{4} &= \frac{4k}{4} && \text{Divide both sides by 4.} \\
-\frac{7}{4} &= k && \text{Simplify.}
\end{aligned}
$$

Thus, $k = -7/4$.

15. Substitute $(x, y) = (-1, k)$ into the equation $y = -5x + 3$.

$$
\begin{aligned}
y &= -5x + 3 && \text{Original equation.} \\
k &= -5(-1) + 3 && \text{Substitute: } -1 \text{ for } x,\ k \text{ for } y. \\
k &= 5 + 3 && \text{Multiply: } -5(-1) = 5. \\
k &= 8 && \text{Add: } 5 + 3 = 8.
\end{aligned}
$$

Thus, $k = 8$.

17. An equation is linear if and only if it has the form $y = mx + b$. Of the offered choices, only $y = 6x + 4$ has this form, where $m = 6$ and $b = 4$. The remaining choices do not have the form $y = mx + b$, so they are not linear equations.

19. An equation is linear if and only if it has the form $y = mx + b$. Of the offered choices, only $y = x + 7$ has this form, where $m = 1$ and $b = 7$. The remaining choices do not have the form $y = mx + b$, so they are not linear equations.

21. An equation is linear if and only if it has the form $y = mx + b$. The equation $y = -2x - 2$ can be written

$$y = -2x + (-2),$$

which has the form $y = mx + b$, where $m = -2$ and $b = -2$. Hence, this equation is linear. The remaining choices do not have the form $y = mx + b$, so they are not linear equations.

23. An equation is linear if and only if it has the form $y = mx + b$. The equation $y = 7x - 3$ can be written

$$y = 7x + (-3),$$

which has the form $y = mx + b$, where $m = 7$ and $b = -3$. Hence, this equation is linear. The remaining choices do not have the form $y = mx + b$, so they are not linear equations.

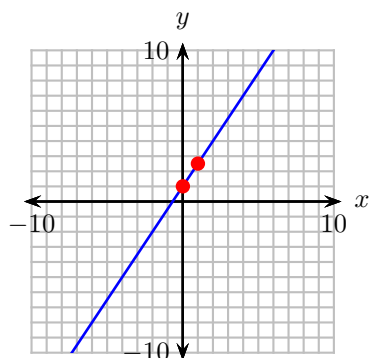**25.** Consider the equation $y = -3x + 1$. Let's calculate two points that satisfy this equation. First, substitute 0 for x.

$$\begin{aligned} y &= -3x + 1 \\ &= -3(0) + 1 \\ &= 1 \end{aligned}$$

This computation tells us that $(0, 1)$ satisfies the linear equation $y = -3x + 1$ and is a point on the graph. Now, let's substitute 1 for x.

$$\begin{aligned} y &= -3x + 1 \\ &= -3(1) + 1 \\ &= -2 \end{aligned}$$

This computation tells us that $(1, -2)$ satisfies the linear equation $y = -3x + 1$ and is a point on the graph.

The equation $y = -3x + 1$ is linear. We need only plot our two points $(0, 1)$ and $(1, -2)$ and draw a line through them. The result follows.

Note that this line is identical to the given graph.

For contrast, consider the equation $y = -\frac{3}{2}x + 2$. Let's calculate two points that satisfy this equation. First, substitute 0 for x.
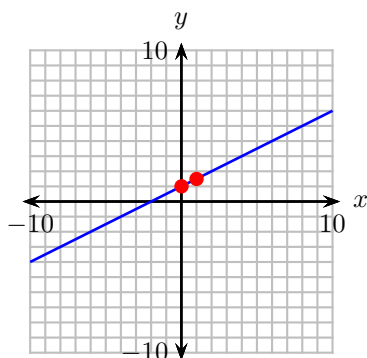
$$
\begin{aligned}
y &= -\frac{3}{2}x + 2 \\
&= -\frac{3}{2}(0) + 2 \\
&= 2
\end{aligned}
$$

This computation tells us that $(0, 2)$ satisfies the linear equation $y = -\frac{3}{2}x + 2$ and is a point on the graph. Now, let's substitute 1 for x.

$$
\begin{aligned}
y &= -\frac{3}{2}x + 2 \\
&= -\frac{3}{2}(1) + 2 \\
&= \frac{1}{2}
\end{aligned}
$$

This computation tells us that $(1, 1/2)$ satisfies the linear equation $y = -\frac{3}{2}x + 2$ and is a point on the graph.

The equation $y = -\frac{3}{2}x + 2$ is linear. We need only plot our two points $(0, 2)$ and $(1, 1/2)$ and draw a line through them. The result follows.

Note that this line differs from the given graph. Readers should use the same procedure to show that the lines produced by the remaining two equations are also different from the original.
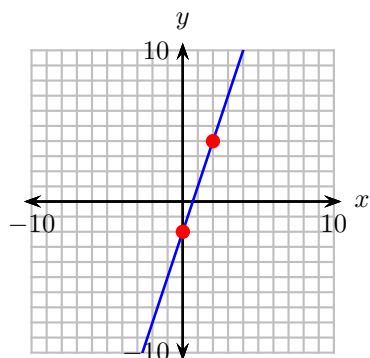
27. Consider the equation $y = \frac{3}{2}x + 1$. Let's calculate two points that satisfy this equation. First, substitute 0 for x.

$$y = \frac{3}{2}x + 1$$
$$= \frac{3}{2}(0) + 1$$
$$= 1$$

This computation tells us that $(0, 1)$ satisfies the linear equation $y = \frac{3}{2}x + 1$ and is a point on the graph. Now, let's substitute 1 for x.

$$y = \frac{3}{2}x + 1$$
$$= \frac{3}{2}(1) + 1$$
$$= \frac{5}{2}$$

This computation tells us that $(1, 5/2)$ satisfies the linear equation $y = \frac{3}{2}x + 1$ and is a point on the graph.

The equation $y = \frac{3}{2}x + 1$ is linear. We need only plot our two points $(0, 1)$ and $(1, 5/2)$ and draw a line through them. The result follows.

Note that this line is identical to the given graph.

For contrast, consider the equation $y = \frac{1}{2}x + 1$. Let's calculate two points that satisfy this equation. First, substitute 0 for x.

$$\begin{aligned} y &= \frac{1}{2}x + 1 \\ &= \frac{1}{2}(0) + 1 \\ &= 1 \end{aligned}$$

This computation tells us that $(0, 1)$ satisfies the linear equation $y = \frac{1}{2}x + 1$ and is a point on the graph. Now, let's substitute 1 for x.

$$\begin{aligned} y &= \frac{1}{2}x + 1 \\ &= \frac{1}{2}(1) + 1 \\ &= \frac{3}{2} \end{aligned}$$

This computation tells us that $(1, 3/2)$ satisfies the linear equation $y = \frac{1}{2}x + 1$ and is a point on the graph.

The equation $y = \frac{1}{2}x + 1$ is linear. We need only plot our two points $(0, 1)$ and $(1, 3/2)$ and draw a line through them. The result follows.

Note that this line differs from the given graph. Readers should use the same procedure to show that the lines produced by the remaining two equations are also different from the original.
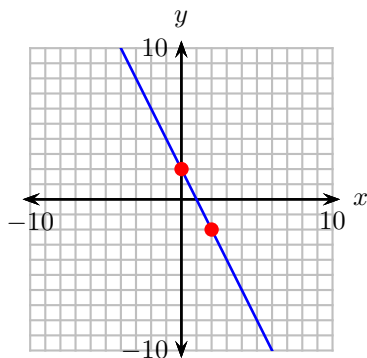
29. The equation $y = 3x - 2$ is linear. Thus, to plot the equation, we need only calculate two points that satisfy the equation, plot them, then draw a line through them. First, substitute 0 for x.

$$y = 3x - 2$$
$$= 3(0) - 2$$
$$= -2$$

This computation tells us that $(0, -2)$ satisfies the linear equation $y = 3x - 2$ and is a point on the graph. Now, let's substitute 2 for x.

$$y = 3x - 2$$
$$= 3(2) - 2$$
$$= 4$$

This computation tells us that $(2, 4)$ satisfies the linear equation $y = 3x - 2$ and is a point on the graph. Finally, plot these points and draw a line through them. The graph of $y = 3x - 2$ follows.
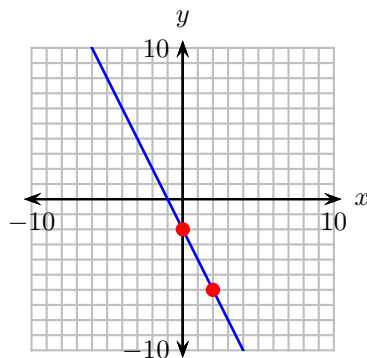
31. The equation $y = -2x - 1$ is linear. Thus, to plot the equation, we need only calculate two points that satisfy the equation, plot them, then draw a line through them. First, substitute 0 for x.

$$y = -2x - 1$$
$$= -2(0) - 1$$
$$= -1$$

This computation tells us that $(0, -1)$ satisfies the linear equation $y = -2x - 1$ and is a point on the graph. Now, let's substitute 2 for x.

$$y = -2x - 1$$
$$= -2(2) - 1$$
$$= -5$$

This computation tells us that $(2, -5)$ satisfies the linear equation $y = -2x - 1$ and is a point on the graph. Finally, plot these points and draw a line through them. The graph of $y = -2x - 1$ follows.
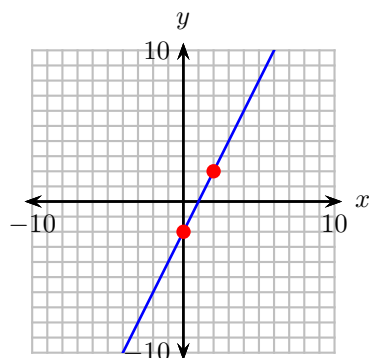
33. The equation $y = -2x + 2$ is linear. Thus, to plot the equation, we need only calculate two points that satisfy the equation, plot them, then draw a line through them. First, substitute 0 for x.

$$
\begin{aligned}
y &= -2x + 2 \\
&= -2(0) + 2 \\
&= 2
\end{aligned}
$$

This computation tells us that $(0, 2)$ satisfies the linear equation $y = -2x + 2$ and is a point on the graph. Now, let's substitute 2 for x.

$$
\begin{aligned}
y &= -2x + 2 \\
&= -2(2) + 2 \\
&= -2
\end{aligned}
$$

This computation tells us that $(2, -2)$ satisfies the linear equation $y = -2x + 2$ and is a point on the graph. Finally, plot these points and draw a line through them. The graph of $y = -2x + 2$ follows.
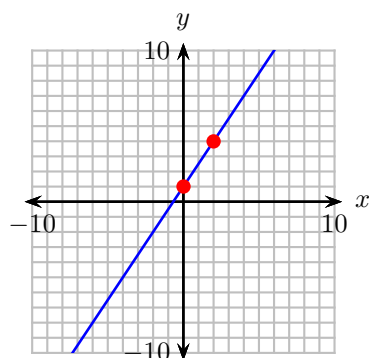

35. The equation $y = -2x - 2$ is linear. Thus, to plot the equation, we need only calculate two points that satisfy the equation, plot them, then draw a line through them. First, substitute 0 for x.

$$
\begin{aligned}
y &= -2x - 2 \\
&= -2(0) - 2 \\
&= -2
\end{aligned}
$$

This computation tells us that $(0, -2)$ satisfies the linear equation $y = -2x - 2$ and is a point on the graph. Now, let's substitute 2 for x.

$$
\begin{aligned}
y &= -2x - 2 \\
&= -2(2) - 2 \\
&= -6
\end{aligned}
$$

This computation tells us that $(2, -6)$ satisfies the linear equation $y = -2x - 2$ and is a point on the graph. Finally, plot these points and draw a line through them. The graph of $y = -2x - 2$ follows.
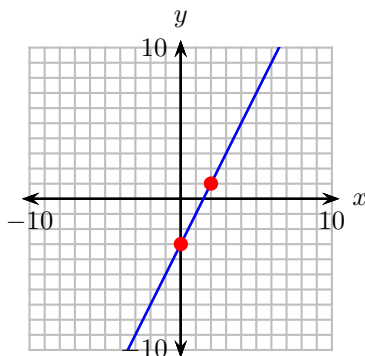

37. The equation $y = 2x - 2$ is linear. Thus, to plot the equation, we need only calculate two points that satisfy the equation, plot them, then draw a line through them. First, substitute 0 for x.

$$
\begin{aligned}
y &= 2x - 2 \\
&= 2(0) - 2 \\
&= -2
\end{aligned}
$$

This computation tells us that $(0, -2)$ satisfies the linear equation $y = 2x - 2$ and is a point on the graph. Now, let's substitute 2 for x.

$$
\begin{aligned}
y &= 2x - 2 \\
&= 2(2) - 2 \\
&= 2
\end{aligned}
$$

This computation tells us that $(2, 2)$ satisfies the linear equation $y = 2x - 2$ and is a point on the graph. Finally, plot these points and draw a line through them. The graph of $y = 2x - 2$ follows.
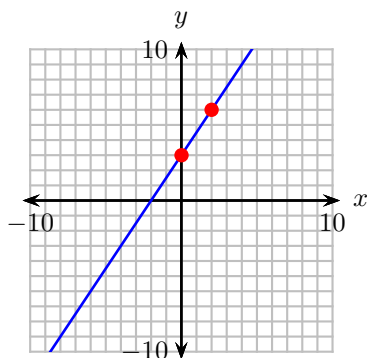
39. The equation $y = \frac{3}{2}x + 1$ is linear. Thus, to plot the equation, we need only calculate two points that satisfy the equation, plot them, then draw a line through them. First, substitute 0 for x.

$$y = \frac{3}{2}x + 1$$
$$= \frac{3}{2}(0) + 1$$
$$= 1$$

This computation tells us that $(0, 1)$ satisfies the linear equation $y = \frac{3}{2}x + 1$ and is a point on the graph. Now, let's substitute 2 for x.

$$y = \frac{3}{2}x + 1$$
$$= \frac{3}{2}(2) + 1$$
$$= 4$$

This computation tells us that $(2, 4)$ satisfies the linear equation $y = \frac{3}{2}x + 1$ and is a point on the graph. Finally, plot these points and draw a line through them. The graph of $y = \frac{3}{2}x + 1$ follows.

41. The equation $y = 2x - 3$ is linear. Thus, to plot the equation, we need only calculate two points that satisfy the equation, plot them, then draw a line through them. First, substitute 0 for x.

$$\begin{aligned} y &= 2x - 3 \\ &= 2(0) - 3 \\ &= -3 \end{aligned}$$

This computation tells us that $(0, -3)$ satisfies the linear equation $y = 2x - 3$ and is a point on the graph. Now, let's substitute 2 for x.

$$\begin{aligned} y &= 2x - 3 \\ &= 2(2) - 3 \\ &= 1 \end{aligned}$$

This computation tells us that $(2, 1)$ satisfies the linear equation $y = 2x - 3$ and is a point on the graph. Finally, plot these points and draw a line through them. The graph of $y = 2x - 3$ follows.

43. The equation $y = \frac{3}{2}x + 3$ is linear. Thus, to plot the equation, we need only calculate two points that satisfy the equation, plot them, then draw a line through them. First, substitute 0 for x.

$$\begin{aligned} y &= \frac{3}{2}x + 3 \\ &= \frac{3}{2}(0) + 3 \\ &= 3 \end{aligned}$$

This computation tells us that $(0, 3)$ satisfies the linear equation $y = \frac{3}{2}x + 3$ and is a point on the graph. Now, let's substitute 2 for x.

$$y = \frac{3}{2}x + 3$$
$$= \frac{3}{2}(2) + 3$$
$$= 6$$

This computation tells us that $(2, 6)$ satisfies the linear equation $y = \frac{3}{2}x + 3$ and is a point on the graph. Finally, plot these points and draw a line through them. The graph of $y = \frac{3}{2}x + 3$ follows.

45. The equations $y = \frac{1}{2}x - 1$ and $y = \frac{5}{2}x - 2$ are linear. Thus, to plot these equations, we need only calculate two points that satisfy each equation, plot them, then draw a line through them. First, substitute 0 for x in $y = \frac{1}{2}x - 1$.

$$y = \frac{1}{2}x - 1$$
$$= \frac{1}{2}(0) - 1$$
$$= -1$$

This computation tells us that $(0, -1)$ satisfies the linear equation $y = \frac{1}{2}x - 1$ and is a point on its graph. Now, let's substitute 2 for x in $y = \frac{1}{2}x - 1$.

$$y = \frac{1}{2}x - 1$$
$$= \frac{1}{2}(2) - 1$$
$$= 0$$

This computation tells us that $(2, 0)$ satisfies the linear equation $y = \frac{1}{2}x - 1$ and is a point on its graph. Plot these points and draw a line through them.
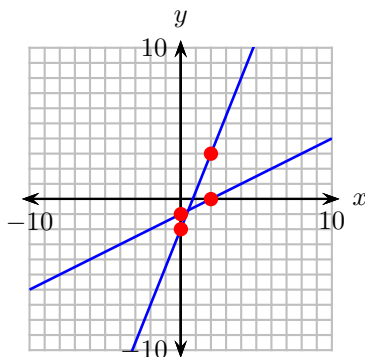
Make similar computations with $y = \frac{5}{2}x - 2$. First substitute 0 for x.

$$
\begin{aligned}
y &= \frac{5}{2}x - 2 \\
&= \frac{5}{2}(0) - 2 \\
&= -2
\end{aligned}
$$

This computation tells us that $(0, -2)$ satisfies the linear equation $y = \frac{5}{2}x - 2$ and is a point on the graph. Now, let's substitute 2 for x in $y = \frac{5}{2}x - 2$.

$$
\begin{aligned}
y &= \frac{5}{2}x - 2 \\
&= \frac{5}{2}(2) - 2 \\
&= 3
\end{aligned}
$$

This computation tells us that $(2, 3)$ satisfies the linear equation $y = \frac{5}{2}x - 2$ and is a point on the graph.

Note that the graph of $y = \frac{5}{2}x - 2$ rises more quickly than does the graph of $y = \frac{1}{2}x - 1$.

47. The equations $y = -\frac{1}{2}x + 1$ and $y = -3x + 3$ are linear. Thus, to plot these equations, we need only calculate two points that satisfy each equation, plot them, then draw a line through them. First, substitute 0 for x in $y = -\frac{1}{2}x + 1$.

$$
\begin{aligned}
y &= -\frac{1}{2}x + 1 \\
&= -\frac{1}{2}(0) + 1 \\
&= 1
\end{aligned}
$$

This computation tells us that $(0, 1)$ satisfies the linear equation $y = -\frac{1}{2}x + 1$ and is a point on its graph. Now, let's substitute 2 for x in $y = -\frac{1}{2}x + 1$.

$$y = -\frac{1}{2}x + 1$$
$$= -\frac{1}{2}(2) + 1$$
$$= 0$$

This computation tells us that $(2, 0)$ satisfies the linear equation $y = -\frac{1}{2}x + 1$ and is a point on its graph. Plot these points and draw a line through them.
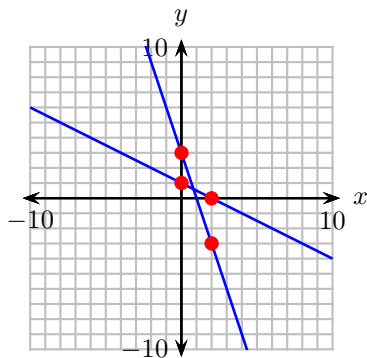
Make similar computations with $y = -3x + 3$. First substitute 0 for x.

$$y = -3x + 3$$
$$= -3(0) + 3$$
$$= 3$$

This computation tells us that $(0, 3)$ satisfies the linear equation $y = -3x + 3$ and is a point on the graph. Now, let's substitute 2 for x in $y = -3x + 3$.

$$y = -3x + 3$$
$$= -3(2) + 3$$
$$= -3$$

This computation tells us that $(2, -3)$ satisfies the linear equation $y = -3x + 3$ and is a point on the graph.

Note that the graph of $y = -3x + 3$ falls more quickly than does the graph of $y = -\frac{1}{2}x + 1$.

49. The equations $y = -3x - 1$ and $y = -\frac{1}{2}x - 2$ are linear. Thus, to plot these equations, we need only calculate two points that satisfy each equation, plot them, then draw a line through them. First, substitute 0 for x in $y = -3x - 1$.

$$y = -3x - 1$$
$$= -3(0) - 1$$
$$= -1$$

This computation tells us that $(0, -1)$ satisfies the linear equation $y = -3x - 1$ and is a point on its graph. Now, let's substitute 2 for x in $y = -3x - 1$.

$$y = -3x - 1$$
$$= -3(2) - 1$$
$$= -7$$

This computation tells us that $(2, -7)$ satisfies the linear equation $y = -3x - 1$ and is a point on its graph. Plot these points and draw a line through them.
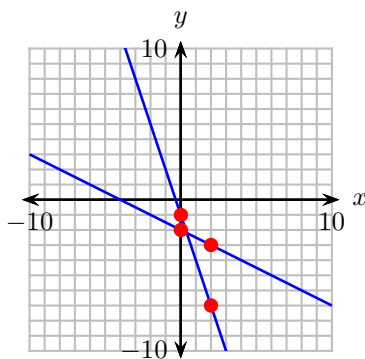
Make similar computations with $y = -\frac{1}{2}x - 2$. First substitute 0 for x.

$$y = -\frac{1}{2}x - 2$$
$$= -\frac{1}{2}(0) - 2$$
$$= -2$$

This computation tells us that $(0, -2)$ satisfies the linear equation $y = -\frac{1}{2}x - 2$ and is a point on the graph. Now, let's substitute 2 for x in $y = -\frac{1}{2}x - 2$.

$$y = -\frac{1}{2}x - 2$$
$$= -\frac{1}{2}(2) - 2$$
$$= -3$$

This computation tells us that $(2, -3)$ satisfies the linear equation $y = -\frac{1}{2}x - 2$ and is a point on the graph.

Note that the graph of $y = -3x - 1$ falls more quickly than does the graph of $y = -\frac{1}{2}x - 2$.

51. The equations $y = \frac{3}{2}x - 2$ and $y = 3x + 1$ are linear. Thus, to plot these equations, we need only calculate two points that satisfy each equation, plot them, then draw a line through them. First, substitute 0 for x in $y = \frac{3}{2}x - 2$.

$$
y = \frac{3}{2}x - 2
$$
$$
= \frac{3}{2}(0) - 2
$$
$$
= -2
$$

This computation tells us that $(0, -2)$ satisfies the linear equation $y = \frac{3}{2}x - 2$ and is a point on its graph. Now, let's substitute 2 for x in $y = \frac{3}{2}x - 2$.

$$
y = \frac{3}{2}x - 2
$$
$$
= \frac{3}{2}(2) - 2
$$
$$
= 1
$$

This computation tells us that $(2, 1)$ satisfies the linear equation $y = \frac{3}{2}x - 2$ and is a point on its graph. Plot these points and draw a line through them.
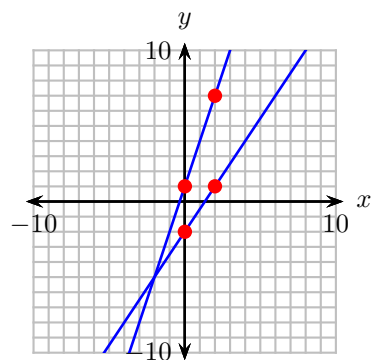
Make similar computations with $y = 3x + 1$. First substitute 0 for x.

$$
y = 3x + 1
$$
$$
= 3(0) + 1
$$
$$
= 1
$$

This computation tells us that $(0, 1)$ satisfies the linear equation $y = 3x + 1$ and is a point on the graph. Now, let's substitute 2 for x in $y = 3x + 1$.

$$
y = 3x + 1
$$
$$
= 3(2) + 1
$$
$$
= 7
$$

This computation tells us that $(2, 7)$ satisfies the linear equation $y = 3x + 1$ and is a point on the graph.

Note that the graph of $y = 3x + 1$ rises more quickly than does the graph of $y = \frac{3}{2}x - 2$.