

Student: _____
Date: _____

Instructor: Alfredo Alvarez
Course: 2413 Cal I

Assignment:
 calmath2413alvarez150next

1. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$ for the given function.

$$f(x) = 3x^2 - 4x + 1$$

$$f(x+h) = \boxed{}$$

$$\frac{f(x+h) - f(x)}{h} = \boxed{}$$

(Simplify your answer.)

Answers $3(x+h)^2 - 4(x+h) + 1$
 $6x + 3h - 4$

2. Simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$ for the given function.

$$f(x) = \frac{21}{x}$$

$$\frac{f(x+h) - f(x)}{h} = \boxed{} \text{ (Simplify your answer.)}$$

Answer: $-\frac{21}{x(x+h)}$

3. Simplify the difference quotient $\frac{f(x) - f(a)}{x - a}$ for the given function.

$$f(x) = 2 - 5x - x^2$$

$$\frac{f(x) - f(a)}{x - a} = \boxed{} \text{ (Simplify your answer.)}$$

Answer: $-a - x - 5$

4. The function $s(t)$ represents the position of an object at time t moving along a line. Suppose $s(3) = 155$ and $s(5) = 205$. Find the average velocity of the object over the interval of time $[3, 5]$.

The average velocity over the interval $[3, 5]$ is $v_{av} = \boxed{}$. (Simplify your answer.)

Answer: 25

5. What is the slope of the secant line between the points $(a, f(a))$ and $(b, f(b))$ on the graph of f ?

Which of the following is the correct formula for the slope of the secant line?

☐ A. $m_{\text{sec}} = \frac{f(b) + f(a)}{b - a}$

☐ B. $m_{\text{sec}} = \frac{f(b) - f(a)}{b - a}$

☐ C. $m_{\text{sec}} = \frac{f(b) - f(a)}{b + a}$

☐ D. $m_{\text{sec}} = \frac{f(a) - f(b)}{b - a}$

Answer: B. $m_{\text{sec}} = \frac{f(b) - f(a)}{b - a}$

6. What is the slope of the line tangent to the graph of f at $(a, f(a))$?

Which of the following is the correct formula for the slope of the tangent line?

☐ A. $m_{\text{tan}} = \lim_{t \rightarrow a} \frac{f(b) - f(t)}{b - t}$

☐ B. $m_{\text{tan}} = \lim_{t \rightarrow a} \frac{f(b) + f(t)}{b - t}$

☐ C. $m_{\text{tan}} = \lim_{t \rightarrow a} \frac{f(t) - f(a)}{t - a}$

☐ D. $m_{\text{tan}} = \lim_{t \rightarrow a} \frac{f(t) + f(a)}{t - a}$

Answer: C. $m_{\text{tan}} = \lim_{t \rightarrow a} \frac{f(t) - f(a)}{t - a}$

7. The position of an object moving along a line is given by the function $s(t) = -20t^2 + 160t$. Find the average velocity of the object over the following intervals.

(a) $[1, 4]$ (b) $[1, 3]$ (c) $[1, 2]$ (d) $[1, 1 + h]$ where $h > 0$ is any real number.(a) The average velocity of the object over the interval $[1, 4]$ is .(b) The average velocity of the object over the interval $[1, 3]$ is .(c) The average velocity of the object over the interval $[1, 2]$ is .(d) The average velocity of the object over the interval $[1, 1 + h]$ is .

Answers 60

80

100

 $-20h + 120$

8. For the position function $s(t) = -16t^2 + 102t$, complete the following table with the appropriate average velocities. Then make a conjecture about the value of the instantaneous velocity at $t = 1$.

Time Interval	$[1, 2]$	$[1, 1.5]$	$[1, 1.1]$	$[1, 1.01]$	$[1, 1.001]$
Average Velocity	_____	_____	_____	_____	_____

Complete the following table.

Time Interval	$[1, 2]$	$[1, 1.5]$	$[1, 1.1]$	$[1, 1.01]$	$[1, 1.001]$
Average Velocity	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

(Type exact answers. Type integers or decimals.)

The value of the instantaneous velocity at $t = 1$ is .

(Round to the nearest integer as needed.)

Answers 54

62

68.4

69.84

69.984

70

9. For the function $f(x) = 17x^3 - x$, make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at $x = 1$.

Complete the table.

(Round the final answer to three decimal places as needed. Round all intermediate values to four decimal places as needed.)

Interval	Slope of secant line
[1, 2]	<input type="text"/>
[1, 1.5]	<input type="text"/>
[1, 1.1]	<input type="text"/>
[1, 1.01]	<input type="text"/>
[1, 1.001]	<input type="text"/>

An accurate conjecture for the slope of the tangent line at $x = 1$ is .

(Round to the nearest integer as needed.)

Answers 118.000

79.750

55.270

50.510

50.100

50

10.

Let $f(x) = \frac{x^2 - 16}{x - 4}$. **(a)** Calculate $f(x)$ for each value of x in the following table. **(b)** Make a conjecture about the value of

$$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}.$$

(a) Calculate $f(x)$ for each value of x in the following table.

x	3.9	3.99	3.999	3.9999
$f(x) = \frac{x^2 - 16}{x - 4}$				
x	4.1	4.01	4.001	4.0001
$f(x) = \frac{x^2 - 16}{x - 4}$				

(Type an integer or decimal rounded to four decimal places as needed.)

(b) Make a conjecture about the value of $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$.

$$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4} = \boxed{} \text{ (Type an integer or a decimal.)}$$

Answers 7.9

7.99

7.999

7.9999

8.1

8.01

8.001

8.0001

8

11. Let $g(t) = \frac{t - 49}{\sqrt{t} - 7}$.

a. Make two tables, one showing the values of g for $t = 48.9, 48.99, \text{ and } 48.999$ and one showing values of g for $t = 49.1, 49.01, \text{ and } 49.001$.

b. Make a conjecture about the value of $\lim_{t \rightarrow 49} \frac{t - 49}{\sqrt{t} - 7}$.

a. Make a table showing the values of g for $t = 48.9, 48.99, \text{ and } 48.999$.

t	48.9	48.99	48.999
g(t)			

(Round to four decimal places.)

Make a table showing the values of g for $t = 49.1, 49.01, \text{ and } 49.001$.

t	49.1	49.01	49.001
g(t)			

(Round to four decimal places.)

b. Make a conjecture about the value of $\lim_{t \rightarrow 49} \frac{t - 49}{\sqrt{t} - 7}$. Select the correct choice below and fill in any answer boxes in your choice.

☐ A. $\lim_{t \rightarrow 49} \frac{t - 49}{\sqrt{t} - 7} =$ _____ (Simplify your answer.)

☐ B. The limit does not exist.

Answers 13.9929

13.9993

13.9999

14.0071

14.0007

14.0001

A. $\lim_{t \rightarrow 49} \frac{t - 49}{\sqrt{t} - 7} =$ (Simplify your answer.)

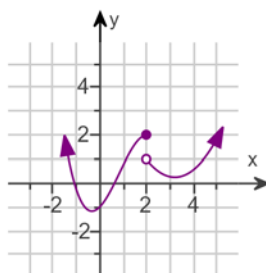
12. If $\lim_{x \rightarrow a^-} f(x) = L$ and $\lim_{x \rightarrow a^+} f(x) = M$, where L and M are finite real numbers, then what must be true about L and M in order for $\lim_{x \rightarrow a} f(x)$ to exist?
-

Choose the correct answer below.

- ☐ A. $L < M$
- ☐ B. $L = M$
- ☐ C. $L > M$
- ☐ D. $L \neq M$

Answer: B. $L = M$

13. Use the graph to find the following limits and function value.



- a. $\lim_{x \rightarrow 2^-} f(x)$
 b. $\lim_{x \rightarrow 2^+} f(x)$
 c. $\lim_{x \rightarrow 2} f(x)$
 d. $f(2)$

a. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

☐ A. $\lim_{x \rightarrow 2^-} f(x) =$ _____ (Type an integer.)

☐ B. The limit does not exist.

b. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

☐ A. $\lim_{x \rightarrow 2^+} f(x) =$ _____ (Type an integer.)

☐ B. The limit does not exist.

c. Find the limit. Select the correct choice below and fill in any answer boxes in your choice.

☐ A. $\lim_{x \rightarrow 2} f(x) =$ _____ (Type an integer.)

☐ B. The limit does not exist.

d. Find the function value. Select the correct choice below and fill in any answer boxes in your choice.

☐ A. $f(2) =$ _____ (Type an integer.)

☐ B. The answer is undefined.

Answers A. $\lim_{x \rightarrow 2^-} f(x) =$ (Type an integer.)

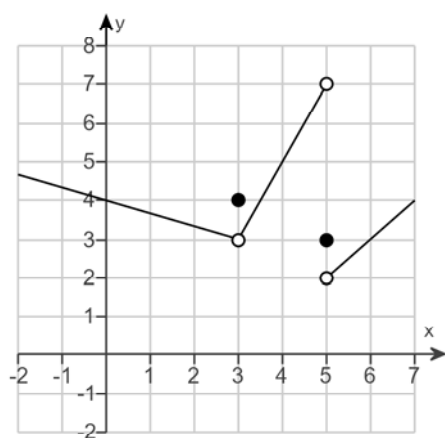
A. $\lim_{x \rightarrow 2^+} f(x) =$ (Type an integer.)

B. The limit does not exist.

A. $f(2) =$ (Type an integer.)

14.

Use the graph of f to complete parts (a) through (i). If a limit does not exist, explain why.



(a) Find $f(3)$. Select the correct choice below, and fill in the answer box if necessary.

- ☐ A. $f(3) =$ _____
(Type an integer or a fraction.)
- ☐ B. The value of $f(3)$ is undefined.

(b) Find $\lim_{x \rightarrow 3^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- ☐ A. $\lim_{x \rightarrow 3^-} f(x) =$ _____
(Type an integer or a fraction.)
- ☐ B. The limit does not exist because $f(x)$ is not defined for all $x < 3$.

(c) Find $\lim_{x \rightarrow 3^+} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- ☐ A. $\lim_{x \rightarrow 3^+} f(x) =$ _____
(Type an integer or a fraction.)
- ☐ B. The limit does not exist because $f(x)$ is not defined for all $x > 3$.

(d) Find $\lim_{x \rightarrow 3} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- ☐ A. $\lim_{x \rightarrow 3} f(x) =$ _____
(Type an integer or a fraction.)
- ☐ B. The limit does not exist because
 $\lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x)$.

(e) Find $f(5)$. Select the correct choice below, and fill in the answer box if necessary.

- ☐ A. $f(5) =$ _____
(Type an integer or a fraction.)
- ☐ B. The value of $f(5)$ is undefined.

(f) Find $\lim_{x \rightarrow 5^-} f(x)$. Select the correct choice below, and fill in the answer box if necessary.

- ☐ A. $\lim_{x \rightarrow 5^-} f(x) =$ _____
(Type an integer or a fraction.)
- ☐ B. The limit does not exist because $f(x)$ is not defined for all $x < 5$.

Answers A. $f(3) =$ (Type an integer or a fraction.)

A. $\lim_{x \rightarrow 3^-} f(x) =$ (Type an integer or a fraction.)

A. $\lim_{x \rightarrow 3^+} f(x) =$ (Type an integer or a fraction.)

A. $\lim_{x \rightarrow 3} f(x) =$ (Type an integer or a fraction.)

A. $f(5) =$ (Type an integer or a fraction.)

A. $\lim_{x \rightarrow 5^-} f(x) =$ (Type an integer or a fraction.)

A. $\lim_{x \rightarrow 5^+} f(x) =$ (Type an integer or a fraction.)

B. The limit does not exist because $\lim_{x \rightarrow 5^-} f(x) \neq \lim_{x \rightarrow 5^+} f(x)$.

A. $f(4) =$ (Type an integer or a fraction.)

A. $\lim_{x \rightarrow 4^-} f(x) =$ (Type an integer or a fraction.)

A. $\lim_{x \rightarrow 4^+} f(x) =$ (Type an integer or a fraction.)

A. $\lim_{x \rightarrow 4} f(x) =$ (Type an integer or a fraction.)

15. Explain why $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x - 3} = \lim_{x \rightarrow 3} (x - 4)$, and then evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x - 3}$.

Choose the correct answer below.

- ☐ A. The numerator of the expression $\frac{x^2 - 7x + 12}{x - 3}$ simplifies to $x - 4$ for all x , so the limits are equal.
- ☐ B. The limits $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x - 3}$ and $\lim_{x \rightarrow 3} (x - 4)$ equal the same number when evaluated using direct substitution.
- ☐ C. Since $\frac{x^2 - 7x + 12}{x - 3} = x - 4$ whenever $x \neq 3$, it follows that the two expressions evaluate to the same number as x approaches 3.
- ☐ D. Since each limit approaches 3, it follows that the limits are equal.

Now evaluate the limit.

$$\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x - 3} = \boxed{} \text{ (Simplify your answer.)}$$

Answers C.

Since $\frac{x^2 - 7x + 12}{x - 3} = x - 4$ whenever $x \neq 3$, it follows that the two expressions evaluate to the same number as x approaches 3.

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16. Find the following limit or state that it does not exist.

$$\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x + 1}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x + 1} = \underline{\hspace{2cm}}$ (Type an exact answer.)
- ☐ B. The limit does not exist.

Answer: A. $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x + 1} = \boxed{1}$ (Type an exact answer.)

17. Find the following limit or state that it does not exist.

$$\lim_{x \rightarrow 289} \frac{\sqrt{x} - 17}{x - 289}$$

Simplify the given limit.

$$\lim_{x \rightarrow 289} \frac{\sqrt{x} - 17}{x - 289} = \lim_{x \rightarrow 289} \left(\boxed{} \right) \text{ (Simplify your answer.)}$$

Evaluate the limit, if possible. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow 289} \frac{\sqrt{x} - 17}{x - 289} = \underline{\hspace{2cm}}$ (Type an exact answer.)
- ☐ B. The limit does not exist.

Answers $\frac{1}{\sqrt{x} + 17}$

A. $\lim_{x \rightarrow 289} \frac{\sqrt{x} - 17}{x - 289} = \boxed{\frac{1}{34}}$ (Type an exact answer.)

18. Determine the following limit.

$$\lim_{x \rightarrow -\infty} 6x^{17}$$

$$\lim_{x \rightarrow -\infty} 6x^{17} = \boxed{}$$

Answer: $-\infty$

19. Determine the following limit.

$$\lim_{x \rightarrow \infty} x^{-7}$$

$$\lim_{x \rightarrow \infty} x^{-7} = \boxed{}$$

Answer: 0

20. Determine the following limit at infinity.

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} \text{ if } f(x) \rightarrow 100,000 \text{ and } g(x) \rightarrow \infty \text{ as } x \rightarrow \infty$$

Find the limit. Choose the correct answer below.

☐ A. $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \frac{1}{100,000}$

☐ B. $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 100,000$

☐ C. $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \infty$

☐ D. $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$

Answer: D. $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$

21. Determine the following limit at infinity.

$$\lim_{x \rightarrow \infty} \frac{4 + 9x + 9x^4}{x^4}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

☐ A. $\lim_{x \rightarrow \infty} \frac{4 + 9x + 9x^4}{x^4} = \underline{\hspace{2cm}}$

☐ B. The limit does not exist and is neither $-\infty$ nor ∞ .

Answer: A. $\lim_{x \rightarrow \infty} \frac{4 + 9x + 9x^4}{x^4} = \boxed{9}$

22. Determine the following limit.

$$\lim_{x \rightarrow \infty} \frac{\sin 10x}{3x}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow \infty} \frac{\sin 10x}{3x} =$ _____ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither $-\infty$ nor ∞ .

Answer: A. $\lim_{x \rightarrow \infty} \frac{\sin 10x}{3x} =$ (Simplify your answer.)

23. Determine the following limit.

$$\lim_{x \rightarrow \infty} (3x^5 - 5x^4 + 1)$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow \infty} (3x^5 - 5x^4 + 1) =$ _____
- ☐ B. The limit does not exist and is neither $-\infty$ nor ∞ .

Answer: A. $\lim_{x \rightarrow \infty} (3x^5 - 5x^4 + 1) =$

24. Determine the following limit.

$$\lim_{w \rightarrow \infty} \frac{20w^2 + 3w + 3}{\sqrt{25w^4 + 5w^3}}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{w \rightarrow \infty} \frac{20w^2 + 3w + 3}{\sqrt{25w^4 + 5w^3}} =$ _____ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Answer: A. $\lim_{w \rightarrow \infty} \frac{20w^2 + 3w + 3}{\sqrt{25w^4 + 5w^3}} =$ (Simplify your answer.)

25. Determine the following limit.

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{49x^2 + x}}{x}$$

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow -\infty} \frac{\sqrt{49x^2 + x}}{x} = \underline{\hspace{2cm}}$ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Answer: A. $\lim_{x \rightarrow -\infty} \frac{\sqrt{49x^2 + x}}{x} =$ (Simplify your answer.)

26. Determine $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ for the following function. Then give the horizontal asymptotes of f , if any.

$$f(x) = \frac{2x}{10x + 4}$$

Evaluate $\lim_{x \rightarrow \infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow \infty} \frac{2x}{10x + 4} = \underline{\hspace{2cm}}$ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Evaluate $\lim_{x \rightarrow -\infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow -\infty} \frac{2x}{10x + 4} = \underline{\hspace{2cm}}$ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Give the horizontal asymptotes of f , if any. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- ☐ A. The function has one horizontal asymptote, $\underline{\hspace{2cm}}$.
(Type an equation.)
- ☐ B. The function has two horizontal asymptotes. The top asymptote is $\underline{\hspace{2cm}}$ and the bottom asymptote is $\underline{\hspace{2cm}}$.
(Type equations.)
- ☐ C. The function has no horizontal asymptotes.

Answers A. $\lim_{x \rightarrow \infty} \frac{2x}{10x + 4} = \boxed{\frac{1}{5}}$ (Simplify your answer.)

A. $\lim_{x \rightarrow -\infty} \frac{2x}{10x + 4} = \boxed{\frac{1}{5}}$ (Simplify your answer.)

A. The function has one horizontal asymptote, $\boxed{y = \frac{1}{5}}$. (Type an equation.)

27. Evaluate $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ for the following rational function. Use ∞ or $-\infty$ where appropriate. Then give the horizontal asymptote of f (if any).

$$f(x) = \frac{9x^2 - 8x + 7}{3x^2 + 1}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow \infty} f(x) =$ _____ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow -\infty} f(x) =$ _____ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Identify the horizontal asymptote. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The horizontal asymptote is $y =$ _____.
- ☐ B. There are no horizontal asymptotes.

Answers A. $\lim_{x \rightarrow \infty} f(x) =$ (Simplify your answer.)

A. $\lim_{x \rightarrow -\infty} f(x) =$ (Simplify your answer.)

A. The horizontal asymptote is $y =$.

28. Evaluate $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ for the rational function. Then give the horizontal asymptote of f (if any).

$$f(x) = \frac{10x^5 - 7}{2x^5 - 6x^4}$$

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

☐ A. $\lim_{x \rightarrow -\infty} \frac{10x^5 - 7}{2x^5 - 6x^4} = \underline{\hspace{2cm}}$ (Simplify your answer.)

- ☐ B. The limit does not exist and is neither ∞ or $-\infty$.

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

☐ A. $\lim_{x \rightarrow \infty} \frac{10x^5 - 7}{2x^5 - 6x^4} = \underline{\hspace{2cm}}$ (Simplify your answer.)

- ☐ B. The limit does not exist and is neither ∞ or $-\infty$.

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The function has a horizontal asymptote at $y = \underline{\hspace{2cm}}$.
(Simplify your answer.)

- ☐ B. The function has no horizontal asymptote.

Answers A. $\lim_{x \rightarrow -\infty} \frac{10x^5 - 7}{2x^5 - 6x^4} = \boxed{5}$ (Simplify your answer.)

A. $\lim_{x \rightarrow \infty} \frac{10x^5 - 7}{2x^5 - 6x^4} = \boxed{5}$ (Simplify your answer.)

A. The function has a horizontal asymptote at $y = \boxed{5}$. (Simplify your answer.)

29. Determine $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ for the following function. Then give the horizontal asymptotes of f , if any.

$$f(x) = \frac{3x^5 - 6}{x^6 + 4x^4}$$

Evaluate $\lim_{x \rightarrow \infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

☐ A. $\lim_{x \rightarrow \infty} \frac{3x^5 - 6}{x^6 + 4x^4} = \underline{\hspace{2cm}}$ (Simplify your answer.)

☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Evaluate $\lim_{x \rightarrow -\infty} f(x)$. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

☐ A. $\lim_{x \rightarrow -\infty} \frac{3x^5 - 6}{x^6 + 4x^4} = \underline{\hspace{2cm}}$ (Simplify your answer.)

☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Identify the horizontal asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- ☐ A. The function has two horizontal asymptotes. The top asymptote is $\underline{\hspace{2cm}}$ and the bottom asymptote is $\underline{\hspace{2cm}}$.
(Type equations.)
- ☐ B. The function has one horizontal asymptote, $\underline{\hspace{2cm}}$.
(Type an equation.)
- ☐ C. The function has no horizontal asymptotes.

Answers A. $\lim_{x \rightarrow \infty} \frac{3x^5 - 6}{x^6 + 4x^4} = \boxed{0}$ (Simplify your answer.)

A. $\lim_{x \rightarrow -\infty} \frac{3x^5 - 6}{x^6 + 4x^4} = \boxed{0}$ (Simplify your answer.)

B. The function has one horizontal asymptote, $\boxed{y = 0}$. (Type an equation.)

30. Evaluate $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.

$$f(x) = \frac{9x^3 + 3}{1 - 2x^3}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow \infty} \frac{9x^3 + 3}{1 - 2x^3} = \underline{\hspace{2cm}}$ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow -\infty} \frac{9x^3 + 3}{1 - 2x^3} = \underline{\hspace{2cm}}$ (Simplify your answer.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Answers A. $\lim_{x \rightarrow \infty} \frac{9x^3 + 3}{1 - 2x^3} = \boxed{-\frac{9}{2}}$ (Simplify your answer.)

A. $\lim_{x \rightarrow -\infty} \frac{9x^3 + 3}{1 - 2x^3} = \boxed{-\frac{9}{2}}$ (Simplify your answer.)

31. Find all the asymptotes of the function.

$$f(x) = \frac{4x^2 + 8}{2x^2 + x - 10}$$

Find the horizontal asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- ☐ A. The function has one horizontal asymptote, _____.
(Type an equation using y as the variable.)
- ☐ B. The function has two horizontal asymptotes. The top asymptote is _____ and the bottom asymptote is _____.
(Type equations using y as the variable.)
- ☐ C. The function has no horizontal asymptotes.

Find the vertical asymptotes. Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- ☐ A. The function has one vertical asymptote, _____.
(Type an equation using x as the variable.)
- ☐ B. The function has two vertical asymptotes. The leftmost asymptote is _____ and the rightmost asymptote is _____.
(Type equations using x as the variable.)
- ☐ C. The function has no vertical asymptotes.

Find the slant asymptote(s). Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- ☐ A. The function has one slant asymptote, _____.
(Type an equation using x and y as the variables.)
- ☐ B. The function has two slant asymptotes. The asymptote with the larger slope is _____ and the asymptote with the smaller slope is _____.
(Type equations using x and y as the variables.)
- ☐ C. The function has no slant asymptotes.

Answers A. The function has one horizontal asymptote, . (Type an equation using y as the variable.)

B.

The function has two vertical asymptotes. The leftmost asymptote is and the rightmost asymptote is .

(Type equations using x as the variable.)

C. The function has no slant asymptotes.

32. Complete the following sentences.

(a) A function is continuous from the left at a if _____.

(b) A function is continuous from the right at a if _____.

(a) A function is continuous from the left at a if (1) .

(b) A function is continuous from the right at a if (2) .

(1) ☐ $\lim_{x \rightarrow a^+} f(x) = f(a)$

☐ $\lim_{x \rightarrow a^-} f(x) = f(a)$

☐ $\lim_{x \rightarrow a^-} f(x) = -f(a)$

☐ $\lim_{x \rightarrow a^+} f(x) = -f(a)$

(2) ☐ $\lim_{x \rightarrow a^-} f(x) = f(a)$

☐ $\lim_{x \rightarrow a^-} f(x) = -f(a)$

☐ $\lim_{x \rightarrow a^+} f(x) = -f(a)$

☐ $\lim_{x \rightarrow a^+} f(x) = f(a)$

Answers (1) $\lim_{x \rightarrow a^-} f(x) = f(a)$

(2) $\lim_{x \rightarrow a^+} f(x) = f(a)$

33. Determine whether the following function is continuous at a. Use the continuity checklist to justify your answer.

$$f(x) = \frac{5x^2 + 18x + 9}{x^2 - 2x}, a = 2$$

Select all that apply.

☐ A. The function is continuous at $a = 2$.

☐ B. The function is not continuous at $a = 2$ because $f(2)$ is undefined.

☐ C. The function is not continuous at $a = 2$ because $\lim_{x \rightarrow 2} f(x)$ does not exist.

☐ D. The function is not continuous at $a = 2$ because $\lim_{x \rightarrow 2} f(x) \neq f(2)$.

Answer: B. The function is not continuous at $a = 2$ because $f(2)$ is undefined. , C.

The function is not continuous at $a = 2$ because $\lim_{x \rightarrow 2} f(x)$ does not exist. , D.

The function is not continuous at $a = 2$ because $\lim_{x \rightarrow 2} f(x) \neq f(2)$.

34. Determine whether the following function is continuous at a . Use the continuity checklist to justify your answer.

$$f(x) = \begin{cases} \frac{x^2 - 64}{x - 8} & \text{if } x \neq 8 \\ 7 & \text{if } x = 8 \end{cases}; a = 8$$

Select all that apply.

- ☐ A. The function is continuous at $a = 8$.
- ☐ B. The function is not continuous at $a = 8$ because $f(8)$ is undefined.
- ☐ C. The function is not continuous at $a = 8$ because $\lim_{x \rightarrow 8} f(x)$ does not exist.
- ☐ D. The function is not continuous at $a = 8$ because $\lim_{x \rightarrow 8} f(x) \neq f(8)$.

Answer: D. The function is not continuous at $a = 8$ because $\lim_{x \rightarrow 8} f(x) \neq f(8)$.

35. Determine the intervals on which the following function is continuous.

$$f(x) = \frac{x^2 - 3x + 2}{x^2 - 1}$$

On what interval(s) is f continuous?

(Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)

Answer: $(-\infty, -1), (-1, 1), (1, \infty)$

36. Evaluate the following limit.

$$\lim_{x \rightarrow 7} \sqrt{x^2 + 15}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☒ A. $\lim_{x \rightarrow 7} \sqrt{x^2 + 15} =$, because $x^2 + 15$ is continuous for all x and the square root function is continuous for all $x \geq 0$.
(Type an integer or a fraction.)
- ☐ B. The limit does not exist and is neither ∞ nor $-\infty$.

Answer: A.

$\lim_{x \rightarrow 7} \sqrt{x^2 + 15} =$, because $x^2 + 15$ is continuous for all x and the square root function is continuous for all $x \geq 0$.
(Type an integer or a fraction.)

37. Suppose x lies in the interval $(5,9)$ with $x \neq 7$. Find the smallest positive value of δ such that the inequality $0 < |x - 7| < \delta$ is true for all possible values of x .

The smallest positive value of δ is . (Type an integer or a fraction.)

Answer: 2

38. Suppose $|f(x) - 7| < 0.3$ whenever $0 < x < 7$. Find all values of $\delta > 0$ such that $|f(x) - 7| < 0.3$ whenever $0 < |x - 1| < \delta$.

The values of δ are $0 < \delta \leq$. (Type an integer or a fraction.)

Answer: 1

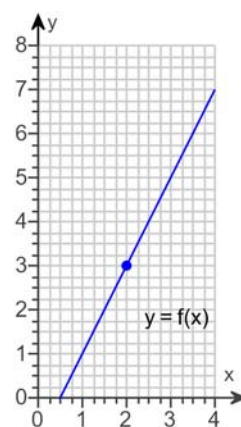
39. The function f in the figure satisfies $\lim_{x \rightarrow 2} f(x) = 3$. Determine the largest value of $\delta > 0$ satisfying each statement.

a. If $0 < |x - 2| < \delta$, then $|f(x) - 3| < 2$.

b. If $0 < |x - 2| < \delta$, then $|f(x) - 3| < 1$.

a. $\delta =$ (Simplify your answer.)

b. $\delta =$ (Simplify your answer.)



Answers 1

$\frac{1}{2}$

40. Use the precise definition of a limit to prove the following limit. Specify a relationship between ε and δ that guarantees the limit exists.

$$\lim_{x \rightarrow 0} (4x - 8) = -8$$

Let $\varepsilon > 0$ be given. Choose the correct proof below.

- ☐ A. Choose $\delta = 4\varepsilon$. Then, $|(4x - 8) - (-8)| < \varepsilon$ whenever $0 < |x - 0| < \delta$.
- ☐ B. Choose $\delta = \varepsilon$. Then, $|(4x - 8) - (-8)| < \varepsilon$ whenever $0 < |x - 0| < \delta$.
- ☐ C. Choose $\delta = \frac{\varepsilon}{4}$. Then, $|(4x - 8) - (-8)| < \varepsilon$ whenever $0 < |x - 0| < \delta$.
- ☐ D. Choose $\delta = \frac{\varepsilon}{8}$. Then, $|(4x - 8) - (-8)| < \varepsilon$ whenever $0 < |x - 0| < \delta$.
- ☐ E. None of the above proofs is correct.

Answer:

C. Choose $\delta = \frac{\varepsilon}{4}$. Then, $|(4x - 8) - (-8)| < \varepsilon$ whenever $0 < |x - 0| < \delta$.

41. Find the value of the derivative of the function at the given point.

$$f(x) = 4x^2 - 3x; (-1, 7)$$

$$f'(-1) = \boxed{} \text{ (Type an integer or a simplified fraction.)}$$

Answer: -11

42. a. Use the definition $m_{\tan} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ to find the slope of the line tangent to the graph of f at P .
- b. Determine an equation of the tangent line at P .

$$f(x) = x^2 - 5, P(4, 11)$$

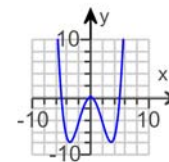
a. $m_{\tan} = \boxed{}$

b. $y = \boxed{}$

Answers 8

$$8x - 21$$

43. Match the graph of the function on the right with the graph of its derivative.



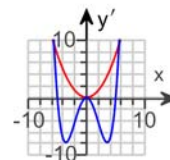
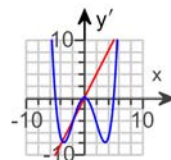
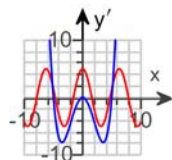
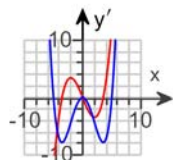
Choose the correct graph of the function (in blue) and its derivative (in red) below.

☐ A.

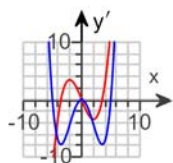
☐ B.

☐ C.

☐ D.



Answer:



A.

44. A line perpendicular to another line or to a tangent line is called a normal line. Find an equation of the line perpendicular to the line that is tangent to the following curve at the given point P.

$$y = 5x + 11; P(-2, 1)$$

The equation of the normal line at P(-2, 1) is .

Answer: $y = -\frac{1}{5}x + \frac{3}{5}$

45. Use the Quotient Rule to evaluate and simplify $\frac{d}{dx} \left(\frac{x-3}{6x-5} \right)$.

$$\frac{d}{dx} \left(\frac{x-3}{6x-5} \right) = \text{$$

Answer: $\frac{13}{(6x-5)^2}$

46.

Use the Quotient Rule to find $g'(1)$ given that $g(x) = \frac{3x^2}{4x+1}$.

$$g'(1) = \boxed{}$$

(Simplify your answer.)

Answer: $\frac{18}{25}$

47. a. Use the Product Rule to find the derivative of the given function.

b. Find the derivative by expanding the product first.

$$f(x) = (x-5)(3x+3)$$

a. Use the product rule to find the derivative of the function. Select the correct answer below and fill in the answer box(es) to complete your choice.

- ☐ A. The derivative is $(x-5)(3x+3)()$.
- ☐ B. The derivative is $(x-5)(3x+3) + ()$.
- ☐ C. The derivative is $()x(3x+3)$.
- ☐ D. The derivative is $(x-5)() + (3x+3)()$.
- ☐ E. The derivative is $()(x-5)$.

b. Expand the product.

$$(x-5)(3x+3) = \boxed{} \text{ (Simplify your answer.)}$$

Using either approach, $\frac{d}{dx}(x-5)(3x+3) = \boxed{}$.

Answers D. The derivative is $(x-5)(\boxed{3}) + (3x+3)(\boxed{1})$.

$$3x^2 - 12x - 15$$

$$6x - 12$$

48. Use the quotient rule to find the derivative of the given function. Then find the derivative by first simplifying the function. Are the results the same?

$$h(w) = \frac{3w^3 - w}{w}$$

What is the immediate result of applying the quotient rule? Select the correct answer below.

- ☐ A. $\frac{w(9w^2 - 1) - (3w^3 - w)(1)}{w^2}$
- ☐ B. $3w^2 - 1$
- ☐ C. $(9w^2 - 1)(w) + (3w^3 - w)(1)$
- ☐ D. $6w$

What is the fully simplified result of applying the quotient rule?

What is the result of first simplifying the function, then taking the derivative? Select the correct answer below.

- ☐ A. $(9w^2 - 1)(w) + (3w^3 - w)(1)$
- ☐ B. $6w$
- ☐ C. $3w^2 - 1$
- ☐ D. $\frac{w(9w^2 - 1) - (3w^3 - w)(1)}{w^2}$

Are the two results the same?

- ☐ No
- ☐ Yes

Answers

A. $\frac{w(9w^2 - 1) - (3w^3 - w)(1)}{w^2}$

$6w$

B. $6w$

Yes

49. Evaluate the limit.

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 2x}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 2x} = \underline{\hspace{2cm}}$
- ☐ B. The limit is undefined.

Answer: A. $\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 2x} = \boxed{\frac{7}{2}}$

50. Evaluate the limit.

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\tan x}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\tan x} = \underline{\hspace{2cm}}$
- ☐ B. The limit is undefined.

Answer: A. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\tan x} = \boxed{2}$

51. Use the trigonometric limits $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ and/or $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$ to evaluate the following limit.

$$\lim_{\theta \rightarrow 0} \frac{\cos^2 \theta - 1}{\theta}$$

Select the correct choice below, and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $\lim_{\theta \rightarrow 0} \frac{\cos^2 \theta - 1}{\theta} = \underline{\hspace{2cm}}$ (Simplify your answer.)
- ☐ B. The limit is undefined.

Answer: A. $\lim_{\theta \rightarrow 0} \frac{\cos^2 \theta - 1}{\theta} = \boxed{0}$ (Simplify your answer.)

52. Find $\frac{dy}{dx}$ for the following function.

$$y = 6 \sin x + 8 \cos x$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $6 \cos x - 8 \sin x$

53. Find the derivative of the following function.

$$y = e^{-x} \sin x$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $e^{-x}(\cos x - \sin x)$

54. Find an equation of the line tangent to the following curve at the given point.

$$y = 15x^2 + 8 \sin x; P(0,0)$$

The equation for the tangent line is $\boxed{}$.

Answer: $y = 8x$

55. Let $h(x) = f(g(x))$ and $p(x) = g(f(x))$. Use the table below to compute the following derivatives.

a. $h'(1)$

b. $p'(3)$

x	1	2	3	4
$f(x)$	1	2	4	3
$f'(x)$	-9	-8	-4	-6
$g(x)$	3	1	2	4
$g'(x)$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{5}{7}$	$\frac{6}{7}$

$h'(1) = \boxed{}$ (Simplify your answer.)

$p'(3) = \boxed{}$ (Simplify your answer.)

Answers $-\frac{8}{7}$
 $-\frac{24}{7}$

56. Calculate the derivative of the following function.

$$y = (5x - 8)^3$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $15(5x - 8)^2$

57. Calculate the derivative of the following function.

$$y = 2(5x^4 + 2)^{-9}$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $\frac{-360x^3}{(5x^4 + 2)^{10}}$

58. Calculate the derivative of the following function.

$$y = \cos(9t - 16)$$

$$\frac{dy}{dt} = \boxed{}$$

Answer: $-9 \sin(9t - 16)$

59. Calculate the derivative of the following function.

$$y = \tan(e^x)$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $e^x \sec^2 e^x$

60. Calculate the derivative of the following function.

$$y = (\csc x + \cot x)^{11}$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $-11 \csc x (\csc x + \cot x)^{11}$

61. Calculate the derivative of the following function.

$$y = \cos(3 \sin x)$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $-3 \sin(3 \sin x) \cdot \cos x$

62. For some equations, such as $x^2 + y^2 = 1$ or $x - y^2 = 1$, it is possible to solve for y and then calculate $\frac{dy}{dx}$. Even in these cases, explain why implicit differentiation is usually a more efficient method for calculating the derivative.

Choose the correct answer below.

- ☐ A. Because implicit differentiation gives a single unified derivative.
- ☐ B. Because it produces $\frac{dy}{dx}$ in terms of x only.
- ☐ C. Because implicit differentiation gives two or more derivatives.
- ☐ D. Because it produces $\frac{dy}{dx}$ in terms of y only.

Answer: A. Because implicit differentiation gives a single unified derivative.

63. Calculate $\frac{dy}{dx}$ using implicit differentiation.

$$16x = y^2$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $\frac{8}{y}$

64. Calculate $\frac{dy}{dx}$ using implicit differentiation.

$$\cos(y) + 9 = x$$

$$\frac{dy}{dx} = \boxed{}$$

Answer: $-\csc y$

65. Consider the curve $x = y^3$. Use implicit differentiation to verify that $\frac{dy}{dx} = \frac{1}{3y^2}$ and then find $\frac{d^2y}{dx^2}$.

Use implicit differentiation to find the derivative of each side of the equation.

$$\frac{d}{dx}x = \boxed{} \text{ and } \frac{d}{dx}y^3 = \boxed{} \frac{dy}{dx}$$

Solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \boxed{}$$

Find $\frac{d^2y}{dx^2}$.

$$\frac{d^2y}{dx^2} = \boxed{}$$

Answers 1

$$3y^2$$

$$\frac{1}{3y^2}$$

$$-\frac{2}{9y^5}$$

66. Carry out the following steps for the given curve.

- a. Use implicit differentiation to find $\frac{dy}{dx}$.
 b. Find the slope of the curve at the given point.

$$x^5 + y^5 = 244; (3, 1)$$

- a. Use implicit differentiation to find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \boxed{}$$

- b. Find the slope of the curve at the given point.

The slope of $x^5 + y^5 = 244$ at $(3, 1)$ is $\boxed{}$.
 (Simplify your answer.)

Answers
$$\frac{-x^4}{y^4}$$

$$-81$$

67. Use implicit differentiation to find $\frac{dy}{dx}$.

$$\sin(y) + \sin(x) = 5y$$

$$\frac{dy}{dx} = \boxed{}$$

Answer:
$$\frac{\cos x}{5 - \cos y}$$

68. Use implicit differentiation to find $\frac{dy}{dx}$.

$$4 \sin(xy) = 9x + 5y$$

$$\frac{dy}{dx} = \boxed{}$$

Answer:
$$\frac{9 - 4y \cos(xy)}{4x \cos(xy) - 5}$$

69. Use implicit differentiation to find $\frac{dy}{dx}$.

$$e^{4xy} = 5y$$

$$\frac{dy}{dx} = \boxed{}$$

Answer:
$$\frac{4y e^{4xy}}{5 - 4x e^{4xy}}$$

70. Use implicit differentiation to find $\frac{dy}{dx}$ for the following equation.

$$6x^5 + 7y^5 = 13xy$$

$$\frac{dy}{dx} = \boxed{}$$

Answer:
$$\frac{30x^4 - 13y}{13x - 35y^4}$$

71. State the rule of differentiation for the logarithmic function $f(x) = \log_b x$. How does it differ from the derivative formula for $\ln x$?

Which of the following is the rule of differentiation for the logarithmic function $f(x) = \log_b x$?

- ☐ A. If $b > 0$ and $b \neq 1$, then $\frac{d}{dx}(\log_b x) = x \cdot \ln b$ for $x > 0$, and $\frac{d}{dx}(\log_b |x|) = x \cdot \ln b$ for $x \neq 0$.
- ☐ B. If $b > 0$ and $b \neq 1$, then $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$ for $x > 0$, and $\frac{d}{dx}(\log_b |x|) = \frac{1}{x \ln b}$ for $x \neq 0$.
- ☐ C. If $b > 0$ and $b \neq 1$, then $\frac{d}{dx}(\log_b x) = \frac{\ln b}{x}$ for $x > 0$, and $\frac{d}{dx}(\log_b |x|) = \frac{\ln b}{x}$ for $x \neq 0$.
- ☐ D. If $b > 0$ and $b \neq 1$, then $\frac{d}{dx}(\log_b x) = \frac{x}{\ln b}$ for $x > 0$, and $\frac{d}{dx}(\log_b |x|) = \frac{x}{\ln b}$ for $x \neq 0$.

How does the rule of differentiation for the function $f(x) = \log_b x$ differ from the derivative formula for $\ln x$?

- ☐ A. $\frac{d}{dx}(\log_b x) = \left(\frac{d}{dx} \ln x \right) \div \ln b$
- ☐ B. $\frac{d}{dx}(\log_b x) = \frac{d}{dx} \ln(bx)$
- ☐ C. $\frac{d}{dx}(\log_b x) = \left(\frac{d}{dx} \ln x \right) \cdot b$
- ☐ D. $\frac{d}{dx}(\log_b x) = \left(\frac{d}{dx} \ln x \right) \cdot \ln b$

Answers

B. If $b > 0$ and $b \neq 1$, then $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$ for $x > 0$, and $\frac{d}{dx}(\log_b |x|) = \frac{1}{x \ln b}$ for $x \neq 0$.

A. $\frac{d}{dx}(\log_b x) = \left(\frac{d}{dx} \ln x \right) \div \ln b$

72. Find $\frac{d}{dx}(\ln \sqrt{x^2 + 16})$.

$$\frac{d}{dx}(\ln \sqrt{x^2 + 16}) = \boxed{}$$

Answer: $\frac{x}{x^2 + 16}$

73. Express the function $f(x) = g(x)^{h(x)}$ in terms of the natural logarithmic and natural exponential functions (base e).

$$f(x) = \boxed{}$$

Answer: $e^{h(x) \ln g(x)}$

74. Find the derivative.

$$\frac{d}{dx} (\ln (6x^2 + 5))$$

$$\frac{d}{dx} (\ln (6x^2 + 5)) = \boxed{}$$

Answer: $\frac{12x}{6x^2 + 5}$

75. Evaluate the derivative.

$$y = 4x^{5\pi}$$

$$y' = \boxed{} \text{ (Type an exact answer.)}$$

Answer: $20\pi x^{(5\pi - 1)}$

76. Find $\frac{dy}{dx}$ for the function $y = 7^x$.

$$\frac{dy}{dx} = \boxed{}$$

Answer: $7^x \ln 7$

77. Calculate the derivative of the following function.

$$y = 7 \log_2 (x^4 - 2)$$

$$\frac{d}{dx} 7 \log_2 (x^4 - 2) = \boxed{}$$

Answer: $\frac{28x^3}{(x^4 - 2) \ln 2}$

78. Differentiate.

$$y = \log_3 x$$

$$\frac{d}{dx} \log_3 x = \boxed{}$$

Answer: $\frac{1}{x \cdot \ln 3}$

79. Use logarithmic differentiation to evaluate $f'(x)$.

$$f(x) = \frac{(x+4)^{12}}{(4x-12)^{10}}$$

$f'(x) =$

Answer: $\frac{(x+4)^{12}}{(4x-12)^{10}} \left[\frac{12}{x+4} - \frac{10}{x-3} \right]$

80. State the derivative formulas for $\sin^{-1}x$, $\tan^{-1}x$, and $\sec^{-1}x$.

What is the derivative of $\sin^{-1}x$?

- ☐ A. $-\frac{1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$
- ☐ B. $-\frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$
- ☐ C. $\frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$
- ☐ D. $\frac{1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$

What is the derivative of $\tan^{-1}x$?

- ☐ A. $\frac{1}{1+x^2}$ for $-\infty < x < \infty$
- ☐ B. $-\frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$
- ☐ C. $-\frac{1}{1+x^2}$ for $-\infty < x < \infty$
- ☐ D. $-\frac{1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$

What is the derivative of $\sec^{-1}x$?

- ☐ A. $\frac{1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$
- ☐ B. $\frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$
- ☐ C. $-\frac{1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$
- ☐ D. $-\frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$

Answers C. $\frac{1}{\sqrt{1-x^2}}$ for $-1 < x < 1$

A. $\frac{1}{1+x^2}$ for $-\infty < x < \infty$

A. $\frac{1}{|x|\sqrt{x^2-1}}$ for $|x| > 1$

81. Evaluate the derivative of the function.

$$f(x) = \sin^{-1}(6x^4)$$

$$f'(x) = \boxed{}$$

Answer: $\frac{24x^3}{\sqrt{1-36x^8}}$

82. Find the derivative of the function $y = 7 \tan^{-1}(3x)$.

$$\frac{dy}{dx} = \boxed{}$$

Answer: $\frac{21}{1+(3x)^2}$

83. Evaluate the derivative of the following function.

$$f(s) = \cot^{-1}(e^s)$$

$$\frac{d}{ds} \cot^{-1}(e^s) = \boxed{}$$

Answer: $-\frac{e^s}{1+e^{2s}}$

84. The sides of a square increase in length at a rate of 4 m/sec.

- a. At what rate is the area of the square changing when the sides are 20 m long?
 b. At what rate is the area of the square changing when the sides are 25 m long?

a. Write an equation relating the area of a square, A , and the side length of the square, s .

Differentiate both sides of the equation with respect to t .

$$\frac{dA}{dt} = \left(\frac{\quad}{\quad} \right) \frac{ds}{dt}$$

The area of the square is changing at a rate of (1) when the sides are 20 m long.

b. The area of the square is changing at a rate of (2) when the sides are 25 m long.

- | | |
|-----------------------------------|-------------------------------|
| (1) <input type="radio"/> m^2/s | (2) <input type="radio"/> m |
| <input type="radio"/> m/s | <input type="radio"/> m^2/s |
| <input type="radio"/> m | <input type="radio"/> m/s |
| <input type="radio"/> m^3/s | <input type="radio"/> m^3/s |

Answers $A = s^2$

$2s$

160

(1) m^2/s

200

(2) m^2/s

85. The area of a circle increases at a rate of $2 \text{ cm}^2 / \text{s}$.

- a. How fast is the radius changing when the radius is 4 cm?
 b. How fast is the radius changing when the circumference is 6 cm?

a. Write an equation relating the area of a circle, A , and the radius of the circle, r .

(Type an exact answer, using π as needed.)

Differentiate both sides of the equation with respect to t .

$$\frac{dA}{dt} = \left(\frac{\quad}{\quad} \right) \frac{dr}{dt}$$

(Type an exact answer, using π as needed.)

When the radius is 4 cm, the radius is changing at a rate of (1)
 (Type an exact answer, using π as needed.)

b. When the circumference is 6 cm, the radius is changing at a rate of (2)
 (Type an exact answer, using π as needed.)

- (1) ☐ cm^2 / s . (2) ☐ cm^3 / s .
☐ cm^3 / s . ☐ cm^2 / s .
☐ cm . ☐ cm / s .
☐ cm / s . ☐ cm .

Answers $A = \pi r^2$

$$2\pi r$$

$$\frac{1}{4\pi}$$

(1) cm / s .

$$\frac{1}{3}$$

(2) cm / s .

86. A rope passing through a capstan on a dock is attached to a boat offshore. The rope is pulled in at a constant rate of 3 ft/s, and the capstan is 4 ft vertically above the water. How fast is the boat traveling when it is 7 ft from the dock?

Let x be the horizontal distance from the boat to the dock and z be the length of the rope. Write an equation relating x and z .

Differentiate both sides of the equation with respect to t .

$$\left(\text{ } \right) \frac{dx}{dt} = \left(\text{ } \right) \frac{dz}{dt}$$

When the boat is 7 ft from the dock, it is traveling at about (1)
(Round to two decimal places as needed.)

- (1) ☒ ft^3/s .
☐ ft^2/s .
☐ ft .
☐ ft/s .

Answers $x^2 + 4^2 = z^2$

$2x$

$2z$

3.46

(1) ft/s.

87. A 13-foot ladder is leaning against a vertical wall (see figure) when Jack begins pulling the foot of the ladder away from the wall at a rate of 0.7 ft/s . How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 ft from the wall?



Let x be the distance from the foot of the ladder to the wall and let y be the distance from the top of the ladder to the ground. Write an equation relating x and y .

Differentiate both sides of the equation with respect to t .

$$\left(\frac{\quad}{\quad} \right) \frac{dx}{dt} + \left(\frac{\quad}{\quad} \right) \frac{dy}{dt} = \frac{\quad}{\quad}$$

When the foot of the ladder is 5 ft from the wall, the top of the ladder is sliding down the wall at a rate of

(1)

(Round to two decimal places as needed.)

- (1) ☐ ft^2/s .
☐ ft^3/s .
☐ ft .
☐ ft/s .

Answers $x^2 + y^2 = 169$

$2x$

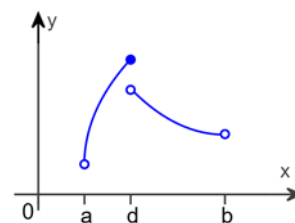
$2y$

0

0.29

$(1) \text{ ft/s}$.

88. Determine from the graph whether the function has any absolute extreme values on $[a, b]$.



Where do the absolute extreme values of the function occur on $[a, b]$?

- ☐ A. The absolute maximum occurs at $x = d$ and the absolute minimum occurs at $x = a$ on $[a, b]$.
- ☐ B. The absolute maximum occurs at $x = d$ and there is no absolute minimum on $[a, b]$.
- ☐ C. There is no absolute maximum and the absolute minimum occurs at $x = a$ on $[a, b]$.
- ☐ D. There is no absolute maximum and there is no absolute minimum on $[a, b]$.

Answer: B. The absolute maximum occurs at $x = d$ and there is no absolute minimum on $[a, b]$.

89. Find the critical points of the following function.

$$f(x) = 2x^2 + 3x + 1$$

What is the derivative of $f(x) = 2x^2 + 3x + 1$?

$$f'(x) = \boxed{}$$

Find the critical points, if any, of f on the domain. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The critical point(s) occur(s) at $x = $.
(Use a comma to separate answers as needed.)
- ☐ B. There are no critical points for $f(x) = 2x^2 + 3x + 1$ on the domain.

Answers $4x + 3$

A. The critical point(s) occur(s) at $x = \boxed{-\frac{3}{4}}$. (Use a comma to separate answers as needed.)

90. Find the critical points of the following function.

$$f(x) = 2x^2 + 5x + 1$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The critical point(s) occur(s) at $x = $.
(Use a comma to separate answers as needed.)
- ☐ B. There are no critical points.

Answer: A. The critical point(s) occur(s) at $x = \boxed{-\frac{5}{4}}$. (Use a comma to separate answers as needed.)

91. Find the critical points of the following function.

$$f(x) = -\frac{x^3}{3} + 144x$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The critical point(s) occur(s) at $x =$ _____ .
(Use a comma to separate answers as needed.)
- ☐ B. There are no critical points.

Answer: A. The critical point(s) occur(s) at $x =$. (Use a comma to separate answers as needed.)

92. Determine the location and value of the absolute extreme values of f on the given interval, if they exist.

$$f(x) = x^2 - 11 \text{ on } [-3, 4]$$

What is/are the absolute maximum/maxima of f on the given interval? Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- ☐ A. The absolute maximum/maxima is/are _____ at $x =$ _____ .
(Use a comma to separate answers as needed.)
- ☐ B. There is no absolute maximum of f on the given interval.

What is/are the absolute minimum/minima of f on the given interval? Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- ☐ A. The absolute minimum/minima is/are _____ at $x =$ _____ .
(Use a comma to separate answers as needed.)
- ☐ B. There is no absolute minimum of f on the given interval.

Answers A. The absolute maximum/maxima is/are at $x =$.
(Use a comma to separate answers as needed.)

A. The absolute minimum/minima is/are at $x =$.
(Use a comma to separate answers as needed.)

93. All boxes with a square base, an open top, and a volume of 210 ft^3 have a surface area given by $S(x) = x^2 + \frac{840}{x}$, where x is the length of the sides of the base. Find the absolute minimum of the surface area function on the interval $(0, \infty)$. What are the dimensions of the box with minimum surface area?

Determine the derivative of the given function $S(x)$.

$$S'(x) = \boxed{}$$

The absolute minimum value of the surface area function is $\boxed{} \text{ ft}^2$.
(Round to three decimal places as needed.)

The dimensions of the box with minimum surface area are a base of length $\boxed{} \text{ ft}$ and a height of $\boxed{} \text{ ft}$.
(Round to three decimal places as needed.)

Answers $2x - \frac{840}{x^2}$

168.250

7.489

3.744

94. A stone is launched vertically upward from a cliff 48 ft above the ground at a speed of 32 ft / s. Its height above the ground t seconds after the launch is given by $s = -16t^2 + 32t + 48$ for $0 \leq t \leq 3$. When does the stone reach its maximum height?

Find the derivative of s .

$$s' = \boxed{}$$

The stone reaches its maximum height at $\boxed{} \text{ s}$.
(Simplify your answer.)

Answers $-32t + 32$

1

95. Suppose a tour guide has a bus that holds a maximum of 82 people. Assume his profit (in dollars) for taking n people on a city tour is $P(n) = n(41 - 0.5n) - 82$. (Although P is defined only for positive integers, treat it as a continuous function.)

- a. How many people should the guide take on a tour to maximize the profit?
 b. Suppose the bus holds a maximum of 34 people. How many people should be taken on a tour to maximize the profit?

- a. Find the derivative of the given function $P(n)$.

$$P'(n) = \boxed{}$$

If the bus holds a maximum of 82 people, the guide should take $\boxed{}$ people on a tour to maximize the profit.

- b. If the bus holds a maximum of 34 people, the guide should take $\boxed{}$ people on a tour to maximize the profit.

Answers $-n + 41$

41

34

96. At what points c does the conclusion of the Mean Value Theorem hold for $f(x) = x^3$ on the interval $[-6, 6]$?

The conclusion of the Mean Value Theorem holds for $c = \boxed{}$.

(Use a comma to separate answers as needed. Type an exact answer, using radicals as needed.)

Answer: $2\sqrt{3}, -2\sqrt{3}$

97. a. Determine whether the Mean Value Theorem applies to the function $f(x) = -6 + x^2$ on the interval $[-2, 1]$.
 b. If so, find the point(s) that are guaranteed to exist by the Mean Value Theorem.

- a. Choose the correct answer below.

- ☐ A. Yes, because the function is continuous on the interval $[-2, 1]$ and differentiable on the interval $(-2, 1)$.
☐ B. No, because the function is not continuous on the interval $[-2, 1]$, and is not differentiable on the interval $(-2, 1)$.
☐ C. No, because the function is differentiable on the interval $(-2, 1)$, but is not continuous on the interval $[-2, 1]$.
☐ D. No, because the function is continuous on the interval $[-2, 1]$, but is not differentiable on the interval $(-2, 1)$.

- b. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The point(s) is/are $x = \underline{\hspace{2cm}}$.
 (Simplify your answer. Use a comma to separate answers as needed.)
☐ B. The Mean Value Theorem does not apply in this case.

Answers A. Yes, because the function is continuous on the interval $[-2, 1]$ and differentiable on the interval $(-2, 1)$.

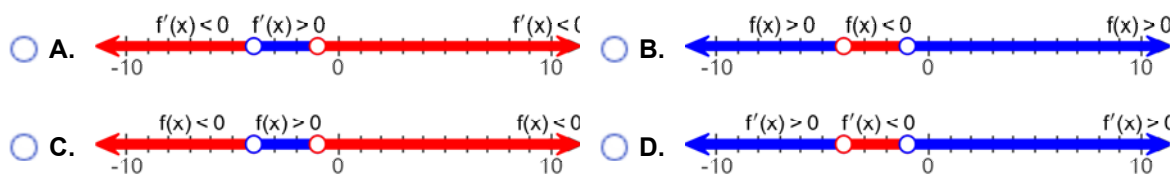
A. The point(s) is/are $x = \boxed{-\frac{1}{2}}$.

(Simplify your answer. Use a comma to separate answers as needed.)

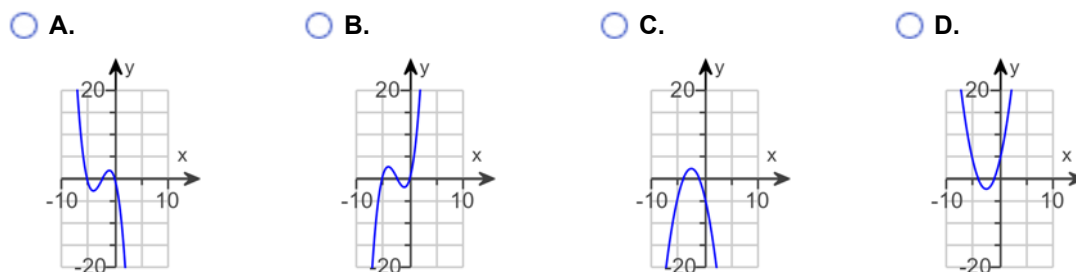
98. Sketch a function that is continuous on $(-\infty, \infty)$ and has the following properties. Use a number line to summarize information about the function.

$$f'(x) < 0 \text{ on } (-\infty, -4); f'(x) > 0 \text{ on } (-4, -1); f'(x) < 0 \text{ on } (-1, \infty).$$

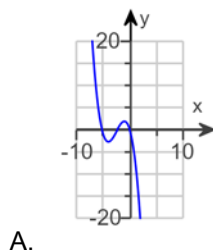
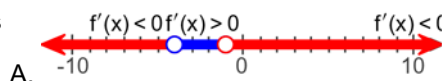
Which number line summarizes the information about the function?



Which of the following graphs matches the description of the given properties?



Answers



99. Find the intervals on which f is increasing and the intervals on which it is decreasing.

$$f(x) = 7 - x^2$$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- ☐ A. The function is increasing on _____ and decreasing on _____.
(Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)
- ☐ B. The function is decreasing on _____. The function is never increasing.
(Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- ☐ C. The function is increasing on _____. The function is never decreasing.
(Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- ☐ D. The function is never increasing nor decreasing.

Answer: A. The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.

(Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)

100. Find the intervals on which f is increasing and the intervals on which it is decreasing.

$$f(x) = -6 + x - x^2$$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- ☐ A. The function is increasing on _____ and decreasing on _____.
(Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)
- ☐ B. The function is decreasing on _____. The function is never increasing.
(Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- ☐ C. The function is increasing on _____. The function is never decreasing.
(Simplify your answer. Type your answer in interval notation. Use a comma to separate answers as needed.)
- ☐ D. The function is never increasing nor decreasing.

Answer: A. The function is increasing on $\left(-\infty, \frac{1}{2}\right)$ and decreasing on $\left(\frac{1}{2}, \infty\right)$.

(Simplify your answers. Type your answers in interval notation. Use a comma to separate answers as needed.)

101. Locate the critical points of the following function. Then use the Second Derivative Test to determine whether they correspond to local maxima, local minima, or neither.

$$f(x) = 7 - 4x^2$$

What is(are) the critical point(s) of f ? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The critical point(s) is(are) $x =$ _____.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- ☐ B. There are no critical points for f .

What is/are the local maximum/maxima of f ? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The local maximum/maxima of f is/are at $x =$ _____.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- ☐ B. There is no local maximum of f .

What is/are the local minimum/minima of f ? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The local minimum/minima of f is/are at $x =$ _____.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- ☐ B. There is no local minimum of f .

Answers A. The critical point(s) is(are) $x =$.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local maximum/maxima of f is/are at $x =$.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

B. There is no local minimum of f .

102. Locate the critical points of the following function. Then use the Second Derivative Test to determine whether they correspond to local maxima, local minima, or neither.

$$f(x) = -3x^3 - 18x^2 + 12$$

What is(are) the critical point(s) of f ? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The critical point(s) is(are) $x =$ _____.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- ☐ B. There are no critical points for f .

What is/are the local maximum/maxima of f ? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The local maximum/maxima of f is/are at $x =$ _____.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- ☐ B. There is no local maximum of f .

What is/are the local minimum/minima of f ? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. The local minimum/minima of f is/are at $x =$ _____.
(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- ☐ B. There is no local minimum of f .

Answers A. The critical point(s) is(are) $x =$.

(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local maximum/maxima of f is/are at $x =$.

(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

A. The local minimum/minima of f is/are at $x =$.

(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

103. Fill in the blanks: The goal of an optimization problem is to find the maximum or minimum value of the _____ function subject to the _____.

The goal of an optimization problem is to find the maximum or minimum value of the (1) function subject to the (2) .

- | | |
|---|---|
| (1) <input type="radio"/> optimization function | (2) <input type="radio"/> extreme values. |
| <input type="radio"/> objective function | <input type="radio"/> variables. |
| <input type="radio"/> constraint function | <input type="radio"/> constraints. |
| <input type="radio"/> subjective function | <input type="radio"/> optimizations. |

Answers (1) objective function

(2) constraints.

104. Use a linear approximation to estimate the following quantity. Choose a value of a to produce a small error.

$$\ln(1.05)$$

What is the value found using the linear approximation?

$$\ln(1.05) \approx \boxed{} \quad (\text{Round to two decimal places as needed.})$$

Answer: 0.05

105. Consider the following function and express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)dx$.

$$f(x) = e^{11x}$$

$$dy = \left(\boxed{\phantom{11e^{11x}}} \right) dx \quad (\text{Type an exact answer in terms of } e.)$$

Answer: $11e^{11x}$

106. Consider the following function and express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x)dx$.

$$f(x) = 2x^3 - 3x$$

$$dy = \left(\boxed{} \right) dx$$

Answer: $6x^2 - 3$

107. Consider the following function and express the relationship between a small change in x and the corresponding change in y in the form $dy = f'(x) dx$.

$$f(x) = \cot 3x$$

$$dy = \boxed{} dx$$

Answer: $-3\csc^2(3x)$

108. Evaluate the following limit. Use l'Hôpital's Rule when it is convenient and applicable.

$$\lim_{x \rightarrow 0} \frac{9 \sin 6x}{5x}$$

Use l'Hôpital's Rule to rewrite the given limit so that it is not an indeterminate form.

$$\lim_{x \rightarrow 0} \frac{9 \sin 6x}{5x} = \lim_{x \rightarrow 0} \left(\frac{\quad}{\quad} \right)$$

Evaluate the limit.

$$\lim_{x \rightarrow 0} \frac{9 \sin 6x}{5x} = \frac{\quad}{\quad} \text{ (Type an exact answer.)}$$

Answers $\frac{54 \cos(6x)}{5}$

$$\frac{54}{5}$$

109. Evaluate the following limit. Use l'Hôpital's Rule when it is convenient and applicable.

$$\lim_{u \rightarrow \frac{\pi}{4}} \frac{\tan u - \cot u}{4u - \pi}$$

Use l'Hôpital's Rule to rewrite the given limit so that it is not an indeterminate form.

$$\lim_{u \rightarrow \frac{\pi}{4}} \frac{\tan u - \cot u}{4u - \pi} = \lim_{u \rightarrow \frac{\pi}{4}} \left(\frac{\quad}{\quad} \right)$$

Evaluate the limit.

$$\lim_{u \rightarrow \frac{\pi}{4}} \frac{\tan u - \cot u}{4u - \pi} = \frac{\quad}{\quad} \text{ (Type an exact answer.)}$$

Answers $\frac{\sec^2 u + \csc^2 u}{4}$

$$1$$

110. Use a calculator or program to compute the first 10 iterations of Newton's method when they are applied to the following function with the given initial approximation.

$$f(x) = x^2 - 23; x_0 = 5$$

k	x_k	k	x_k
0	<input type="text"/>	6	<input type="text"/>
1	<input type="text"/>	7	<input type="text"/>
2	<input type="text"/>	8	<input type="text"/>
3	<input type="text"/>	9	<input type="text"/>
4	<input type="text"/>	10	<input type="text"/>
5	<input type="text"/>		

(Round to six decimal places as needed.)

Answers 5.000000

4.795832

4.800000

4.795832

4.795833

4.795832

4.795832

4.795832

4.795832

4.795832

4.795832

111. Use a calculator or program to compute the first 10 iterations of Newton's method for the given function and initial approximation.

$$f(x) = 2 \sin x + x + 1, x_0 = 1.1$$

Complete the table.

(Do not round until the final answer. Then round to six decimal places as needed.)

k	x_k	k	x_k
1	<input type="text"/>	6	<input type="text"/>
2	<input type="text"/>	7	<input type="text"/>
3	<input type="text"/>	8	<input type="text"/>
4	<input type="text"/>	9	<input type="text"/>
5	<input type="text"/>	10	<input type="text"/>

Answers - 0.935670

- 0.337584

- 0.228777

- 0.337584

- 0.336532

- 0.337584

- 0.337584

- 0.337584

- 0.337584

- 0.337584

112. Determine the following indefinite integral. Check your work by differentiation.

$$\int (7x^{13} - 11x^{21}) \, dx$$

$$\int (7x^{13} - 11x^{21}) \, dx = \boxed{} \text{ (Use } C \text{ as the arbitrary constant.)}$$

Answer: $\frac{x^{14}}{2} - \frac{x^{22}}{2} + C$

113. Evaluate the following indefinite integral.

$$\int \left(\frac{8}{\sqrt{x}} + 8\sqrt{x} \right) dx$$

$$\int \left(\frac{8}{\sqrt{x}} + 8\sqrt{x} \right) dx = \boxed{}$$

(Use C as the arbitrary constant.)

Answer:

$$16\sqrt{x} + \frac{16}{3}x^{\frac{3}{2}} + C$$

114. Determine the following indefinite integral.

$$\int \left(\frac{5}{w^2} - 6w^6 \right) dw$$

$$\int \left(\frac{5}{w^2} - 6w^6 \right) dw = \boxed{}$$

(Use C as the arbitrary constant.)

Answer:

$$-5w^{-1} - \frac{6}{7}w^7 + C$$

115. Find $\int (6x + 5)^2 dx$.

$$\int (6x + 5)^2 dx = \boxed{}$$

(Use C as the arbitrary constant.)

$$\text{Answer: } 12x^3 + 30x^2 + 25x + C$$

116. Determine the following indefinite integral. Check your work by differentiation.

$$\int 4m(5m^2 - 7m) dm$$

$$\int 4m(5m^2 - 7m) dm = \boxed{} \text{ (Use C as the arbitrary constant.)}$$

Answer:

$$5m^4 - \frac{28m^3}{3} + C$$

117. Determine the following indefinite integral. Check your work by differentiation.

$$\int \left(4x^{\frac{1}{3}} + 2x^{-\frac{1}{3}} + 6 \right) dx$$

$$\int \left(4x^{\frac{1}{3}} + 2x^{-\frac{1}{3}} + 6 \right) dx = \boxed{} \text{ (Use C as the arbitrary constant.)}$$

Answer: $\frac{4}{3}x^{\frac{4}{3}} + 2x^{\frac{2}{3}} + 6x + C$

118. Determine the following indefinite integral. Check your work by differentiation.

$$\int 4\sqrt[5]{x} \, dx$$

$$\int 4\sqrt[5]{x} \, dx = \boxed{} \text{ (Use C as the arbitrary constant.)}$$

Answer: $\frac{10}{3}x^{\frac{6}{5}} + C$

119. Determine the following indefinite integral. Check your work by differentiation.

$$\int (4x + 5)(3 - x) \, dx$$

$$\int (4x + 5)(3 - x) \, dx = \boxed{} \text{ (Use C as the arbitrary constant.)}$$

Answer: $-\frac{4}{3}x^3 + \frac{7}{2}x^2 + 15x + C$

120. Determine the following indefinite integral. Check your work by differentiation.

$$\int \left(\frac{6}{x^4} + 2 - \frac{8}{x^2} \right) dx$$

$$\int \left(\frac{6}{x^4} + 2 - \frac{8}{x^2} \right) dx = \boxed{} \text{ (Use C as the arbitrary constant.)}$$

Answer: $-\frac{2}{x^3} + 2x + \frac{8}{x} + C$

121. Determine the following indefinite integral.

$$\int \frac{3x^5 - 6x^4}{x^3} dx$$

$$\int \frac{3x^5 - 6x^4}{x^3} dx = \boxed{}$$

(Use C as the arbitrary constant.)

Answer: $x^3 - 3x^2 + C$

122. Determine the following indefinite integral. Check your work by differentiation.

$$\int (\csc^2 2\theta + 8) d\theta$$

$$\int (\csc^2 2\theta + 8) d\theta = \boxed{} \text{ (Use C as the arbitrary constant.)}$$

Answer: $-\frac{1}{2} \cot(2\theta) + 8\theta + C$

123. Determine the following indefinite integral. Check your work by differentiation.

$$\int (\sec^2 x - 4) dx$$

$$\int (\sec^2 x - 4) dx = \boxed{} \text{ (Use C as the arbitrary constant.)}$$

Answer: $\tan x - 4x + C$

124. For the following function f, find the antiderivative F that satisfies the given condition.

$$f(x) = 8x^3 + 3 \sin x, F(0) = 2$$

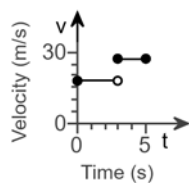
The antiderivative that satisfies the given condition is $F(x) = \boxed{}$.

Answer: $2x^4 - 3 \cos x + 5$

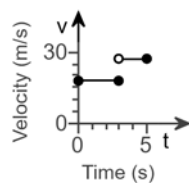
125. Suppose an object moves along a line at 18 m/s for $0 \leq t \leq 3$ s and at 27 m/s for $3 < t \leq 5$ s. Sketch the graph of the velocity function and find the displacement of the object for $0 \leq t \leq 5$.

Sketch the graph of the velocity function. Choose the correct graph below.

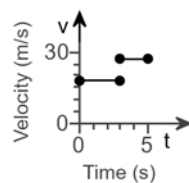
☐ A.



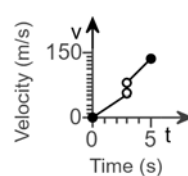
☐ B.



☐ C.

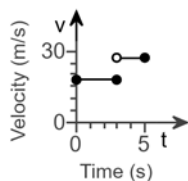


☐ D.



The displacement of the object for $0 \leq t \leq 5$ is m. (Simplify your answer.)

Answers

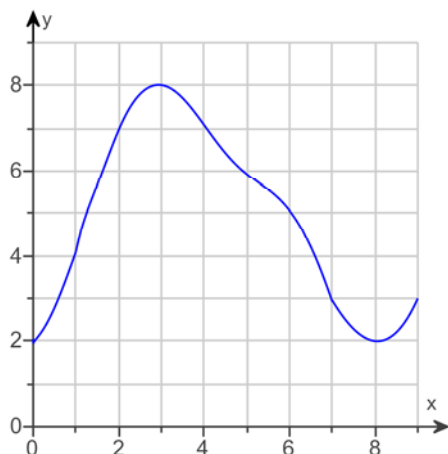


B.

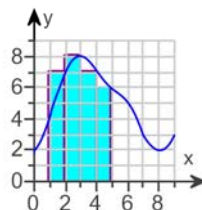
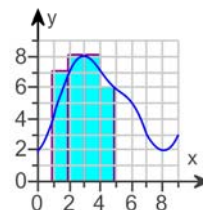
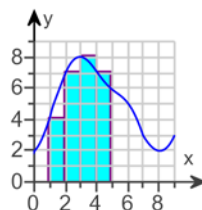
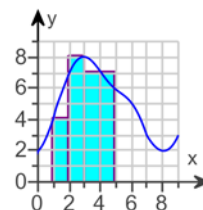
108

126.

Approximate the area of the region bounded by the graph of $f(x)$ (shown below) and the x -axis by dividing the interval $[1, 5]$ into $n = 4$ subintervals. Use a left and right Riemann sum to obtain two different approximations. Draw the approximating rectangles.

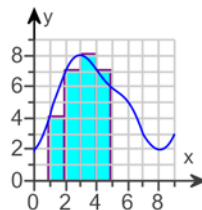
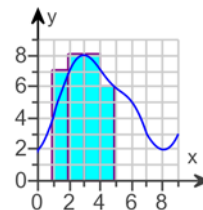
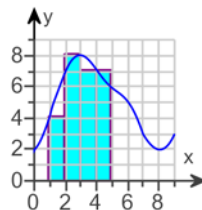
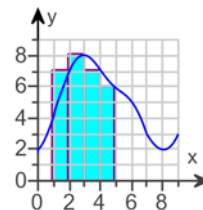


In which graph below are the selected points the left endpoints of the 4 approximating rectangles?

☐ A.

☐ B.

☐ C.

☐ D.


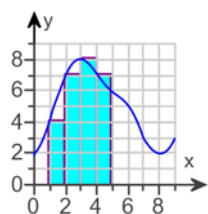
Using the specified rectangles, approximate the area.

In which graph below are the selected points the right endpoints of the 4 approximating rectangles?

☐ A.

☐ B.

☐ C.

☐ D.


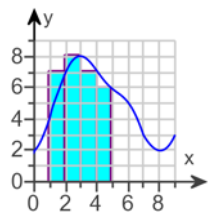
Using the specified rectangles, approximate the area.

Answers



C.

26



D.

28

-
127. Does the right Riemann sum underestimate or overestimate the area of the region under the graph of a positive decreasing function? Explain.

Choose the correct answer below.

- ☐ A. Overestimate; the rectangles do not fit under the curve.
- ☐ B. Overestimate; the rectangles all fit under the curve.
- ☐ C. Underestimate; the rectangles do not fit under the curve.
- ☐ D. Underestimate; the rectangles all fit under the curve.

Answer: D. Underestimate; the rectangles all fit under the curve.

128. The following function is negative on the given interval.

$$f(x) = -5 - x^2; [3, 7]$$

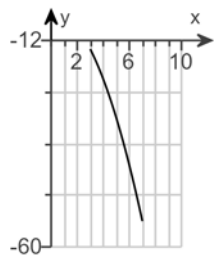
- a. Sketch the function on the given interval.
 b. Approximate the net area bounded by the graph of f and the x -axis on the interval using a left, right, and midpoint Riemann sum with $n = 4$.

a. Choose the correct graph below.

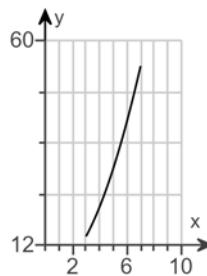
☐ A.



☐ B.



☐ C.

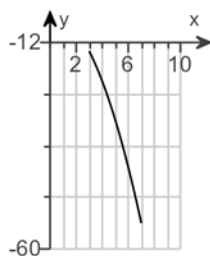


b. The approximate net area using a left Riemann sum is .
 (Type an integer or a decimal.)

The approximate net area using a midpoint Riemann sum is .
 (Type an integer or a decimal.)

The approximate net area using a right Riemann sum is .
 (Type an integer or a decimal.)

Answers



B.

- 106

- 125

- 146

129. Use the definition of the definite integral to evaluate $\int_0^3 (3x + 2) \, dx$. Use right Riemann sums and theorems.

$$\int_0^3 (3x + 2) \, dx = \text{} \quad (\text{Simplify your answer.})$$

Answer: 19.5

130.

Evaluate $\frac{d}{dx} \int_a^x f(t) dt$ and $\frac{d}{dx} \int_a^b f(t) dt$, where a and b are constants.

$$\frac{d}{dx} \int_a^x f(t) dt = \boxed{} \text{ (Simplify your answer.)}$$

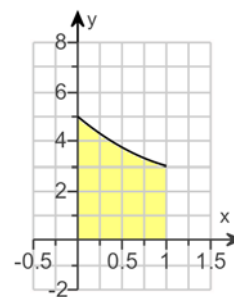
$$\frac{d}{dx} \int_a^b f(t) dt = \boxed{} \text{ (Simplify your answer.)}$$

Answers $f(x)$

0

131. Evaluate the following integral using the Fundamental Theorem of Calculus. Discuss whether your result is consistent with the figure.

$$\int_0^1 (x^2 - 3x + 5) dx$$



$$\int_0^1 (x^2 - 3x + 5) dx = \boxed{}$$

Is your result consistent with the figure?

- ☐ A. Yes, because the definite integral is negative and the graph of f lies below the x -axis.
- ☐ B. Yes, because the definite integral is positive and the graph of f lies above the x -axis.
- ☐ C. No, because the definite integral is positive and the graph of f lies below the x -axis.
- ☐ D. No, because the definite integral is negative and the graph of f lies above the x -axis.

Answers $\frac{23}{6}$

B. Yes, because the definite integral is positive and the graph of f lies above the x -axis.

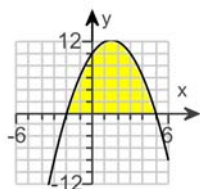
132. Evaluate the following integral using the fundamental theorem of calculus. Sketch the graph of the integrand and shade the region whose net area you have found.

$$\int_{-2}^5 (x^2 - 3x - 10) dx$$

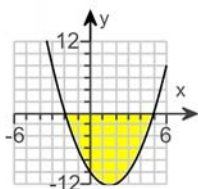
$$\int_{-2}^5 (x^2 - 3x - 10) dx = \boxed{}$$

Choose the correct sketch below.

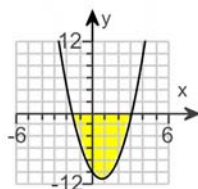
☐ A.



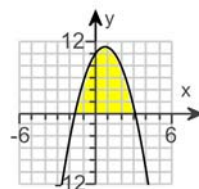
☐ B.



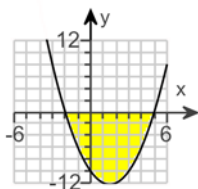
☐ C.



☐ D.



Answers $-\frac{343}{6}$



B.

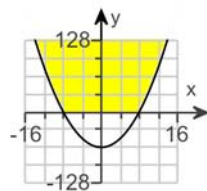
133. Find (i) the net area and (ii) the area of the region above the x-axis bounded by $y = 64 - x^2$. Graph the function and indicate the region in question.

Set up the integral (or integrals) needed to compute this area. Select the correct choice below and fill in the answer boxes to complete your answer.

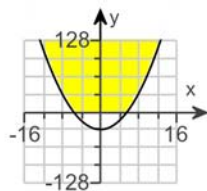
- ☐ A. $\int_{-8}^{\quad} (\quad) dx$
- ☐ B. $\int_{\quad}^0 (\quad) dx + \int_0^{\quad} (\quad) dx$

Choose the correct graph below.

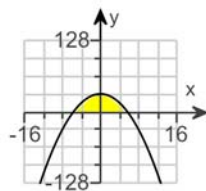
☐ A.



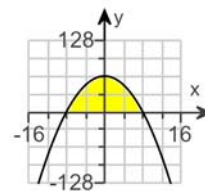
☐ B.



☐ C.



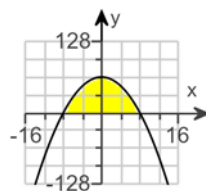
☐ D.



(i) The net area is .

(ii) The area is .

Answers A. $\int_{-8}^{\quad} (\quad) dx$



D.

$$\frac{2048}{3}$$

$$\frac{2048}{3}$$

134. Is x^{37} an even or odd function? Is $\cos^3(x)$ an even or odd function?

Is x^{37} an even or odd function?

- ☐ odd function
☐ even function

Is $\cos^3(x)$ an even or odd function?

- ☐ odd function
☐ even function

Answers odd function

even function

135. On which derivative rule is the Substitution Rule based?

Choose the correct answer below.

- ☐ A. The Constant Multiple Rule
☐ B. The Product Rule
☐ C. The Quotient Rule
☐ D. The Chain Rule

Answer: D. The Chain Rule

136. Use the substitution $u = x^2 + 20$ to find the following indefinite integral. Check your answer by differentiation.

$$\int 2x(x^2 + 20)^{12} dx$$

$$\int 2x(x^2 + 20)^{12} dx = \boxed{}$$

(Use C as the arbitrary constant.)

Answer: $\frac{1}{13}(x^2 + 20)^{13} + C$

137. Use the substitution $u = 5x^2 + 1$ to find the following indefinite integral. Check your answer by differentiation.

$$\int 10x \cos(5x^2 + 1) dx$$

$$\int 10x \cos(5x^2 + 1) dx = \boxed{}$$

(Use C as the arbitrary constant.)

Answer: $\sin(5x^2 + 1) + C$

138. Use the substitution $u = 2x^2 + 5x$ to evaluate the indefinite integral below.

$$\int (4x + 5)\sqrt{2x^2 + 5x} \, dx$$

Write the integrand in terms of u .

$$\int (4x + 5)\sqrt{2x^2 + 5x} \, dx = \int (\boxed{}) \, du$$

Evaluate the integral.

$$\int (4x + 5)\sqrt{2x^2 + 5x} \, dx = \boxed{}$$

(Use C as the arbitrary constant.)

Answers \sqrt{u}

$$\frac{2}{3} (2x^2 + 5x)^{\frac{3}{2}} + C$$

139. Use the substitution $u = x^7 + 6x$ to find the following indefinite integral. Check your answer by differentiation.

$$\int (7x^6 + 6)\sqrt{x^7 + 6x} \, dx$$

$$\int (7x^6 + 6)\sqrt{x^7 + 6x} \, dx = \boxed{}$$

(Use C as the arbitrary constant.)

Answer:

$$\frac{2}{3} (x^7 + 6x)^{\frac{3}{2}} + C$$

140. Find an antiderivative of the following function by trial and error. Check your answer by differentiation.

$$e^{7x+3}$$

$$\int e^{7x+3} \, dx = \boxed{}$$

(Use C as the arbitrary constant.)

$$\text{Answer: } \frac{1}{7} e^{7x+3} + C$$

141. Use a change of variables or the table to evaluate the following indefinite integral.

$$\int x^7 e^{x^8} dx$$

¹ Click the icon to view the table of general integration formulas.

$$\int x^7 e^{x^8} dx = \boxed{} \text{ (Use } C \text{ as the arbitrary constant.)}$$

1: General Integration Formulas

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C$$

$$\int \csc ax \cot ax \, dx = -\frac{1}{a} \csc ax + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int b^x dx = \frac{1}{\ln b} b^x + C, b > 0, b \neq 1$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C, a > 0$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C, a > 0$$

Answer: $\frac{1}{8} e^{x^8} + C$

142. Use a change of variables or the table to evaluate the following indefinite integral.

$$\int (x^2 + x)^8 (2x + 1) dx$$

² Click the icon to view the table of general integration formulas.

$$\int (x^2 + x)^8 (2x + 1) dx = \boxed{} \text{ (Use } C \text{ as the arbitrary constant.)}$$

2: General Integration Formulas

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C$$

$$\int \csc ax \cot ax \, dx = -\frac{1}{a} \csc ax + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int b^x dx = \frac{1}{\ln b} b^x + C, b > 0, b \neq 1$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C, a > 0$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C, a > 0$$

Answer: $\frac{(x^2 + x)^9}{9} + C$

143. Use a change of variables or the accompanying table to evaluate the following indefinite integral.

$$\int \frac{e^{8x}}{e^{8x} + 5} dx$$

³ Click the icon to view the table of general integration formulas.

Determine a change of variables from x to u . Choose the correct answer below.

- ☐ A. $u = \frac{1}{e^{8x} + 5}$
- ☐ B. $u = e^{8x}$
- ☐ C. $u = e^{8x} + 5$
- ☐ D. $u = 8x$

Write the integral in terms of u .

$$\int \frac{e^{8x}}{e^{8x} + 5} dx = \int (\text{ }) du$$

Evaluate the integral.

$$\int \frac{e^{8x}}{e^{8x} + 5} dx = \text{ }$$

(Use C as the arbitrary constant.)

3: General Integration Formulas

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C$$

$$\int \csc ax \cot ax \, dx = -\frac{1}{a} \csc ax + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int b^x dx = \frac{1}{\ln b} b^x + C, b > 0, b \neq 1$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C, a > 0$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C, a > 0$$

Answers C. $u = e^{8x} + 5$

$$\frac{1}{8u}$$

$$\frac{1}{8} \ln |e^{8x} + 5| + C$$

144. Use a change of variables or the table to evaluate the following definite integral.

$$\int_0^{\pi/15} \cos 5x \, dx$$

⁴ Click to view the table of general integration formulas.

$$\int_0^{\pi/15} \cos 5x \, dx = \boxed{} \text{ (Type an exact answer.)}$$

4: General Integration Formulas

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C$$

$$\int \csc ax \cot ax \, dx = -\frac{1}{a} \csc ax + C$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$$

$$\int b^x \, dx = \frac{1}{\ln b} b^x + C, b > 0, b \neq 1$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C, a > 0$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C, a > 0$$

Answer: $\frac{\sqrt{3}}{10}$

145. Use a change of variables or the table to evaluate the following definite integral.

$$\int_0^2 12 e^{3x} dx$$

⁵ Click to view the table of general integration formulas.

$$\int_0^2 12 e^{3x} dx = \boxed{} \text{ (Type an exact answer.)}$$

5: General Integration Formulas

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C$$

$$\int \csc ax \cot ax \, dx = -\frac{1}{a} \csc ax + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int b^x dx = \frac{1}{\ln b} b^x + C, b > 0, b \neq 1$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C, a > 0$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C, a > 0$$

Answer: $4e^6 - 4$

146. Use a change of variables or the table to evaluate the following definite integral.

$$\int_0^1 \frac{2x}{(x^2 + 2)^2} dx$$

⁶ Click to view the table of general integration formulas.

$$\int_0^1 \frac{2x}{(x^2 + 2)^2} dx = \boxed{} \text{ (Type an exact answer.)}$$

6: General Integration Formulas

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

$$\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C$$

$$\int \csc ax \cot ax \, dx = -\frac{1}{a} \csc ax + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int b^x dx = \frac{1}{\ln b} b^x + C, b > 0, b \neq 1$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C, a > 0$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C, a > 0$$

Answer: $\frac{1}{6}$

147. Evaluate the following integral.

$$\int \frac{dx}{x^2 - 2x + 10}$$

$$\int \frac{dx}{x^2 - 2x + 10} = \boxed{}$$

(Use C as the arbitrary constant as needed.)

Answer: $\frac{1}{3} \tan^{-1} \frac{x-1}{3} + C$

148. Evaluate the following integral using integration by parts.

$$\int 17t e^t dt$$

Use the integration by parts formula so that the new integral is simpler than the original one. Choose the correct answer below.

- ☐ A. $17 e^t - \int (17t e^t) dt$
- ☐ B. $17 e^t + \int (17 e^t) dt$
- ☐ C. $17t e^t + \int (17t e^t) dt$
- ☐ D. $17t e^t - \int (17 e^t) dt$

Evaluate the integral.

$$\int 17t e^t dt = \boxed{}$$

(Use C as the arbitrary constant as needed.)

Answers

D. $17t e^t - \int (17 e^t) dt$

$17t e^t - 17 e^t + C$

149. Evaluate the following integral using integration by parts.

$$\int 8x \ln 6x dx$$

$$\int 8x \ln 6x dx = \boxed{}$$

(Use C as the arbitrary constant as needed.)

Answer: $4x^2 \ln 6x - 2x^2 + C$

150. If the general solution of a differential equation is $y(t) = C e^{-3t} + 13$, what is the solution that satisfies the initial condition $y(0) = 10$?

$$y(t) = \boxed{}$$

Answer: $-3 e^{-3t} + 13$