1

M1314
062016

MATH 1314 Free Response

1. Solve $x - 5 = \sqrt{10 - 2x}$

2. Graph and find the min and max of $f(x) = 2x^3 - 6x^2 + 2$

3. Graph $f(x) = \begin{cases} -x - 1 & \text{if } x < 1 \\ 2x + 4 & \text{if } x \geq 1 \end{cases}$

4. For $f(x) = x^2 - 3x - 7$ find $\dfrac{f(x+h) - f(x)}{h}$

5. Graph $f(x) = -1(x - 4)^2 + 9$

6. Find the domain of $f(x) = \sqrt{2x + 8}$

7. For $f(x) = 3x^2 + 4x + 5$ and $g(x) = 2x - 8$ find $(f \cdot g)(x)$

8. For $f(x) = 3x^2 + 4x + 5$ and $g(x) = 2x + 3$ find $(f \circ g)(x)$

9. Find the distance between $(-7, -8)$ and $(-3, -5)$

10. Find the midpoint of $(-7, -8)$ and $(-3, -5)$

11. Graph $x^2 + y^2 - 8x + 16y + 71 = 0$

12. Solve $2x^3 - 11x^2 + 10x + 8 = 0$

13. Find the vertical asymptote of $R(x) = \dfrac{x^2 + 6x + 8}{x^2 - 8x + 12}$

14. Find the horizontal asymptote of $R(x) = \dfrac{4x^2 + 3x + 1}{12x^2 - x - 11}$

15. Find the slant asymptote of $R(x) = \dfrac{4x^2 + 8x + 11}{x - 1}$

16. Expand $\log_2 \left(\dfrac{16\sqrt[5]{x + 11}}{(x + 3)^{10}} \right)$

17. Solve $3^{8-2x} = \frac{1}{81}$

18. Graph $f(x) = -x^2 - 6x - 8$

19. Solve $\log_4(x) + \log_4(x-6) = 2$

20. Solve $\log_3(x+2) + \log_3(x-4) = \log_3(16)$

21. Solve $6000 = 3000\left(1 + \frac{0.10}{4}\right)^{4x}$

22. Solve $6000 = 3000e^{0.10x}$

23. Solve the system
$$\begin{aligned} x + y + z &= 6 \\ x + y - z &= 4 \\ 3x + y + z &= 12 \end{aligned}$$

24. $\sum_{x=2}^{4}(2x^2 + 6)$

25. Expand $(2x-5)^3$

26. Solve $2^{x+4} = 1024$

27. Solve $\log(4+x) - \log(x-5) = \log(4)$

28. Solve $-10x^2 + 70x + 180 = 0$

29. Solve $2x^2 - 11x - 6 = 0$

① $x-5=\sqrt{10-2x}$ Solve

$(x-5)^2=(\sqrt{10-2x})^2$

$(x-5)(x-5)=10-2x$

$x^2-5x-5x+25=10-2x$

$x^2-10x+25=10-2x$

$x^2-10x+25-10+2x=0$

$x^2-8x+15=0$

$(x-3)(x-5)=0$

15.1
3.5

3

Let $x-3=0$ OR $x-5=0$

$x-3+3=0+3$ OR $x-5+5=0+5$

$x=3$ BAD OR $x=5$ Good

Ck $x-5=\sqrt{10-2x}$

$(3)-5=\sqrt{10-2(3)}$

$3-5=\sqrt{10-6}$

$-2=\sqrt{4}$

$-2\neq 2$ x

Ck $x-5=\sqrt{10-2x}$

$(5)-5=\sqrt{10-2(5)}$

$5-5=\sqrt{10-10}$

$0=\sqrt{0}$

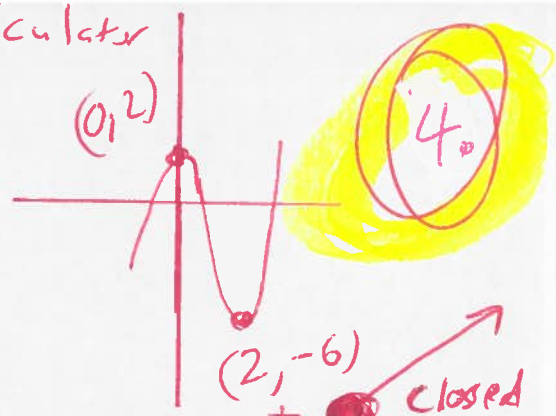
$0=0$ ✓

{5}

② $f(x) = 2x^3 - 6x^2 + 2$ use Graphing Calculator graph

$y_1 = 2x\wedge 3 - 6x\wedge 2 + 2$

Max = (0, 2) Min = (2, −6)

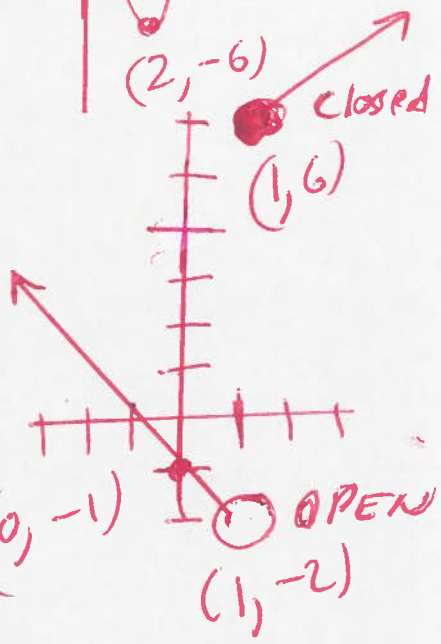


③ $f(x) = \begin{cases} -x-1 & \text{if } x < 1 \quad \text{OPEN} \\ 2x+4 & \text{if } x \geq 1 \quad \text{closed} \end{cases}$

$y_1 = -x - 1 \div (x < 1)$

$y_2 = 2x + 4 \div (x \geq 1)$

use graphing calculator



④ $f(x) = x^2 - 3x - 7$ simplify

$$\frac{f(x+h) - f(x)}{h} =$$

$$\frac{((x+h)^2 - 3(x+h) - 7) - (x^2 - 3x - 7)}{h} =$$

$$\frac{(x+h)(x+h) - 3x - 3h - 7 - x^2 + 3x + 7}{h} =$$

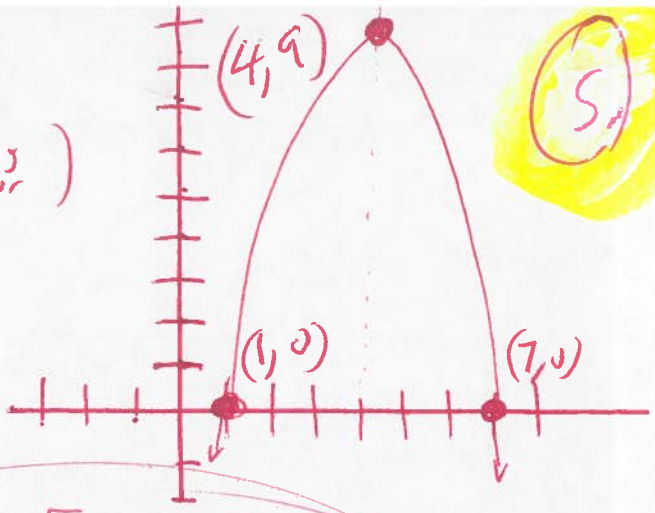
$$\frac{x^2 + xh + xh + h^2 - 3x - 3h - h - x^2 + 3x + 7}{h} =$$

$$\frac{2xh + h^2 - 3h}{h} =$$

$$2x + h - 3 =$$

(5) $f(x) = -1(x-4)^2 + 9$ Graph

$y = -1(x-4)^2 + 9$ (use graphing calculator)



5

(6) $f(x) = \sqrt{2x+8}$ find domain

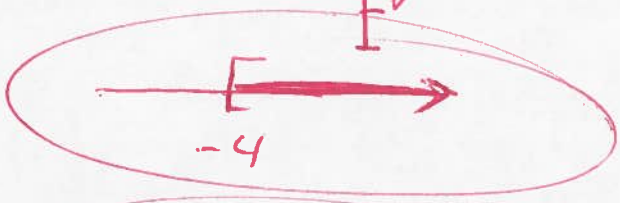
$2x+8 \geq 0$

$2x+8-8 \geq 0-8$

$2x \geq -8$

$\frac{2x}{2} \geq \frac{-8}{2}$

$x \geq -4$



$[-4, \infty)$

(7) $f(x) = 3x^2 + 4x + 5$ and $g(x) = 2x - 8$

Find $(f \circ g)(x) =$

$f(x) \cdot g(x) =$

$(3x^2 + 4x + 5)(2x - 8)$

$6x^3 - 24x^2 + 8x^2 - 32x + 10x - 40 =$

$6x^3 - 16x^2 - 22x - 40 =$

(8) $f(x) = 3x^2 + 4x + 5$ and $g(x) = 2x + 3$

Find $(f \circ g)(x) =$

$f(g(x)) =$

$3(2x+3)^2 + 4(2x+3) + 5 =$

$3(2x+3)(2x+3) + 8x + 12 + 5 =$

$3(4x^2 + 6x + 6x + 9) + 8x + 12 + 5 =$

$3(4x^2 + 12x + 9) + 8x + 12 + 5 =$

$12x^2 + 36x + 27 + 8x + 12 + 5 =$

$12x^2 + 44x + 44 =$

⑨ Find distance $(-7, -8)$ and $(-3, -5)$
$x_1 \; y_1$ $x_2 \; y_2$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{((-7) - (-3))^2 + ((-8) - (-5))^2}$$

$$d = \sqrt{(-7+3)^2 + (-8+5)^2}$$

$$d = \sqrt{(-4)^2 + (-3)^2}$$

$$d = \sqrt{16 + 9}$$

$$d = \sqrt{25}$$

$$d = 5$$

⑩ Find midpoint $(-7, -8)$ and $(-3, -5)$
$x_1 \; y_1$ $x_2 \; y_2$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Midpoint} = \left(\frac{(-7) + (-3)}{2}, \frac{(-8) + (-5)}{2} \right)$$

$$\text{Mid point} = \left(\frac{-7-3}{2}, \frac{-8-5}{2} \right)$$

$$\text{Mid point} = \left(\frac{-10}{2}, \frac{-13}{2} \right)$$

$$\text{Midpoint} = \left(-5, -\frac{13}{2} \right)$$

11. $x^2+y^2-8x+16y+71=0$ Graph

$x^2-8x+y^2+16y=-71$

$x^2-8x+\left(\frac{1}{2}(-8)\right)^2+y^2+16y+\left(\frac{1}{2}(16)\right)^2=-71+\left(\frac{1}{2}(-8)\right)^2+\left(\frac{1}{2}(16)\right)^2$

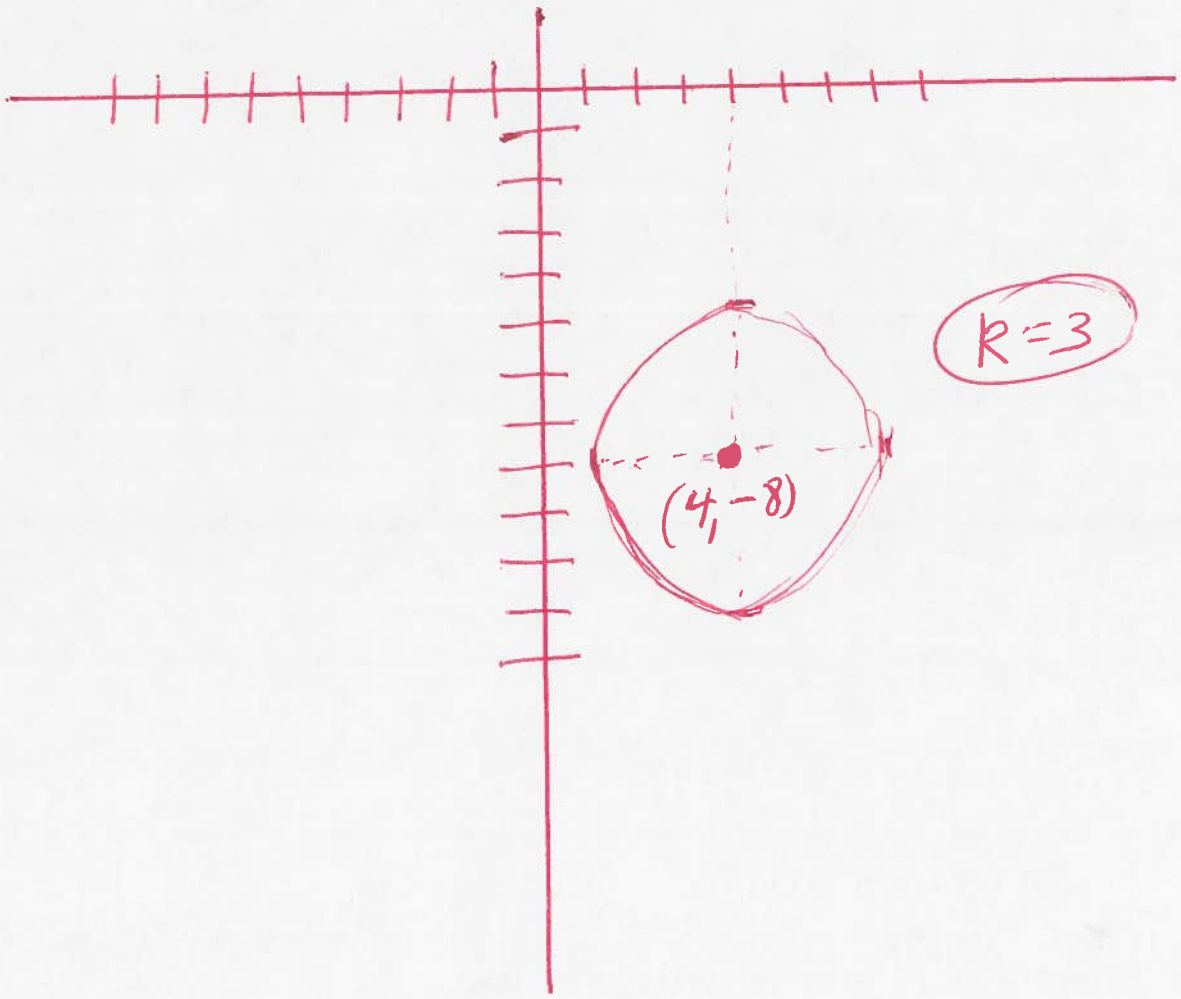
$x^2-8x+(-4)^2+y^2+16y+(8)^2=-71+(-4)^2+(8)^2$

$x^2-8x+16+y^2+16y+64=-71+16+64$

$(x-4)(x-4)+(y+8)(y+8)=9$

$(x-4)^2+(y+8)^2=9$

CENTER $=(4,-8)$ Radius $=\sqrt{9}=3$



(12) $2x^3 - 11x^2 + 10x + 8 = 0$ Solve

Last Number / First Number = 8

$\frac{\pm 8}{2} = \frac{\pm 8, \pm 4, \pm 2, \pm 1}{2, 1}$

$\frac{\pm 8}{2}, \frac{\pm 4}{2}, \frac{\pm 2}{2}, \frac{\pm 1}{2}, \frac{\pm 8}{1}, \frac{\pm 4}{1}, \frac{\pm 2}{1}, \frac{\pm 1}{1} =$

$\pm 4, \pm 2, \pm 1, \pm \frac{1}{2}, \pm 8, \pm 4, \pm 2, \pm 1 =$

$\pm 8, \pm 4, \pm 2, \pm 1, \pm \frac{1}{2} =$

Use Synthetic division

$2x^3 - 11x^2 + 10x + 8 = 0$

2)

2	−11	10	8
	4	−14	−8
2	−7	−4	0

Rem

4)

2	−7	−4	
	8	4	
2	1	0	

Rem

Let $2x + 1 = 0$

$2x + 1 - 1 = 0 - 1$

$2x = -1$

$\frac{2x}{2} = -\frac{1}{2}$

$x = -1/2$

$\left\{2, 4, -\frac{1}{2}\right\}$

(13) Find Vertical Asymptote

$$R(x) = \frac{x^2+6x+8}{x^2-8x+12}$$

Set $x^2-8x+12=0$

$(x-2)(x-6)=0$

$x-2=0$ OR $x-6=0$

$x-2+2=0+2$ OR $x-6+6=0+6$

$x=2$ OR $x=6$

(14.) Find horizontal Asymptote

$$R(x) = \frac{4x^2+3x+1}{12x^2-x-11}$$

y=horiz Asym $= \frac{4x^2}{12x^2}$

$y = \frac{4}{12}$

$y = \frac{4(1)}{4(3)}$

$y = \frac{1}{3}$

(15.) Find Slant Asymptote

$$R(x) = \frac{4x^2+8x+11}{x-1}$$

Use Synthetic division

1 | 4 8 11

4 12

4 12 (32)

$Y = 4x+12$ SLANT Asymptote

(16) Expand $\log_2\left(\frac{16\sqrt[5]{x+11}}{(x+3)^{10}}\right) =$

10

$\log_2(16\sqrt[5]{x+11}) - \log_2(x+3)^{10} =$

$\log_2(16) + \log_2\sqrt[5]{x+11} - \log_2(x+3)^{10} =$

$\log_2(16) + \log_2(x+11)^{1/5} - \log_2(x+3)^{10} =$

$\log_2(2^4) + \log_2(x+11)^{1/5} - \log_2(x+3)^{10} =$

$4\log_2(2) + \frac{1}{5}\log_2(x+11) - 10\log_2(x+3) =$

$4(1) + \frac{1}{5}\log_2(x+11) - 10\log_2(x+3) =$

$4 + \frac{1}{5}\log_2(x+11) - 10\log_2(x+3) =$

(17) $3^{8-2x} = \frac{1}{81}$ Solve

$3^{8-2x} = \frac{1}{3^4}$

$3^{8-2x} = 3^{-4}$

$8 - 2x = -4$

$-8 - 2x - 8 = -4 - 8$

$-2x = -12$

$\frac{-2x}{-2} = \frac{-12}{-2}$

$x = 6$

(18) Graph $f(x) = -x^2 - 6x - 8$

(11)

Set $-x^2 - 6x - 8 = 0$

$-1(x^2 + 6x + 8) = 0$

$-1(x + 2)(x + 4) = 0$

Set $-1 \neq 0$ or $x + 2 = 0$ or $x + 4 = 0$

$x + 2 - 2 = 0 - 2$ or $x + 4 - 4 = 0 - 4$

$x = -2$ or $x = -4$

$f(x) = -x^2 - 6x - 8$

$f(-4) = -(-4)^2 - 6(-4) - 8$

$f(-4) = -(-4)(-4) - 6(-4) - 8$

$f(-4) = -16 + 24 - 8$

$f(-4) = 0$

$f(-3) = -(-3)^2 - 6(-3) - 8$

$f(-3) = -(-3)(-3) - 6(-3) - 8$

$f(-3) = -9 + 18 - 8$

$f(-3) = 1$

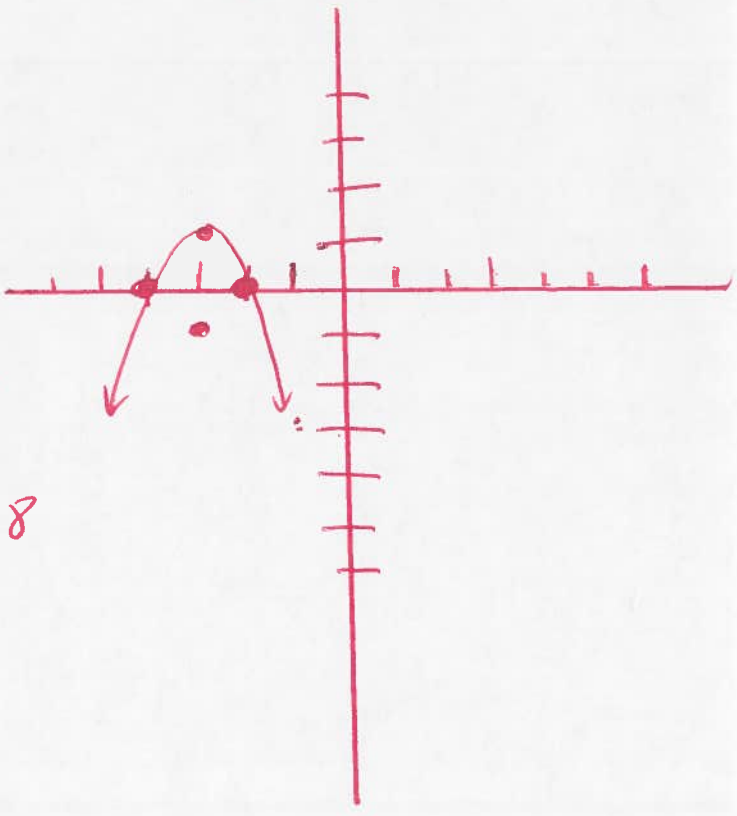
$f(-2) = -(-2)^2 - 6(-2) - 8$

$f(-2) = -(-2)(-2) - 6(-2) - 8$

$f(-2) = -4 + 12 - 8$

$f(-2) = 0$

x	y
-4	0
-3	1
-2	0



19. $\log_4(x) + \log_4(x-6) = 2$ Solve

$\log_4(x)(x-6) = 2$

$4^2 = x(x-6)$

$16 = x^2 - 6x$

$0 = x^2 - 6x - 16$

$0 = (x-2)(x-8)$

Set $x-2=0$ OR $x-8=0$

$x-2+2=0+2$ OR $x-8+8=0+8$

$x=2$ OR $x=8$

BAD ✓ Good

Ck

$\log_4(x) + \log_4(x-6) = 2$

$\log_4(2) + \log_4(2-6) = 2$

$\log_4(2) + \log_4(-4) = 2$

undefined

Ck

$\log_4(x) + \log_4(x-6) = 2$

$\log_4(8) + \log_4(8-6) = 2$

$\log_4(8) + \log_4(2) = 2$

Good Good

$\{8\}$

(20) $\log_3(x+2) + \log_3(x-4) = \log_3(16)$

Solve

$\log_3(x+2)(x-4) = \log_3(16)$

$(x+2)(x-4) = 16$

$x^2 - 4x + 2x - 8 = 16$

$x^2 - 2x - 8 = 16$

$x^2 - 2x - 8 - 16 = 0$

$x^2 - 2x - 24 = 0$

$(x+4)(x-6) = 0$

Set $x+4=0$ OR $x-6=0$

$x+4-4 = 0-4$ OR $x-6+6 = 0+6$

$x = -4$ BAD OR $x = 6$ ✓ Good

Ck $\log_3(x+2) + \log_3(x-4) = \log_3(16)$

$\log_3(-4+2) + \log_3(-4-4) = \log_3(16)$

$\log_3(-2)$ BAD $+ \log_3(-8)$ BAD $= \log_3(16)$

Ck $\log_3(x+2) + \log_3(x-4) = \log_3(16)$

$\log_3(6+2) + \log_3(6-4) = \log_3(16)$

$\log_3(8) + \log_3(2) = \log_3(16)$

Good Good Good

$\{6\}$

(21) $6000 = 3000\left(1 + \frac{.10}{4}\right)^{4x}$ Solve

$6000 = 3000(1 + .025)^{4x}$

$6000 = 3000(1.025)^{4x}$

$\frac{6000}{3000} = \frac{3000(1.025)^{4x}}{3000}$

$2 = (1.025)^{4x}$

$\ln(2) = \ln(1.025)^{4x}$

$\ln(2) = 4x \ln(1.025)$

$\frac{\ln(2)}{\ln(1.025)} = \frac{4x \ln(1.025)}{\ln(1.025)}$

$\frac{\ln(2)}{\ln(1.025)} = 4x$

$\frac{1}{4}\frac{\ln(2)}{\ln(1.025)} = \frac{1}{4}(4x)$

$\frac{\ln(2)}{4\ln(1.025)} = x$

$\ln(2)/(4\ln(1.025)) = x$

$7.017758631 = x$

14.

(22.) $6000 = 3000e^{.10x}$

$\frac{6000}{3000} = \frac{3000e^{.10x}}{3000}$

$2 = e^{.10x}$

$\ln(2) = \ln(e^{.10x})$

$\ln(2) = .10x \ln(e)$

$\ln(2) = .10x(1)$

$\ln(2) = .10x$

$\frac{\ln(2)}{.10} = \frac{.10x}{.10}$

$6.9314471806 = x$

Solve

15

(23.) $x+y+z=6$

$x+y-z=4$

$3x+y+z=12$

use graphing calculator

2ND Matrix Edit

[A]

3X4

$$[A] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 1 & -1 & 4 \\ 3 & 1 & 1 & 12 \end{bmatrix}$$

2ND, Matrix, Math, rref

rref([A]) =

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$(x, y, z) = (3, 2, 1)$

16.

(24) Evaluate

$$\sum_{x=2}^{4} (2x^2+6)$$

$(2(2)^2+6) + (2(3)^2+6) + (2(4)^2+6) =$

$(2(4)+6) + (2(9)+6) + (2(16)+6) =$

$(8+6) + (18+6) + (32+6) =$

$14 + 24 + 38 =$

$76 =$

(25) expand $(2x-5)^3$ ← Use the Binomial Theorem

$$(A+B)^N = {}_{N}C_{0}(A)^N(B)^0 + {}_{N}C_{1}(A)^{N-1}(B)^1 + \ldots + {}_{N}C_{N}(A)^0(B)^N$$

$$(2x-5)^3 = {}_{3}C_{0}(2x)^3(-5)^0 + {}_{3}C_{1}(2x)^2(-5)^1 + {}_{3}C_{2}(2x)^1(-5)^2 + {}_{3}C_{3}(2x)^0(-5)^3$$

$$= (1)(2^3x^3)(1) + (3)(2^2x^2)(-5)^1 + (3)(2x)^1(-5)^2 + (1)(1)(-5)^3$$

$$= (1)(8x^3)(1) + (3)(4x^2)(-5) + (3)(2x)(25) + (1)(1)(-125)$$

$$= 8x^3 - 60x^2 + 150x - 125$$

(26) $2^{x+4} = 1024$

17.

$\ln(2^{x+4}) = \ln(1024)$

$(x+4)\ln(2) = \ln(1024)$

$\frac{(x+4)\ln(2)}{\ln(2)} = \frac{\ln(1024)}{\ln(2)}$

$x+4 = \frac{\ln(1024)}{\ln(2)}$

$x+4-4 = \frac{\ln(1024)}{\ln(2)} - 4$

$x = \frac{\ln(1024)}{\ln(2)} - 4$

$x = 6$

(27) $\log(4+x) - \log(x-5) = \log(4)$

$\log\left(\frac{4+x}{x-5}\right) = \log(4)$

18.

$$\frac{4+x}{x-5} = 4$$

$$\frac{4+x}{x-5} = \frac{4}{1}$$

$1(4+x) = 4(x-5)$ (cross Multiply)

$4 + x = 4x - 20$

$4 + x - 4 = 4x - 20 - 4$

$x = 4x - 24$

$x - 4x = 4x - 24 - 4x$

$1x - 4x = -24$

$-3x = -24$

$$\frac{-3x}{-3} = \frac{-24}{-3}$$

$$x = 8$$

Check $\log(4+8) - \log(8-5) = \log(4)$

$\log(12) - \log(3) = \log(4)$

$\log\left(\frac{12}{3}\right) = \log(4)$ Good

Solve by factoring

(28) $-10x^2+70x+180=0$

19.

divide

possibly

18 : 1
9 : 2
6 : 3

$\frac{-10x^2}{-10}+\frac{70x}{-10}+\frac{180}{-10}=\frac{0}{-10}$

$x^2-7x-18=0$

$(x+2)(x-9)=0$

SA $x+2=0$ OR $x-9=0$

$x+2-2=0-2$ OR $x-9+9=0+9$

$x=-2$ OR $x=9$

29

Solve by factoring

$2x^2 - 11x - 6 = 0$

$(2x+1)(x-6) = 0$

$2x+1 = 0$ OR $x-6 = 0$

$2x+1-1 = 0-1$ OR $x-6+6 = 0+6$

$2x = -1$

$\frac{2x}{2} = \frac{-1}{2}$

$x = -\frac{1}{2}$

$x = 6$

2.1 6.1 2.3 possibilities

20

Solve use Quad form

$2x^2 - 11x - 6 = 0$

$a = 2, \quad b = -11, \quad c = -6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$

$x = \frac{11 \pm \sqrt{121 + 48}}{4}$

$x = \frac{11 \pm \sqrt{169}}{4}$

$x = \frac{11 \pm 13}{4}$

$x = \frac{11-13}{4}$ OR $x = \frac{11+13}{4}$

$x = \frac{-2}{4}$ OR $x = \frac{24}{4}$

$x = \frac{\frac{1}{2}(-1)}{\frac{1}{2}(2)}$ OR $x = 6$

$x = -\frac{1}{2}$